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Effect of shear stress on adhesive contact with a generalized Maugis-Dugdale cohesive zone model

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Abstract

The interplay between interfacial shear stress and adhesion has been an active but controversial subject of adhesive contact mechanics, which is currently plagued by diverse, sometimes contradicting, predictions. Recently, McMeeking et al. showed that a reversible interface slip parameter plays an essential role in determining how interfacial shear stress affects adhesion for a Johnson-Kendall-Roberts (JKR) contact interface. In this paper, adhesive contact between a rigid spherical indenter and an elastic half-space is studied with a generalized Maugis-Dugdale (M-D) model, where a constant frictional shear stress presents in the intimate contact area while a constant adhesive stress exists in a cohesive zone near the contact edge. The model solution predicts that the contact behavior is governed by a non-dimensional reversible shear index $\alpha \tau^*$ as well as the Maugis parameter $\lambda$. More specifically, it is found that the impact of interfacial shear stress on adhesion is most significant when the model approaches the JKR limit, and it gets less pronounced in the transitional regime and eventually becomes negligible in the Derjaguin-Muller-Toporov (DMT) limit. Such behavior is in distinct contrast to Johnson’s phenomenological solution. Finally, the proposed model is experimentally validated by adhesion tests on contact interfaces with varying Maugis parameters, where the reversible slip factor is experimentally extracted for the first time.

Keywords: Adhesion, adhesive contact, frictional slip, shear stress, pull-off force
1. Introduction

Miniaturization of mechanical systems leads to a notable increase in the surface-to-volume ratio. As a consequence, the surface energy relative to the elastic strain energy of these systems (especially for systems with soft/compliant materials) has become more and more prominent (Mate, 2008). The urgent need to understand the influence of adhesion on mechanical systems involving contacting bodies has resulted in vast progress in contact mechanics with adhesion (Chan et al., 2011; Creton and Ciccotti, 2016; Kendall, 2007; Maboudian and Howe, 1997). The revealed mechanisms also promote novel design and applications of many bio-inspired materials and structures with special properties of adhesion (Chen et al., 2020; Gao et al., 2005; Kizilkan and Gorb, 2018; Mazzotta et al., 2020; Silverman and Roberto, 2007; Yao and Gao, 2006). Theoretical advancement in adhesive contact mechanics includes idealized models considering single-asperity contact, e.g. Johnson-Kendall-Roberts (JKR) model (Johnson et al., 1971), Derjaguin-Muller-Toporov (DMT) model (Derjaguin et al., 1975) and Maugis-Dugdale (M-D) model (Maugis, 1992), as well as more realistic models involving interface roughness (Adams, 2005; Ciavarella et al., 2019; Fuller and Tabor, 1975; Maugis, 1996; Persson, 2002). In these classic models, the effect of interfacial shear stress is typically ignored. However, for many mating surfaces, lateral motion and lateral loading are often inevitable, which naturally induce frictional shear stress along the interface. Therefore, how interfacial shear stress influences adhesion
between elastic bodies in contact becomes a significant question to be answered.

When shear stress presents along the contact interface, the equilibrium state of adhesive contact may be altered. A pioneering work looking into this effect was given by Savkoor and Briggs (Savkoor and Briggs, 1977), who investigated the adhesive contact based on the JKR model by assuming a non-slip condition and infinite shear stress near the edge of contact. Their analysis (Savkoor and Briggs, 1977) considered the contribution of the potential energy of the lateral load and the strain energy induced by the interfacial shear stress to the overall energy release rate. It was found that the contact radius under a same normal load and the pull-off force would become smaller when interfacial shear stress was present. We referred to this phenomenon as the adhesion weakening effect of interfacial shear stress. More in-depth analyses (Savkoor, 1992; Thornton, 1991) demonstrated that a large enough lateral load could lead to contact peeling, turning the interface from the JKR configuration to the Hertzian configuration before occurrence of gloss slip. Later on, based on the concept of mode mixity during interface decohesion (Hutchinson and Suo, 1991), several forms of effective adhesion energy, \( w_{\text{eff}} = w_0 \cdot f \left( \frac{K_u}{K_1} \right) \), have been proposed to improve the Savkoor and Briggs’ solution (Ciavarella, 2018; Johnson, 1996; Papangelo and Ciavarella, 2019; Waters and Guduru, 2009). Although when mode mixity is considered the effective adhesion energy \( w_{\text{eff}} \) may become larger than the intrinsic adhesion energy \( w_0 \), these improved models showed that the adhesion weakening effect of interfacial shear stress would still be dominating. In other words,
contact radius under a same normal load would still be smaller and the pull-off force would still be reduced as long as the interfacial shear stress was present.

