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The *J* Integral as a Fracture Criterion

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ABSTRACT: The path independent *J* integral, as formulated by Rice, can be viewed as a parameter which is an average measure of the crack tip elastic-plastic field. This together with the fact that *J* can be evaluated experimentally, makes a critical *J* value an attractive elastic-plastic fracture criterion. The J_{Ic} fracture criterion refers to crack initiation under plane strain conditions from essentially elastic to fully plastic behavior.

Experiments supporting the validity of a J_{Ic} fracture criterion are presented in this paper. Values of the *J* integral were determined experimentally for two steel alloys, one of low and the other of intermediate strength. A review is given of the analytic support for the J_{Ic} fracture criterion. The range of applicability of the J_{Ic} concept, its limitations, and its advantages are also discussed.

KEY WORDS: fracture (materials), failure, cracking (fracturing), crack initiation, elastic theory, plastic theory, tensile properties, stress strain diagrams, bend tests, analyzing, steels, rotor steels, pressure vessel steels

A failure criterion which could accurately predict failure of cracked bodies would be a useful engineering tool both for the evaluation of structural integrity and the selection of materials. Linear-elastic fracture mechanics provides a one parameter failure criterion for a limited class of problems; those of cracked bodies with small scale yielding where the crack tip plastic region is at least an order of magnitude smaller than the physical dimensions of the component [*1*]. It is desirable to have a failure criterion which could predict fracture in structures in cases of both small and large scale plasticity. To do this it is hoped that the concepts of fracture mechanics could be extended to include cases of large scale plastic yielding. The basis of fracture mechanics is the elastic analysis of the crack tip region which shows a unique stress-strain field with a singularity at the crack tip. The strength of the crack tip singularity is the stress intensity

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factor, *K*. The crack tip region then can be characterized by the one parameter *K*. Fracture must occur then for a critical value of *K*. A direct extension of fracture mechanics concepts to cases of large scale yielding would assume again the existence of a crack tip singularity. Work by Hutchinson [*2*] and Rice and Rosengren [*3*] shows that a singularity does exist which is uniquely dependent upon the material flow properties. An analysis of the crack tip stress field for large scale yielding gives a parameter which can only characterize the crack tip singularity to within some scaling constant. An attempt to uniquely characterize the crack tip singularity has required numerical techniques [*4*]. These techniques have focused attention to the region immediately surrounding the crack tip where the accuracy of the analysis becomes uncertain. An attempt to characterize the crack tip region by a crack tip radius [*5*] again relies on numerical techniques applied close to the crack tip. Likewise calculations of crack opening dislocation (COD) centers attention on a region which must be considered uncertain.

A characterization of the crack tip area by a parameter calculated without focusing attention directly at the crack tip would provide a more practical method for analyzing fracture. The path independent *J* integral proposed by Rice [*6*] is such a parameter. Its value depends upon the near tip stress strain field. However, the path independent nature of the integral allows an integration path, taken sufficiently far from the crack tip, to be substituted for a path close to the crack tip region. The *J* integral is truly path independent for linear and nonlinear elastic stress-strain laws; this includes then, the Hencky laws of plasticity. For the physically more appropriate Prandtl-Reuss representation, Hayes [*7*] has shown that the *J* integral is also nearly path independent under situations of monotonic loading. Since *J* can be calculated analytically by using a stress-strain analysis of regions somewhat removed from the crack tip, numerical techniques can be used to calculate *J* quite accurately. Also, an experimental evaluation of *J* can be accomplished quite easily by considering the load deflection curves of identical specimens with varying crack lengths.

The ease with which the *J* integral can be determined and its general applicability to both elastic and plastic behavior makes it an attractive candidate for a failure criterion. For linear elastic behavior the *J* integral is identical to *G*, the energy release rate per unit crack extension. Therefore, a *J* failure criterion for the linear elastic case is identical to the K_{Ic} failure criterion. The use of *J* provides a means of directly extending fracture mechanics concepts from linear elastic behavior to fully plastic behavior.

