

## Fracture of Rubber

A rubber band can be stretched several times its original length. This large deformation may hide its brittleness: the strain to rupture can be markedly reduced by the presence of a crack. This lecture describes fracture mechanics of highly deformable materials, such as rubbers and gels.

Demonstrate in class the effect of a crack on a rubber band. Use a wide rubber band. Show the class that the rubber band can be stretched several times its original length. Then use scissors to cut a crack into the rubber band. Pull the rubber band to rupture. Note that the strain to rupture is markedly reduced by the crack. Pass the scissors and some rubber bands around. Invite every student to try.

**Energy release rate for a crack in a highly deformable elastic body.** For a crack in an elastic body, the energy release rate is defined by using the elastic energy stored in the entire body (<http://imechanica.org/node/7507>). This definition invokes no field theory. Indeed, the energy release rate can be determined experimentally without measuring any field.

This definition is also economic in developing the theory of fracture: the same definition works for any elastic body, so long as we can obtain elastic energy of the body. Following Rivlin and Thomas (1953), we now extend the Griffith approach to bodies capable of large deformation prior to fracture.

Consider a crack in an elastic body subject to a load. The system is characterized by two independent variables:

- $\Delta$ , the displacement of the load, and
- $A$ , the area of the crack.

For a highly deformable material, the area of the crack can change substantially even when the crack does not extend. To specify the change due to the extension of the crack, we take  $A$  to be the area of the crack when the body is undeformed.

For a given cracked body, the load  $P$  can be measured experimentally as a function of the displacement  $\Delta$ . The function can be plotted as the load-displacement curve. When the crack does not extend, the area under the load-displacement curve is the elastic energy  $U$  stored in the body.

The elastic energy stored in the body is a function of the two independent variables, namely,

$$U = U(\Delta, A).$$

The two independent variables may be visualized by imagining many bodies, which are identical in the undeformed states except for the areas of the cracks. We then load these bodies in the same way. For each body, the crack does *not* extend when the load is applied.

When the displacement of the load varies by  $d\Delta$  and the area of the crack varies by  $dA$ , the elastic energy stored in the body varies by

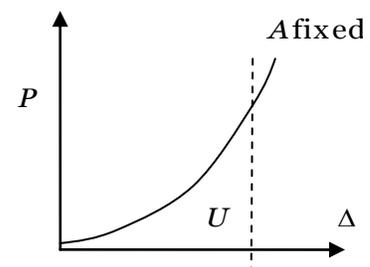
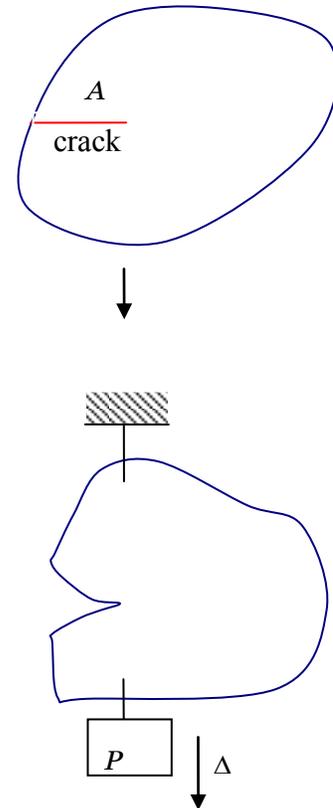
$$dU = \frac{\partial U(\Delta, A)}{\partial \Delta} d\Delta + \frac{\partial U(\Delta, A)}{\partial A} dA.$$