The above theoretical solutions were derived based on the JKR configuration assuming non-slip condition and unlimited frictional shear stress. To consider more realistic situations, Johnson (Johnson, 1997) proposed an empirical solution based on the Maugis-Dugdale approach (Maugis, 1992) by allowing partial interface slip with a constant shear stress within the slipped zone (Savkoor, 1987). In Johnson’s model, it was assumed that friction stress could reduce the effective adhesive stress by a factor determined by a phenomenological function. \( X(g) = \sqrt{(1+g)^2 - 2\beta g - g} \), where \( g = \tau_0 \delta / \sigma_0 h_0 \) and \( \beta \) is a dimensionless interaction factor. In particular, \( \tau_0 \) and \( \delta \) are the frictional shear stress and the mean slip distance at the edge of slip zone, \( \sigma_0 \) and \( h_0 \) are the adhesive stress and the maximum separation of the cohesive interaction in absence of slip. Because the reduction factor was imposed directly on the effective adhesive stress, the adhesion weakening effect persists regardless of the value of the Maugis parameter, \( \lambda \) (Maugis, 1992). Although this phenomenological model is easy to implement, it lacks a sufficiently rigorous justification. Later, Kim et al. (Kim et al., 1998) pointed out that the physically meaningful interpretation of Johnson’s model should lie in identifying the correct reversible energy release rate to supply the true thermodynamically equilibrated adhesion energy, rather than finding the effective adhesion energy that depends on mode mixity.

In contrast, assuming a uniform frictional shear stress along a JKR-type contact...
interface, Menga et al. proposed a new solution originally predicting that the interfacial shear stress would enhance adhesion (Menga et al., 2018). However, in an ensued corrigendum, they concluded that the interfacial shear stress would not affect adhesive contact (Menga et al., 2019). A more comprehensive model considering the JKR-type contact with partial slip and gross slip was presented by McMeeking et al. (McMeeking et al., 2020) based on the framework proposed by Kim et al. (Kim et al., 1998). Through energy analysis, it was shown that variations of the potential energy of the lateral load and the strain energy associated with the shear stress would be cancelled by the dissipative energy in the slip zone when varying the contact size. Consequently, the impact of frictional shear stress is solely determined by the reversible energy stored within the interface or surface microstructures, analogous to the reversible energy stored in the normal cohesive zone. To quantify this reversible shear interface energy, a reversible parameter $\alpha$ was defined as the ratio between the reversible slip displacement and the mean total slip displacement at the periphery of the slipped area (typically $0 \leq \alpha \leq 1$). If the reversible shear interface energy is positive (i.e. $\alpha > 0$), the existence of frictional shear stress would weaken adhesion, which coincides with most experimental reports (Das and Chasiotis, 2020; Mergel et al., 2018; Sahli et al., 2018; Savkoor and Briggs, 1977; Waters and Guduru, 2009). If the reversible shear interface energy is zero (i.e. $\alpha = 0$), the normal adhesive contact would not be affected by frictional shear stress. This behavior is consistent with the prediction of Menga et al. (Menga et al., 2019), who have neglected the reversible
shear interface energy, and is only occasionally observed in experiments (Vorvolakos and Chaudhury, 2003). However, for certain materials with special surface microstructures, it is possible that the slip could eliminate surface microstructures and release the energy originally stored in such surface microstructures (McMeeking et al., 2020). In such special cases, the effective value of $\alpha$ can be effectively regarded as negative, meaning that the existence of interfacial shear stress can enhance adhesion.

It is worth mentioning that, for interfaces with a non-slip assumption, the reversible shear interface energy would be zero and consequently normal adhesion should not be affected by frictional shear stress (McMeeking et al., 2020). This conclusion contradicts the theoretical prediction of adhesion weakening effect by Savkoor and Briggs (Savkoor and Briggs, 1977). The discrepancy lies in the fact that Savkoor and Briggs did not take the dissipative energy in the slip zone into account in their model (Savkoor and Briggs, 1977). Consequently, their resultant energy release rate actually contained two parts based on the analysis by Kim et al. (Kim et al., 1998): A reversible part to compensate the work of adhesion $w_0$, and an irreversible part dissipated due to frictional sliding. If the irreversible part were correctly considered, adhesion would become independent of interfacial stress when non-slip assumption was adopted. It is interesting that Savkoor and Briggs (Savkoor and Briggs, 1977) did observe adhesion weakening effect in experiments, which was likely caused by a positive reversible shear interface energy within the actually slipped interface.

Although the theoretical model by McMeeking et al. (McMeeking et al., 2020)
successfully explains the intricate interplay mechanism between frictional shear stress and normal adhesive contact, the phenomenological parameter \( \alpha \) has never been quantitatively measured and validated. More importantly, as the existing studies are based on the JKR configuration (Ciavarella, 2018; Johnson, 1996; McMeeking et al., 2020; Menga et al., 2018, 2019; Papangelo and Ciavarella, 2019; Savkoor and Briggs, 1977; Waters and Guduru, 2009), it remains unclear how frictional shear stress affects adhesion for more generic contact interfaces. In this work, the adhesive contact behavior between a rigid sphere and an elastic half space is studied when a uniform shear stress is present along the intimate contact zone. By adopting the energy approach (Kim et al., 1998) and the reversible shear interface parameter \( \alpha \) (McMeeking et al., 2020), we derive a solution for contact interfaces with arbitrary Maugis parameter \( \lambda \) (Maugis, 1992). Finally, adhesion tests between spherical borosilicate crown glass lens and Polydimethylsiloxane (PDMS) elastomers confirm that the impact of shear stress on adhesion is indeed determined both by \( \alpha \) and \( \lambda \), as predicted by our theoretical model.