This paper discusses the use of the *J* integral as a failure criterion. The basic concept of the integral is presented along with its advantages and limitations as a failure criterion. Data is presented which experimentally substantiates use of *J* as a failure criterion for the case of through the thickness constraint. It is shown for one material that *J* at failure for fully plastic behavior is equal to the linear elastic value of *G* at failure for extremely large specimens.

Basis for the J_{Ic} Failure Criterion

Definition of the J Integral

The energy line integral, J, is applicable to elastic material or elastic-plastic material when treated by a deformation theory of plasticity. It is defined for two-dimensional problems and is given by the equation [6]

$$J = \int_{\Gamma} Wdy - T \left(\frac{\partial u}{\partial x}\right) ds \qquad (1)$$

As illustrated in Fig. 1, Γ is any contour surrounding the crack tip. The quantity

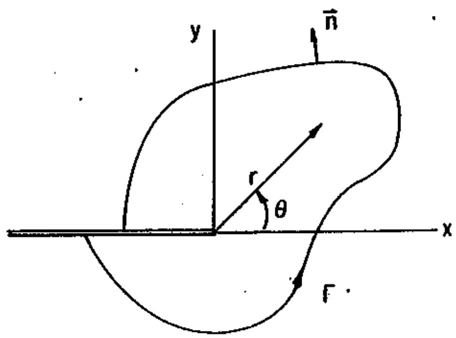

FIG. 1—*Crack tip coordinate system and arbitrary line integral contour.*

W is the strain energy density

$$W = W(\epsilon_{mn}) = \int_0^{\epsilon_{mn}} \sigma_{ij}\, d\epsilon_{ij} \qquad (2)$$

T is the traction vector defined by the outward normal n along Γ, $T_i = \sigma_{ij}n_j$, u is the displacement vector and s is the arc length along Γ.

Rice [6] has proven the path independence of the J integral and this together with an energy interpretation, to be discussed in a subsequent section, makes the J integral a valuable analytic tool. Since paths can be chosen close to the crack tip, the energy line integral represents some average measure of the near tip deformation field. But aside from aiding the approximate analysis of strain concentration at notches and cracks the J integral has utility as a failure criterion. The assumptions which permit this interpretation are presented below.

The J Integral and the Near Tip Field

For the J integral to have validity as a failure criterion it must be assumed that there is a crack tip stress-strain singularity for large scale yielding. The work of Hutchinson [2,8] and Rice and Rosengren [3] supports this assumption. They indicate that the product of stress and strain approaches a $1/r$ singularity as r tends to zero.

$$\sigma_{ij}\, \epsilon_{ij} \rightarrow \frac{\text{a function of } \theta}{r} \quad \text{as } r \rightarrow 0 \qquad (3)$$

While a complete solution is lacking, the general structure of the crack tip singularity has been indicated. In fact, McClintock [7] has shown that by combining the work of Hutchinson [2] and Rice [6] the crack tip plastic stress and strain singularities can be expressed as a function of J.

$$\sigma_{ij}(r,\theta) = \bar{\sigma}_1 \left(\frac{J}{\bar{\sigma}_1 I_n}\right)^{n/(n+1)} \frac{1}{r^{n/(n+1)}}\, \tilde{\sigma}_{ij}(\theta) \qquad (4)$$

$$\epsilon_{ij}(r,\theta) = \left(\frac{J}{\bar{\sigma}_1 I_n}\right)^{1/(n+1)} \frac{1}{r^{1/(n+1)}}\, \tilde{\epsilon}_{ij}(\theta) \qquad (5)$$

Here, n is the strain hardening exponent in the Ramberg-Osgood relation between equivalent stress, $\bar{\sigma}$, and equivalent plastic strain $\bar{\epsilon}_p$.

$$\bar{\sigma} = \bar{\sigma}_1 (\bar{\epsilon}_p)^n \qquad (6)$$

The term I_n is a function of n and the mode of crack opening. For a plane strain tensile crack I_n has a value close to 5.0 over a wide range of n [2]. It is important to note that the plastic crack tip field, Eqs 4 and 5 can be more simply and appropriately written in terms of the plastic stress and strain intensity factors of Hutchinson [2] rather than J. However, these factors are simply related to J. J was chosen as the parameter to characterize the crack tip environment because it can be evaluated experimentally and calculated with less difficulty than the plastic stress and strain intensity factors.