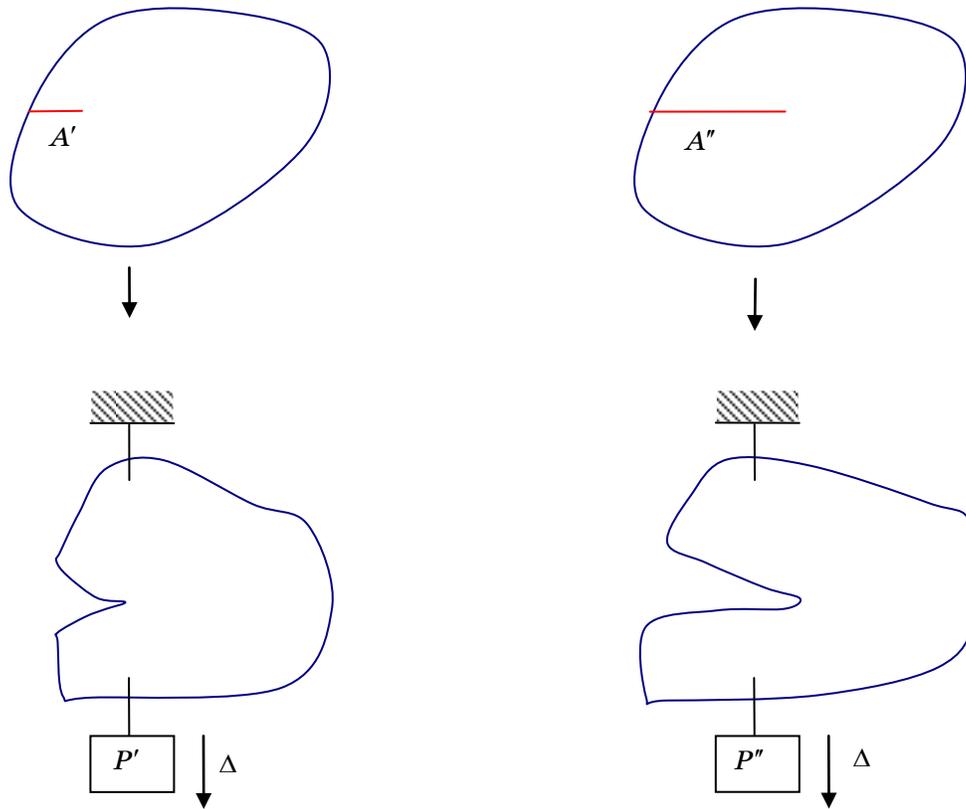
We call one partial derivative the load,

$$P = \frac{\partial U(\Delta, A)}{\partial \Delta},$$

and the other partial derivative the energy release rate,

$$G = -\frac{\partial U(\Delta, A)}{\partial A}.$$

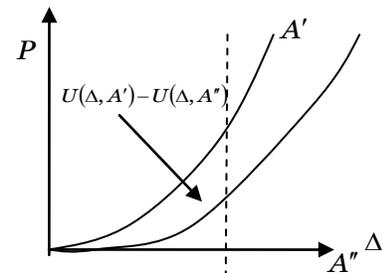




Using these definitions, we write the variation in the elastic energy as

$$dU = Pd\Delta - GdA.$$

The energy release rate can be interpreted in the load-displacement diagram.



**A change of variables.** It is sometimes convenient to use the load  $P$  as an independent variable, rather than the displacement  $\Delta$ . To visualize  $P$  as an independent variable, we represent the load by a weight  $P$ , and regard the elastic body and the weight together as a system. The energy of this system,  $\Pi$ , is the sum of the elastic energy stored in the body and the potential energy stored in the weight, namely,

$$\Pi = U - P\Delta.$$

The quantity  $\Pi$  is known as the Gibbs free energy in thermodynamics, and the potential energy in solid mechanics. When the crack does not extend, the potential energy can be interpreted in the load-displacement diagram.

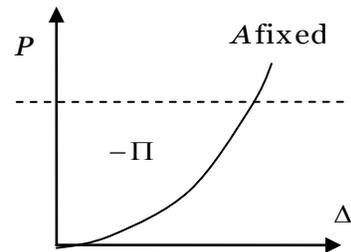
The above definition of  $\Pi$ , together with  $dU = Pd\Delta - GdA$ , leads to

$$d\Pi = -\Delta dP - GdA.$$

On the basis of this expression, we regard  $\Pi$  as a function of the load the area of the crack, namely,

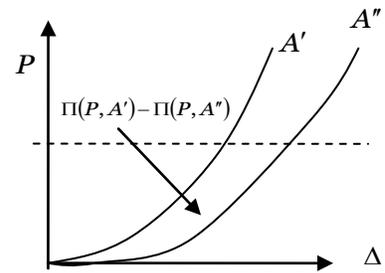
$$\Pi = \Pi(P, A).$$

The partial derivatives of this function are interpreted as



$$\Delta = -\frac{\partial \Pi(P, A)}{\partial P},$$

$$G = -\frac{\partial \Pi(P, A)}{\partial A}.$$



The graphical interpretation makes it evident that the two functions,  $U = U(\Delta, A)$  and  $\Pi(P, A)$ , contain the same information. We should not fool ourselves into believing that we have learned anything really new by a change of variables.