2. Theoretical model

Figure 1 (a) Adhesive contact between a rigid spherical indenter and an incompressible elastic half-space; the indenter is loaded by compression and shear, while rotation is restricted by a constraint...
torque $M$. Constant shear stress is assumed to exist within the intimate contact zone ($r \leq a$). A schematic showing the generic representation of displacement and traction along the interface (upper right panel). (b) The normal cohesive zone follows the Dugdale model, which has a constant traction $\sigma_0$ over a separation distance $h$.

As depicted in Figure 1 (a), the model considers a rigid spherical indenter of radius $R$ pressing against an elastic half-space under a normal force $P$ and a tangential force $T$. Due to the combined loading, the indenter has a normal displacement of $\Delta$ and a lateral displacement of $D$. It is assumed that the interface has slipped grossly under lateral loading, which results in uniformly distributed shear stress $\tau_0$ within the intimate (or inner) contact area $A$ ($r \leq a$). In the meantime, adhesive stress exists within the annulus area $a \leq r \leq c$. Such assumption is based on the observation that, although adhesion and friction may originate from same inter-molecular and surface forces, their magnitudes often decay quite differently upon increase of surface separation. For example, van der Waals attraction forces can persist up to tens of nanometers, and capillary forces and electrostatic force can act from hundreds of nanometers to hundreds of microns. However, friction usually exists only within a few angstroms to a few nanometers (typically near the equilibrium distance of the corresponding interactions) (Israelachvili, 2011; Mate, 2008). Furthermore, it is assumed that the adhesive stress follows the Dugdale-type cohesive zone model even when the interfacial shear stress is present, but the maximum interaction distance $h$ can be altered from its original value $h_0$ in Figure 1 (b), as adopted by McMeeking et al. (McMeeking et al., 2020). For general contact mechanics problems, the normal force $P$ and the tangential force $T$ usually have a mutual effect on displacement fields of the elastic half-space (Menga and Carbone, 2019). Following the previous works (McMeeking et al., 2020; Menga et al., 2018, 2019), we assume that the elastic half-space is incompressible with $v = 0.5$, so that the normal and tangential fields are decoupled.

Based on the energy framework by Kim et al. (Kim et al., 1998), the strain energy
release rate $G^{SE}$, released to the cohesive zone from the contacting bodies of the mechanical system, is given by

$$G^{SE} = \frac{d}{dA} \left( \Phi - \hat{P} \Delta - \hat{T} D \right) = \frac{d\Phi}{dA} - P \frac{d\Delta}{dA} - T \frac{dD}{dA}, \tag{1}$$

where $\Phi$ is the elastic strain energy stored in the elastic half-space and $A$ is the outer contact area (that is $r \leq c$). The caret over a symbol indicates that this term is kept constant when taking derivatives.

The principle of virtual work gives

$$\delta \Phi = \hat{P} \delta \Delta + \hat{T} \delta D + \int_A \hat{i}^+ \cdot \delta \mathbf{u}^+ dS + \int_A \hat{i}^- \cdot \delta \mathbf{u}^- dS, \tag{2}$$

where $\mathbf{t}^+/\mathbf{t}^-$ and $\mathbf{u}^+/-\mathbf{u}^-$ are the traction and displacement of the upper/lower body within the outer contact area $A$, as denoted in the upper right panel of Figure 1 (a).

If we take

$$-\mathbf{t}^+ - \mathbf{t}^- = \mathbf{t} \tag{3}$$

and define the displacement jump $[\mathbf{u}]$ as

$$[\mathbf{u}] = [\mathbf{u}^+] + [\mathbf{u}^-] = \mathbf{u}^+ - \mathbf{u}^- \tag{4},$$

where $[\mathbf{u}^+]$ is the gap between contacting bodies at the beginning of touching and $[\mathbf{u}^-]$ is the surface separation displacement, then we get

$$\delta \Phi - P \delta \Delta - T \delta D = -\int_A \hat{i} \cdot \delta [\mathbf{u}] dS \tag{5}.$$  

Because variation of $[\mathbf{u}^+]$ is zero, we have $\delta [\mathbf{u}^+] = \delta [\mathbf{u}^-]$. Therefore, we can find an alternative form for $G^{SE}$ from Eqs. (1) and (5),

$$G^{SE} = -\int_A \mathbf{t} \cdot \frac{d[\mathbf{u}^+]}{dA} dS. \tag{6}$$

It is noted that $\mathbf{t}$ and $[\mathbf{u}^+]$ can be divided into two parts: $\mathbf{t}_n$ and $[\mathbf{u}^+_n]$ in the normal direction and $\mathbf{t}_t$ and $[\mathbf{u}^+_t]$ in the tangential direction. Because the
normal and tangential fields are decoupled and the shear stress \( t_t \) is only distributed within the inner contact area \( A' (r \leq a) \), Eq. (6) can be written as

\[
G^{SE} = -\int_A t_n \cdot \frac{d[u^+]_n}{dA} dS - \int_A t_i \cdot \frac{d[u^+]_i}{dA} dS .
\]  