From the above, it is not unreasonable to assume that near tip deformation fields and, therefore fracture, will be governed by a characteristic crack tip singularity in the plastic range. Hence, the J integral, being a field parameter whose magnitude depends on the near tip deformation field, is an attractive failure criterion. In terms of the HRR crack tip model stating that fracture initiates at a critical value of J is equivalent to saying the same event will occur in the two bodies when their crack tip environment is identical or nearly so. Since the experiments described in this paper deal only with plane strain the critical value of J is termed J_{Ic}.

The fact that the J integral and the HRR crack tip model are based upon a deformation theory of plasticity rather than the more appropriate incremental theory may lead to some reservations. However, the nature of the crack tip singularity seems to be consistent with the restrictions of the deformation theory, such as proportional loading. In addition, Hayes [*8*] has indicated that the J integral was path independent when evaluated from a finite element solution employing an incremental plasticity type formulation. He indicated that the deformation and incremental theories should yield essentially equivalent results if the loading is monotonic throughout the body.

Energy Rate Interpretation

Energy balance calculations by Irwin [*10*] and Saunders [*11*] show that Griffith type fracture criteria for crack extension in linear elastic solids are dependent on local conditions at the crack tip. Rice [*12*] has pointed out that local conditions also govern inelastic problems. Since the energy line integral reflects the crack tip deformation field it must be related to energy balance considerations. It has been shown by Rice [*6*] that the J integral may be interpreted as the potential energy difference between two identically loaded bodies having neighboring crack sizes. This is stated mathematically as

$$J = -\frac{dU}{d\ell} \qquad (7)$$

where U is the potential energy and ℓ is the crack length. In the linear elastic case and also for small scale yielding, J is therefore equal to G, the crack driving force. For any nonlinear elastic body, J may be interpreted as the energy available for crack extension.

Where deformation is not reversible, the general elastic-plastic problem, J loses its physical significance as a crack driving force. It may still be considered as an energy comparison of two similar bodies with neighboring crack sizes loaded in the same manner. However, because of irreversibility, one cannot relate the energy comparison of neighboring crack sizes to the process of crack

extension. This distinction will perhaps be clarified by a simple graphical illustration presented below which follows that of Rice [*13*].

The potential energy per unit thickness of a two-dimensional elastic body of area, A with a boundary S is given by

$$U = \int_A W dx dy - \int_{S_T} T(u ds) \qquad (8)$$

Here W is the strain energy density and S_T is that portion of the boundary over which the tractions T are prescribed. On a generalized load deflection diagram, as shown in Fig. 2, U is represented by an area. If the boundary conditions are given in terms of the force F° the potential energy is represented by the shaded area above the load deflection curve. In this instance the potential energy is negative and is equal to minus the complementary energy. When the displacements δ° is prescribed, the negative term in Eq 8 drops out since S_T is then