**Fracture energy is specific to a material, independent of types of test-pieces.**

The fracture energy of a rubber  $\Gamma$  is defined as the critical energy release needed to cause a pre-cut crack to extend. (Rivlin and Thomas called the fracture energy the characteristic energy of tearing, and designated the quantity by the letter  $T$ .) The fracture energy can be determined experimentally. Go through the procedure in class.

To ascertain if the fracture energy is independent of test pieces, Rivlin and Thomas tested a specific rubber by using test-pieces of three types. For each type of the test-pieces,

- Introduce cuts of various lengths.
- Record the load-displacement curve.
- Record the point at which tearing starts.

From these experimental data, Rivlin and Thomas estimated the fracture energy. This procedure showed that the point of tearing of all test pieces could be correlated by a single value of the fracture energy:

$$\Gamma = 3.7 \times 10^3 \text{ J/m}^2.$$

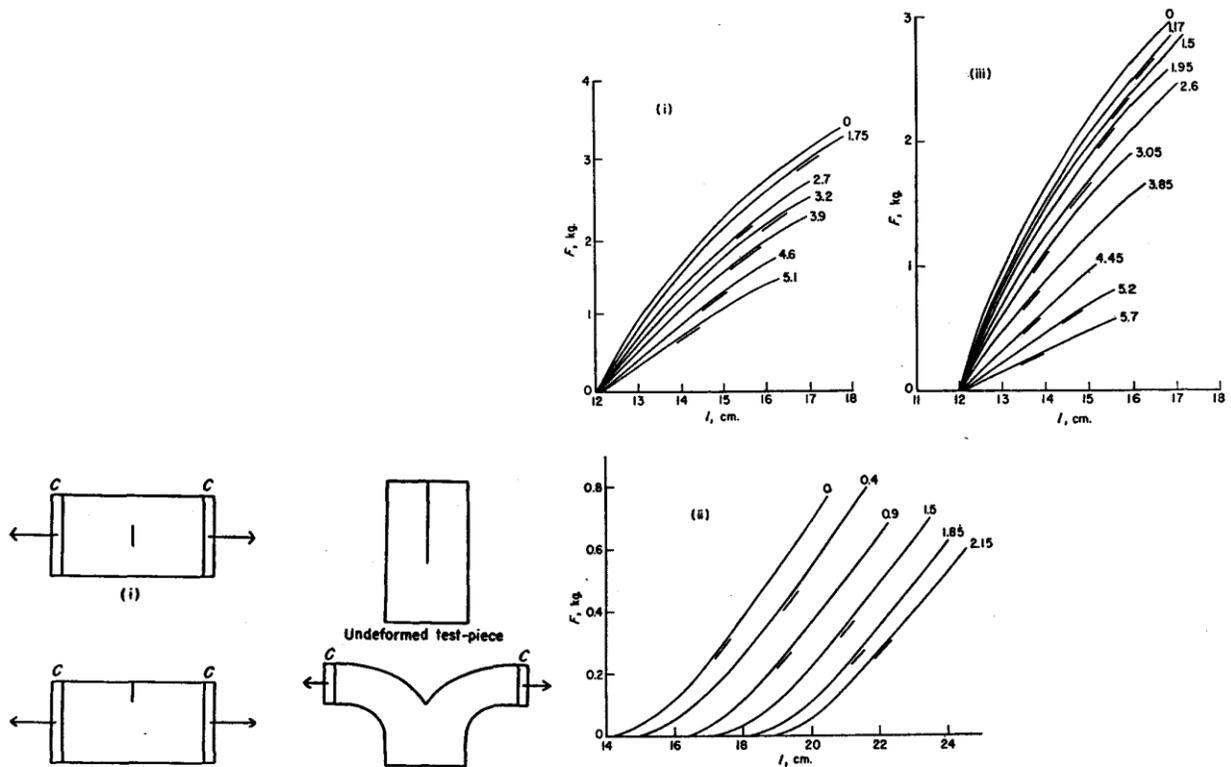


Fig. 1. Schematic diagrams of test-pieces.