(7)

Variation of the separation displacement \( \delta[u^+] \) can be decomposed into a reversible part \( \delta[u^+]_{re} \) and an irreversible part \( \delta[u^+]_a \) for the cohesive response. Typically, the normal adhesion process is treated as fully reversible, but the slip process is regarded as partially reversible and contains an irreversible component (Kim et al., 1998; McMeeking et al., 2020). According to the energy framework by Kim et al. (Kim et al., 1998), the total energy release rate \( G \), from the whole mechanical system to the traction free surfaces, is the sum of \( G^{SE} \) and the energy released from the adhesive zone

\[
G = G^{SE} - \frac{d}{dA} \int_A \left( \int_0^{[u^+]_n} t_n \cdot d[u^+]_{re} \right) dS ,
\]  

(8)

where \( t' \) is the interfacial stress from zero to \( t' \), the equilibrium traction at the given separation state \([u^+]_{re} \). Similarly, \( t' \) and \([u^+]_{re} \) can also be divided into two parts: \( t_n' \) and \([u^+]_{re} \) in the normal direction and \( t_i' \) and \([u^+]_{re} \) in the tangential direction. Because we assume that frictional shear stress only exists in the intimate (inner) contact area (i.e. \( r \leq a \)), no recoverable cohesive energy due to shear stress is stored within the area of \( a \leq r \leq c \), which gives

\[
\frac{d}{dA} \int_A \left( \int_0^{[u^+]_n} t_n \cdot d[u^+]_{re} \right) dS = \frac{d}{dA} \int_A \left( \int_0^{[u^+]_n} t_n \cdot d[u^+]_{re} \right) dS + \frac{d}{dA} \int_A \left( \int_0^{[u^+]_n} t_i \cdot d[u^+]_{re} \right) dS .
\]  

(9)

Substituting Eqs. (7) and (9) into Eq. (8), we have
According to the formula for the differentiation of the integral over a variation

\[
G = \frac{d}{dA} \int_{A_0}^{[s_0]} t_s \cdot d\left[u_s^*\right]_{re} dS - \int_{A} t_s \cdot \frac{d\left[u_s^*\right]_{re}}{dA} dS + \frac{dA}{dA} \left( \int_{A_0}^{[s_0]} t_s \cdot d\left[u_s^*\right]_{re} dS - \int_{A} t_s \cdot \frac{d\left[u_s^*\right]_{re}}{dA} dS \right),
\]  

(10)

According to the formula for the differentiation of the integral over a variation

\[
\frac{d}{dA} \int_{0}^{[s_0]} t \cdot d\left[u^*\right]_{re} = t^* \cdot \frac{d\left[u^*\right]_{re}}{dA}
\]

(11)

and

\[
\frac{d}{dA} \int_{A} \left( \int_{0}^{[s_0]} t \cdot d\left[u^*\right]_{re} \right) dS = \frac{1}{l} \int_{\partial A} \left( \int_{0}^{[s_0]} t \cdot d\left[u^*\right]_{re} \right) dS
\]

\[
+ \int_{A} \frac{d}{dA} \left( \int_{0}^{[s_0]} t \cdot d\left[u^*\right]_{re} \right) dS,
\]

(12)

where \( \partial A \) is the outer boundary of the cohesive zone and \( l \) is the corresponding length, we have an alternative result for \( G \) from Eq. (10)

\[
G = \frac{1}{l} \int_{\partial A} \left( \int_{0}^{[s_0]} t_s \cdot d\left[u_s^*\right]_{re} \right) dS - \int_{A} t_s \cdot \frac{d\left[u_s^*\right]_{re}}{dA} dS + \int_{A} \left( t_s - t_s^0 \right) \frac{d\left[u_s^*\right]_{re}}{dA} dS + \frac{dA}{dA} \left( \int_{A_0}^{[s_0]} t_s \cdot d\left[u_s^*\right]_{re} dS - \int_{A} t_s \cdot \frac{d\left[u_s^*\right]_{re}}{dA} dS - \int_{A} \left( t_s - t_s^0 \right) \frac{d\left[u_s^*\right]_{re}}{dA} dS \right),
\]

(13)

where \( \partial A' \) is the inner boundary of the cohesive zone and \( l' \) is the corresponding length. The first and fourth term in Eq. (13) are the reversible energy release rate \( G^{re} \) to compensate the generation of traction free surfaces (i.e. the work of adhesion \( w_0 = \sigma_0 h_0 \))

\[
G^{re} = \frac{1}{l} \int_{\partial A} \left( \int_{0}^{[s_0]} t_s \cdot d\left[u_s^*\right]_{re} \right) dS + \frac{dA}{dA} \left( \int_{A_0}^{[s_0]} t_s \cdot d\left[u_s^*\right]_{re} dS \right) = w_0.
\]

(14)

The rest of terms in Eq. (13) are the irreversible energy release rate \( G^{ir} \) that represents the energy dissipation rate. The source of dissipation can come from the dissipative sliding and viscous behaviors, whose detailed origins are beyond the scope
of this paper. In the following, we mainly focus on the analytical expression of $G^{re}$.