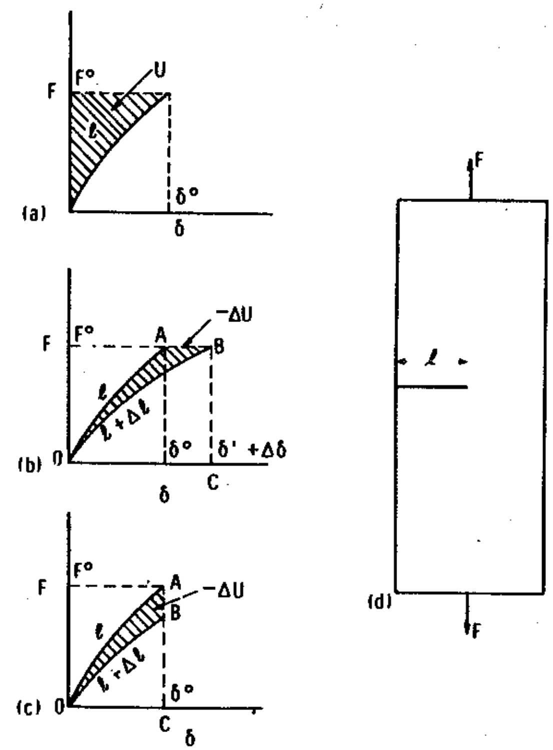

FIG. 2—*Generalized load deflection diagrams.*

nonexistent. The potential energy is then equal to the strain energy, the area under the load deflection curve.

Consider crack extension in a nonlinear elastic body having a crack length ℓ. The load deflection curve is given in Fig. 2*b*. As the crack extends from ℓ to $\ell + \Delta\ell$ under load F^o the total work done on the body is represented by the area OABCO. Because of reversibility, the unloading curve from point B is the same as the loading curve starting with a crack length of $\ell + \Delta\ell$. The strain energy of the body with crack length $\ell + \Delta\ell$ under load F^o is the area OBCO. And the shaded area, OABO, is the difference between the work done on the body to extend the crack to $\ell + \Delta\ell$ and the strain energy of the body at B. Thus it is the energy available for crack extension. But this area also represents minus the potential energy difference between cracks of length $\ell + \Delta\ell$ at load F^o. Considering that here the potential energy is equal to minus the complementary energy makes this readily apparent. Hence, because of reversibility, the J integral, being equal to $-dU/d\ell$ gives the energy available for unit crack extension in elastic materials. In linear elastic fracture mechanics, this crack driving force is termed G. The equality of J and G is evident from a simple graphic illustration. A rigorous derivation of the equality of J and G considering the definition of the J integral, Eq 1, and the linear elastic crack tip stress field equations can be found in Ref *13*. For the case of specified displacements, shown in Fig. 2*c*, the potential energy is equal to the strain energy. The equivalence of the strain energy release rate per unit crack extension and $-dU/d\ell$ is then obvious. It should also be pointed out that the shaded areas in Figs. 2*b* and 2*c*, representing $-\Delta U$ for a constant load and constant displacement are equal. The small additional area in Fig. 2*b*, due to the incremental displacement $\Delta\delta$, is a second order infinitesimal and can be neglected.

Although a deformation type theory of plasticity is essentially a nonlinear elastic theory, one of the restrictions involved in its use is that unloading is not permitted. Plastic deformation is not reversible. As a consequence, the energy interpretation of the J integral cannot be applied to the process of crack extension. It has not been shown that loading a cracked body and then extending the crack under load will give the same result as initially extending the crack and then loading. Therefore, J cannot be identified with the energy available for crack extension in elastic-plastic materials. However, the value of J is still equal to $-dU/d\ell$ and this permits J to be determined experimentally. The physical significance of J for elastic-plastic materials is that it is a measure of the characteristic crack tip elastic-plastic field.

Limitations of the J_{Ic} Approach

Since the Rice energy line integral is expressed only in two dimensions, the J approach is, therefore, limited to problems of plane strain or generalized plane stress. Another limitation is that unloading is not permitted if the deformation theory of plasticity is to be a realistic approximation of elastic-plastic behavior.

This rules out materials which exhibit significant subcritical crack growth prior to fracture. Any crack extension necessarily implies unloading near the crack tip. In general, structures failing in plane stress exhibit some subcritical crack growth. Often this is visible as a flat triangular area ahead of the crack tip denoting the region of shear lip formation. Hence, the J integral failure criterion may be limited to the case of plane strain, which is implied by the subscript I in J_{Ic}. Again as a consequence of the inadmissibility of unloading and subcritical crack growth the J_{Ic} fracture criterion refers to crack initiation rather than propagation. While for a complete treatment of fracture consideration of crack stability is essential, use of the J_{Ic} criterion in an engineering sense is no more restrictive than the use of the K_{Ic} criterion in linear elastic fracture mechanics. It too only refers to crack initiation.