Fig. 4. Measured load-deformation relations for various cut-lengths  $c$ . Bars denote points at which incipient tearing occurred. Numbers against curves give values of  $c$  in cm.

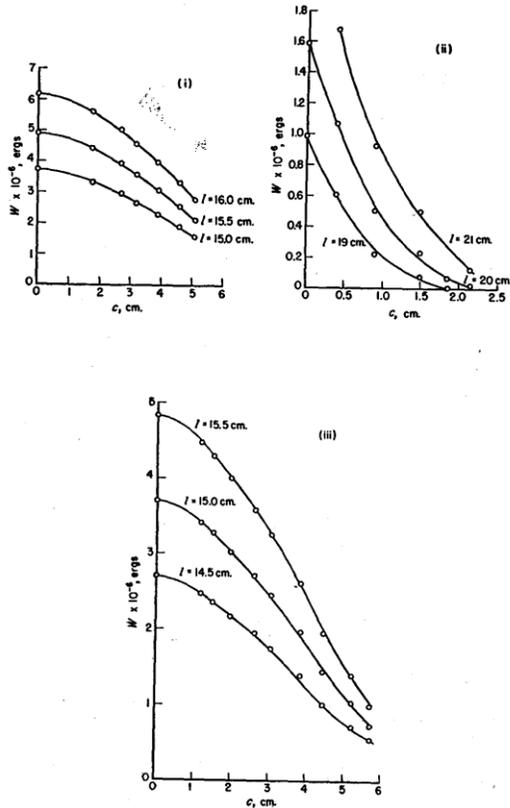


Fig. 5. Relations between  $W$  and  $c$  for given values of the overall deformation. Calculated from the curves of Figure 4.

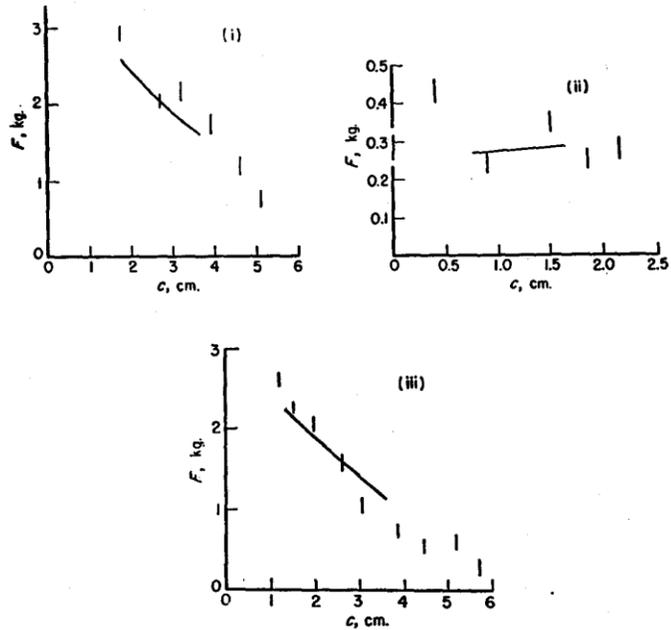


Fig. 6. Values of  $F$  at which tearing occurred for various values of  $c$ . The bars denote experimental values. The curves represent the calculated relations on the assumption that  $T = 3.7 \times 10^8$  ergs/cm<sup>2</sup>.

**Analytical expressions for the energy release rate for some test pieces.** The above procedure to determine the fracture energy is tedious, using many test-pieces. In using the definition to determine the energy release rate from the experimental data, we need to measure load-displacement curves for test-pieces of the same type but with various lengths of the pre-cuts.

Rivlin and Thomas suggested an alternative approach. For some types of test-pieces, the energy release rate can be readily calculated analytically in terms of the stress-stretch curve of homogenous deformation.

- The trousers.  $G = 2Wa - 2\lambda F/t$
- The pure-shear test.  $G = W_0 l_0$

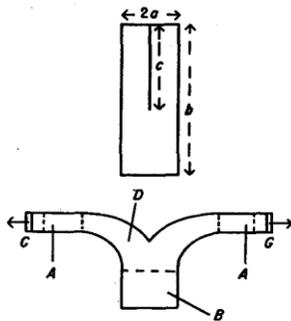


Fig. 7. Schematic diagram of "simple extension" tear test-piece.