From Eq. (14), the reversible energy release rate $G^{re}$ consists of two parts: One due to normal traction and the other due to shear stress. Following the previous works (Kim et al., 1998; McMeeking et al., 2020), we assume that the normal cohesive energy is fully reversible and the corresponding surface separation displacement at $r = c$ is denoted by $h$. While the tangential cohesive energy is assumed to be partially reversible and the reversible displacement is only a fraction of the total slip distance, i.e., $\delta_{re} = \alpha \delta$, where $\delta$ is the total slip distance in $A'$ and $\alpha$ is a reversible interface slip factor ranging between 0 and 1. It is worth noting that $\alpha$ is a constant value, which means that $\delta_{re}$ is proportional to $\delta$. When $\delta$ is very large, $\delta_{re}$ also increases to a very large value. However, in real physical situations, $\delta_{re}$ may have an upper limit that depends on the nature of the tangential interactions. For simplicity, the present work does not consider the saturation of $\delta_{re}$ for the time being. Therefore, $G^{re}$ becomes

$$G^{re} = \frac{1}{l} \int_{l} \left( \int_{0}^{h} \sigma dh \right) dl + \frac{1}{\pi A} \int_{0}^{\delta} \left( \int_{0}^{\delta_{re}} \tau d\delta_{re} \right) dl, \quad (15)$$

where $dA = l \cdot dc = 2 \pi c \cdot dc$ and $dA' = l' \cdot da = 2 \pi a \cdot da$. At the boundary of $r = c$, the normal traction is a constant value $\sigma_0$ and the normal surface separation displacement $h$ has been given from Eq. (5.7) in Maugis’ work (Maugis, 1992). At the boundary of $r = a$, the shear stress is a constant value $\tau_0$ and the slip distance $\delta$ has also been derived from Eq. (45a) in McMeeking’s paper (McMeeking et al., 2020). Therefore we have an alternative result for $G^{re}$ from (15).
\[ G^{re} = \frac{\sigma_o a^2}{\pi R} \left[ \sqrt{m^2 - 1} + (m^2 - 2) \tan^{-1} \sqrt{m^2 - 1} \right] + \]
\[ \frac{16\sigma_o^2 a}{3\pi K} \left[ - \sqrt{m^2 - 1} \tan^{-1} \sqrt{m^2 - 1} - m + 1 \right] + \frac{4\tau_o^2 a^2}{Kc} \left( 1 - \frac{2}{\pi} \right) \frac{da}{dc}, \]

where \( m = a/c \) and \( K = 4E/3(1 - v^2) \). The derivative \( da/dc \) can be determined while holding the load-point displacement fixed.

To eliminate the singularity of normal stresses at the interface, two stress intensity factors, \( K_i \) from the normal force \( P \), and \( K_m \) from the cohesive stress \( \sigma_o \), are necessarily equivalent. Following Maugis’ work (Maugis, 1992), we have

\[ \frac{a^3 K_i / R - P}{2a \sqrt{\pi a}} = \sqrt{\frac{\sigma_o^2 a}{\pi}} \left[ \sqrt{m^2 - 1 + m^2 \tan^{-1} \sqrt{m^2 - 1}} \right] , \]

and the normal displacement \( \Delta \) is given

\[ \Delta = \frac{a^2}{R} - \frac{2\sigma_o}{3Kc} c^2 - a^2 . \]

When the normal displacement is fixed during variation of contact area, \( d\Delta (a,c) = (\partial\Delta / \partial a) \, da + (\partial\Delta / \partial c) \, dc = 0 \) gives

\[ \left( \frac{da}{dc} \right)_{\Delta} = - \frac{\partial\Delta}{\partial c} \frac{\partial\Delta}{\partial a} = \frac{4\sigma_o Rc}{a(3K \sqrt{c^2 - a^2} + 4\sigma_o R)} . \]

By introducing the following dimensionless parameters:

\[ \bar{a} = \frac{a}{(\pi w_o R^2 / K)^{1/2}}, \]
\[ \bar{P} = \frac{P}{\pi w_o R}, \]
\[ \bar{\Delta} = \frac{\Delta}{(\pi^2 w_o^2 R / K^2)^{1/2}}, \]
\[ \bar{\lambda} = \frac{2\sigma_o}{(\pi w_o K^2 / R)^{1/2}}. \]
\[
\overline{\tau} = \frac{\tau_0}{\left(\pi w_0 K^2/R\right)^{1/2}}.
\] (24)

Eqs. (14), (17) and (18) can be written as

\[
\frac{\lambda \overline{a}^2}{2} \left[ \sqrt{m^2 - 1} + (m^2 - 2) \tan^{-1}\sqrt{m^2 - 1} \right] + \frac{4 \lambda \overline{a}^2}{3} \left[ \sqrt{m^2 - 1} \tan^{-1}\sqrt{m^2 - 1 - m + 1} \right],
\]

\[
+ \frac{8(\pi - 2)\alpha \overline{a}^2 \lambda \overline{a}}{2 \lambda + 3\alpha \sqrt{m^2 - 1}} = 1
\]

\[
\overline{P} = \overline{a}^3 - \lambda \overline{a}^2 \left[ \sqrt{m^2 - 1 + m^2 \tan^{-1}\sqrt{m^2 - 1}} \right],
\]

\[
\overline{\Delta} = \overline{a}^3 - \frac{4 \lambda \overline{a}}{3} \sqrt{m^2 - 1}.
\] (27)

It can be clearly seen that the mechanical behavior of the system is determined both by the Maugis parameter, \( \lambda \), and the reversible shear index, \( \alpha \overline{a}^2 \).