In limiting ourselves to problems of plane strain the question arises, What is meant by this term? A strict analytic interpretation requires that displacements through the thickness parallel to the crack tip leading edge be zero. This is never achieved in an absolute sense. For the fracture problem, observed experimental behavior sets the limits for permissible deviation from ideally plane straining. In linear elastic fracture mechanics a thickness criterion was established based on experimental data. From a large number of tests it was found that when

$$B \geqslant 2.5\,\frac{K^2}{\sigma_{yp}^{\,2}} \qquad (9)$$

K_{Ic} was constant and independent of thickness [*1*]. Here B is the thickness, K is the applied stress intensity, and σ_{yp} is the yield strength. Stress intensity loses its normal significance in the presence of large scale yielding. Equation 9 is of no help in setting limits for the J_{Ic} approach. Such limits must be defined on the basis of experiments analogous to the case of K_{Ic} measurements. A simple rule in terms of average thickness direction strain may prove satisfactory.

It is important to note that in terms of the J integral failure criterion plane strain refers to large scale or general yielding behavior. Relatively small specimens may fail in plane strain depending on the in-plane geometry and the thickness compared to the in-plane dimensions. Linear elastic fracture mechanics requires that in-plane dimensions be large compared to the crack tip plastic zone size. This need not be true for the J_{Ic} approach. For example, a precracked Charpy bar undergoing limit load failure at a moderate deflection may fail in plane strain. Constraint is supplied by the essentially elastic material above the crack tip and a compression zone of approximately half the ligament depth. Guides to permissible dimensions may be found in the extensive literature on plane strain limit load problems. However, the real limits should be established experimentally.

The most severe criticism of a one parameter fracture criterion in the general plane strain elastic-plastic regime arises from the fact that radically different limit load slip line fields may develop. On the one hand there is the contained centered fan slip line field typified by the externally deep notched bar. The region ahead of the crack tip in Fig. 3*b* is one of high hydrostatic stress. On the other hand the slip line field for an internally notched bar does not result in hydrostatic stress elevation. McClintock [*14*] has noted that crack tip separation mechanisms should be affected by the two types of macroscopic slip line fields. Hydrostatic stresses will strongly encourage void formation and growth in the first case. In contrast, the slip line field in Fig. 3*a* may promote crack advancement by alternate slip as shown in Fig. 3*c*. These considerations are equivalent to saying remote deformation patterns will destroy any characteristic crack tip singular behavior for limit load failures. This would limit the J_{Ic} approach to contained plasticity.