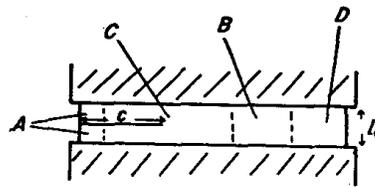


Fig. 8. Schematic diagram of "pure shear" tear test-piece.

That was 1953. Today, of course, we may use the finite element method to relate the energy release rate of any test-piece to the load.

**Why is the fracture energy of rubber so high.** The experimentally measured fracture energy of rubber may be on the order of  $1 \sigma^2 J/m^2$ . This value is much higher than the fracture energy of silica, which is on the order of  $1 J/m^2$ .

- Inelastic deformation
- The process of bond breaking (Lake and Thomas, 1967)

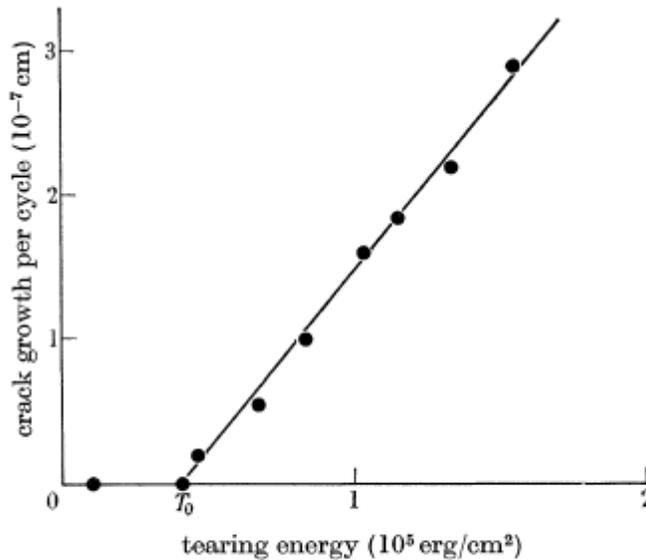


FIGURE 2. A typical crack growth characteristic for a rubber in the vicinity of the crack growth limit  $T_0$ .

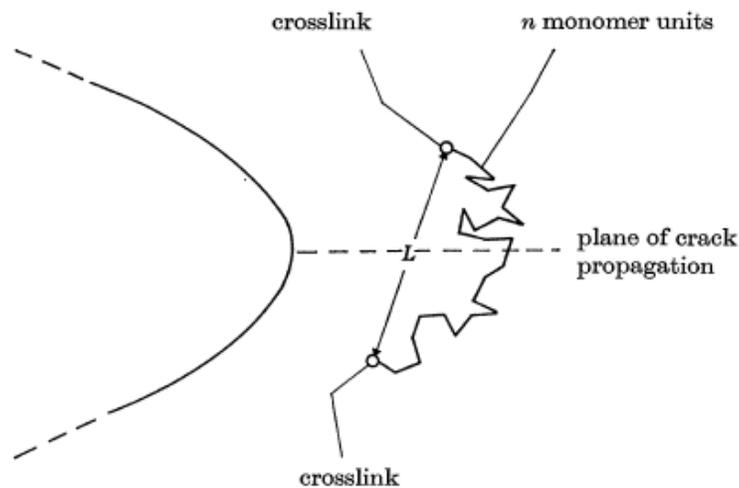


FIGURE 3. Schematic diagram showing a polymer chain lying across the plane of crack propagation.

The effect of viscoelasticity may be reduced by reducing the loading rate or increasing the temperature. Once the effect of viscoelasticity is eliminated, the measured fracture energy,

known as the intrinsic fracture energy, is about  $50 \text{ J/m}^2$ . This value is still much larger than the energy needed to break a single layer of atomic bonds, which is about  $1 \text{ J/m}^2$ . This difference may be understood within the Lake-Thomas picture. The rubber is a network of crosslinked long molecules. At the front of a crack, the chains are stretched. Once a chain breaks, the energy of the entire chain is dissipated.

### References

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