3. Results and discussion

Based on the analysis above, the influence of interfacial shear stress on adhesive contact is manifested by the equivalent adhesion energy

\[
w_e = w_0 \left( 1 - \frac{8(\pi - 2)\alpha \overline{a}^2 \lambda \overline{a}}{2 \lambda + 3\alpha \sqrt{m^2 - 1}} \right).
\] (28)

According to Eq. (28), when the reversible slip factor \( \alpha \) is positive, the existence of interfacial shear stress would result in a reduced equivalent adhesion energy that depends on \( \lambda \). Using Eqs. (25)-(27), we can obtain variations of \( \overline{P} \) with \( \overline{\Delta} \) and \( \overline{a} \) for different values of \( \lambda \) and \( \alpha \overline{a}^2 \). In the following, we will discuss the adhesive contact behavior for three example cases, i.e. the JKR-like regime (\( \lambda = 100 \)), the transitional regime (\( \lambda = 1 \)) and the DMT-like regime (\( \lambda = 0.1 \)).
As shown in Figure 2, for typical JKR-like contacts (λ = 100), interfacial shear stress can noticeably weaken surface adhesion. For example, contact radius \( \vec{a} \) would become smaller at a given \( \vec{P} \) when the interfacial shear stress (equivalently \( \alpha \tau^2 \)) increases. In addition, the pull-off force \( \vec{P_c} \), defined as the maximum tensile force during retraction process, also decreases with increasing interfacial shear stress as indicated by the black dotted line in Figure 2 (a). Similarly, Figure 2 (b) suggests that a higher normal load is needed to maintain the same magnitude of indentation displacement when interfacial shear stress is present. This adhesion weakening effect will continue until the value of \( \alpha \tau^2 \) is high enough to make the equivalent adhesion energy equal to zero, where the contact configuration would jump to Hertzian contact solution as discussed in Appendix A. It is noted that, for the JKR-like contacts with \( \lambda \to \infty \), Eqs. (26) and (27) become

\[
\vec{P} = \vec{a}^2 - \sqrt{6\vec{a}^3 - 24(\pi - 2)\alpha \cdot \vec{a}^4 \tau^2},
\]

\[
\vec{\Delta} = \vec{a}^2 - \sqrt{\frac{8}{3} \vec{a} - \frac{32}{3}(\pi - 2)\alpha \cdot \vec{a}^4 \tau^2},
\]

which are consistent with the solutions of McMeeking et al. (McMeeking et al., 2020) as indicated by the black dashed lines in Figure 2.
Figure 3 (a) Variations of $\bar{P}$ with $\bar{a}$, and (b) variations of $\bar{P}$ and $\bar{\Delta}$ for different values of $\alpha \tau^2$ in the transitional regime ($\lambda = 1$), (c) in the DMT-like regime ($\lambda = 0.1$). (c) Variations of $\bar{P}$ with $\bar{a}$, and (d) variations of $\bar{P}$ and $\bar{\Delta}$ for different values of $\alpha \tau^2$ in the DMT-like regime ($\lambda = 0.1$).

When $\lambda$ becomes smaller, the contact configuration falls into the transitional regime. As shown in Figure 3 (a) and (b), the adhesion weakening effect of interfacial shear stress appears less significant compared with the JKR-like regime. When $\lambda$ further reduces to nearly zero (e.g. when $\lambda = 0.1$), the contact configuration evolves to the DMT-like regime. In this case, the contact radius and the pull-off force are barely affected by interfacial shear stress as suggested by Figure 3 (c) and (d). Therefore, when adhesive contact configuration changes from the JKR-like regime to the DMT-like regime (correspondingly, $\lambda$ changes from $\infty$ to 0), the adhesion weakening effect due to interfacial shear stress would diminish gradually. Eventually, as $\lambda \to 0$, Eqs. (26)-(27) degenerate into
\[ \bar{P} = \bar{a}^3 - 2, \]  \hspace{1cm} (31)
\[ \bar{\Delta} = \bar{a}^2, \]  \hspace{1cm} (32)

which are identical to the DMT model (Derjaguin et al., 1975).

As pull-off force, \( \bar{P} \), is often adopted as a measure to gauge the strength of adhesion, it is informative to see how it changes with interfacial shear stress for different systems. To illustrate this, we plotted \( |\bar{P}| \) as a function of \( \alpha \tau^2 \) for different values of \( \lambda \) in Figure 4. One can see that, for the JKR-like contacts (\( \lambda \to \infty \)), the pull-off force decreases significantly with increasing interfacial shear stress and approaches 0 when the interfacial shear stress is large enough. However, in the transitional regime, the reduction in pull-off force due to interfacial shear stress becomes less and less significant. Eventually, for the DMT-like contacts (\( \lambda \to 0 \)), \( |\bar{P}| \) remains unchanged regardless of the interfacial shear stress. The strong dependence of adhesion weakening effect on Maugis parameter is in sharp contrast to Johnson’s phenomenological model predictions (Johnson, 1997).

![Figure 4 Variations of \( |\bar{P}| \) with \( \alpha \tau^2 \) for different values of \( \lambda \).](image)

4. Experimental results

To validate the influence of interfacial shear stress, we compared the pull-off
force measured from direct pull-off tests and slide-pull-off tests, between a smooth spherical lens indenter and elastic substrates. The spherical lens was made of borosilicate crown glass (SLB-05-10P, Sigma Koki) with a nominal radius of $R = 5.19 \text{ mm}$ and the elastic substrates were made of polydimethylsiloxane (PDMS). Two compositional ratios between matrix material and curing agent in weight ($30:1$ and $5:1$) were adopted to obtain PDMS substrates with different stiffness. The PDMS substrates were cured at $100 \, ^\circ \text{C}$ in plastic dishes for two hours followed by natural cooling in the oven. The solidified surface of the PDMS substrates were flat with a typical root-mean-square roughness about $1 \text{ nm}$ (within an area of $50 \mu \text{m} \times 50 \mu \text{m}$), measured by an atomic force microscope (Ntegra, NT-MDT Inc.).

The Young’s moduli of the PDMS substrates were $0.23 \ \text{MPa}$ for $30:1$ PDMS and $2.06 \ \text{MPa}$ for $5:1$ PDMS, independently measured by indentation tests using a mechanical test instrument (ElectroForce 3100, Bose Inc.) assuming a Poisson’s ratio of 0.5 (Cao et al., 2013; Johnston et al., 2014; Lee et al., 2016; Pritchard et al., 2013).

![Figure 5](image.jpg)

**Figure 5** (a) A schematic showing the experimental setup for the direct pull-off tests. (b) Typical
variation of normal force $P$ with time in a direct pull-off test obtained on a soft PDMS substrate. The insets show the snapshots of the contact interface at four moments marked by i, ii, iii, and iv. The scale bar represents 1 mm. (c) Variations of normal force $P$ with contact radius $a$ measured on a stiff and a soft PDMS substrates. The curves are fitting curves using the Maugis’s model.

For the direct pull-off tests, we used a home-built adhesion test machine based on a tribometer platform (NTR2, CSM Instruments). As schematically shown in Figure 5 (a), the lens was fixed to the force transducer. The PDMS substrate was placed above a transparent rigid box and the contact interface was monitored by a camera through a prism mounted inside the box. The direct pull-off tests involved three sequential steps, i.e. load/compress, dwell, and pull out. The lens was preloaded 0.3 mN against the PDMS substrate, and then dwelled for 60 s to guarantee full adhesive contact. Finally, the lens was pulled out at a fixed rate of 2.9 μm/s. During the whole process, we measured the normal force $P$ and monitored the contact interface as shown in the insets of Figure 5 (b). Variations of $P$ with contact radius $a$ measured on the stiff and soft PDMS substrates are plotted in Figure 5 (c). By fitting the $P \propto a$ data with the Maugis’s model, we estimated the intrinsic work of adhesion $w_0$ and the adhesive stress $\sigma_0$ to be $w_0 = 0.278 J/m^2$ and $\sigma_0 = 118 KPa$ for the soft substrate, $w_0 = 0.0864 J/m^2$ and $\sigma_0 = 41.8 KPa$ for the stiff substrate. Therefore, the value of $\lambda$ was determined to be 7.69 for the soft contact interface and 0.94 for the stiff contact interface.
Figure 6 (a) A schematic showing the experimental setup for the slide-pull-off tests. (b) Typical variations of normal force $P$ and friction force $T$ obtained on the soft PDMS substrate. The insets show the snapshots of the contact interface at four moments marked by i, ii, iii, and iv. The scale bar represents 1mm and the arrow indicates the relative sliding direction of the indenter. (c) The data points are experimental results obtained on the soft substrate (red circles) and the stiff substrate (blue squares). The red solid and the blue dashed lines are fitting curves using Eqs. (25) and (26) for the soft substrate and stiff substrate respectively. All measurements were repeated three times to calculate the uncertainties (standard deviation) of the reported data.

In contrast to the direct pull-off tests, the slide-pull-off tests involved four sequential steps, including load, slide, dwell, and pull out as schematically shown in Figure 6 (a). At first, the lens was preloaded $0.3 \text{ mN}$ against the PDMS substrate. Before dwelling, the lens was slid along the substrate surface for $364 \mu \text{m}$ to induce gross slip. Finally, the lens was pulled out at a fixed rate of $2.9 \mu \text{m/s}$ after dwelling for $60 \text{ s}$. During the dwell and pull-out stages, the tangential displacement of lens was fixed, and thus the interfacial shear stress was maintained throughout the dwell and pull-out periods. During the whole process, we measured the normal force $P$
and friction force $T$, and monitored the contact interface as illustrated in the insets of Figure 6 (b). During the sliding stage, friction force would initially increase to the maximum static friction and then drop slightly to the kinetic friction as shown in Figure 6 (b). As indicated by the snapshots of the interface, the contact area after gross slip (moment ii) is noticeably reduced compared with the initial contact area (moment i). During the pull out stage, both friction and the contact area reduce when the load changes from compressive to tensile and the indenter snaps out of contact at the pull-off load $P_c$. We calculated the average shear stress by dividing friction force by the contact area at moment iii.

As shown in Figure 6 (c), although the presence of interfacial shear stress leads to reduction in pull-off force for both substrates, the reduction is much more significant for the soft substrate (90.5% reduction) than for the stiff substrate (20.2% reduction). This behavior is qualitatively consistent with our model prediction that adhesion weakening effect would be more pronounced for the JKR-like contacts than for the DMT-like contacts. Furthermore, according to the relationship between $|P_c|$ and $\alpha \bar{\tau}^2$ in Figure 4, we could estimate the reversible slip factor $\alpha$ by fitting the experimental data. The fitted value of $\alpha$ is 1 for the soft substrate, while the value is 0.434 for the stiff substrate. The difference in $\alpha$ suggests that the fraction of the reversible interface shear displacement is distinct for these two PDMS surfaces, whose origins may need further explorations at the molecular level. Nevertheless, our experimental results directly confirm that the adhesion weakening effect indeed depends on the magnitude of interfacial shear stress as well as the Maugis parameter $\lambda$.

It is noted that the surfaces of our PDMS samples are very smooth and both exhibit adhesion weakening effect due to interfacial shear stress. When the substrate surface is rough or contains certain microstructures, the contact interface will consist of multiple asperities/microstructures. If shearing the asperities/microstructures does
not release the strain energy originally stored in these asperities/microstructures, the local reversible slip factor $\alpha$ would still be positive. In this case, the adhesion weakening effect would be qualitatively similar. However, for some special roughness/microstructures, if sliding can cause release of the strain energy originally stored in the microstructure, the reversible slip factor $\alpha$ can be effectively negative. In this case, adhesion will be enhanced as speculated by McMeeking et al. (McMeeking et al., 2020). Therefore, rough interface adhesion may exhibit more complicated behavior depending on the detailed deformation mechanisms of the microstructures under compressive and shear loading.

5. Conclusion

In this work, adhesive contact between a rigid spherical indenter and an elastic half-space with frictional shear stress is studied. Based on an energy framework and the Maugis-Dugdale cohesive model, analytical solutions have been derived by assuming a uniform shear stress along the interface. The theoretical predictions suggest that adhesion will be weakened by the interfacial shear stress as long as the reversible slip factor $\alpha$ is positive even when generic contact configurations are considered. More importantly, our model illustrates that the adhesion weakening effect is determined both by the reversible shear index, $\alpha F^2$, and the Maugis parameter, $\lambda$. In particular, adhesion weakening effect is more pronounced for the JKR-like contacts (when $\lambda$ is large) than for the DMT-like contacts (when $\lambda$ is small), and it eventually diminishes when $\lambda \to 0$. To validate the theoretical results, adhesion experiments were conducted for contact systems consisting of a glass lens on elastomer substrates with varying stiffness. The decohesion behavior confirms that the adhesion weakening effect of interfacial shear stress indeed depends on the Maugis parameter. Furthermore, by fitting the experimental data, the reversible slip factors $\alpha$ for the contact interfaces were obtained for the first time. Our model
offers new insight into the interplay mechanism between friction and adhesion, which suggests a strategy for regulating adhesive performance of contact interfaces.

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Appendix A  Transition from adhesive contact to non-adhesive Hertzian contact

As shown in Figure A 1, for typical JKR-like contacts (λ = 100), surface adhesion will be significantly weakened by interfacial shear stress. This weakening effect will continue until the value of $\alpha \tau^2$ is high enough to make the equivalent adhesion energy of Eq. (28) equal to zero, where the adhesive contact would jump to Hertzian solution as shown by the black curves in Figure A 1 (a) and (b).

Figure A 1 (a) Variations of $\bar{P}$ with $\bar{a}$ and (b) variations of $\bar{P}$ with $\bar{\Delta}$ for different values of $\alpha \tau^2$ in the JKR-like regime. The variation curves will jump to Hertzian solution when the equivalent adhesion energy defined in Eq. (28) becomes zero.
Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

CRedit authorship contribution statement

Bo Peng: Methodology, Software, Validation, Writing - Original Draft, Visualization. Qunyang Li: Conceptualization, Methodology, Writing - Review & Editing, Supervision, Funding acquisition. Xi-Qiao Feng: Conceptualization, Writing - Review & Editing. Huajian Gao: Conceptualization, Writing - Review & Editing.