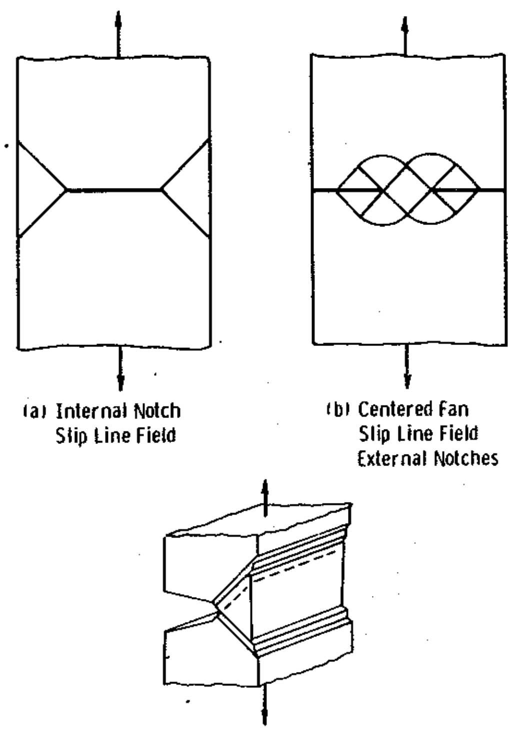

FIG. 3—*Macroscopic slip line fields and alternate slip crack growth pattern.*

Tests are in progress to examine the effect of different slip line fields on the J_{Ic} criteria. It is our speculation that crack tip "blunting" effects will override differences in macroscopic slip line fields. Just as the very presence of a sharp crack is the overwhelming consideration in determining the near tip elastic stress field, the presence of a plastic crack tip singularity and the manner in which the crack physically "blunts" may be the overwhelming considerations in the fracture of elastic-plastic solids. Rice [*13*] has pointed out that the crack tip centered fan slip line field does not result in concentrated strains ahead of the crack tip. Only by considering crack blunting is this physically realistic result obtained. Rice found the region of concentrated strain to be approximately twice the crack opening displacement (COD). Typically this is nearly two orders of magnitude smaller than other component dimensions. From a macroscopic slip line point of view crack tip blunting is negligible, whereas it may actually dominate fracture behavior.

Advantages of the J_{Ic} Approach

Since the path independent J integral is a field parameter, the J_{Ic} fracture criterion is compatible with any criterion based on features specific to the crack tip region. There is no discrepancy between the J_{Ic} approach and fracture criteria based on a COD, a critical strain over some characteristic microstructural distance or other like parameters. The advantage of the J integral over such parameters is that accurate analysis is not needed in the region where such calculations are most difficult. The crack tip area is precisely where analysis is most subject to error. The J integral should be accurately evaluated if the gross features of elastic-plastic behavior away from the crack tip are suitably determined. One would thus expect evaluation of the J integral to be faster and easier than calculation of crack tip features, especially for finite element techniques.

Simple compliance type experiments can be used to determine the value of the J integral. This is a distinct advantage over those parameters which depend only on analysis. Even the COD approach has its experimental pitfalls. With the British method of measurement [*15*] the assumed center of rotation of the bend bar changes with increasing amounts of plasticity. It is only after the full development of plasticity that this center of rotation remains relatively constant.

As with the COD value, J_{Ic} is simply related to the parameters of linear elastic fracture mechanics.

$$J_{Ic} = G_{Ic} = \frac{1-\nu^2}{E} K_{Ic}^2 \tag{10}$$

This relation is unambiguous. In relating COD to G_{Ic}, there is some question as to whether or not the value of the yield strength should be elevated due to constraint. Equation 10 in a sense allows a direct extension of linear elastic fracture mechanics into the elastic-plastic and general yielding range.

Experimental Procedure

Materials and Test Specimens

In order to evaluate the J integral as a failure criterion, a series of tests were performed using a pressure vessel steel, A533B Class 2, and an intermediate strength rotor steel, Ni-Cr-Mo-V alloy heat 1196. The A533B specimens were machined from part of a large test specimen used to evaluate the 02 baseplate in the Heavy Section Steel Technology program. Complete documentation of the mechanical properties of this material can be found in the work of Wessel et al [16] and others [17]. The properties of the Ni-Cr-Mo-V alloy can be found in a paper by Begley and Toolin [18]. For the present, it is sufficient to note that the room temperature 0.2 percent offset yield strengths of A533B, and Ni-Cr-Mo-V steels were 70 and 135 ksi, respectively.

One-inch (1TCT) and two-inch (2TCT) thick compact tension specimens of A533B were tested along with 0.788-in.^{2} bend bars. The test temperature was nominally room temperature. Loading of the compact tension specimens followed ASTM recommendations [19]. Three point loading was used for the bend specimens, with a nominal span of four times the depth.

Only bend bars were tested of the Ni-Cr-Mo-V alloy. The larger size was 0.948 by 0.788 by 4.3 inches. The smaller size was exactly one-half of the larger. Again three point loading was used. The test temperature was 200 F which is the start of upper shelf K_{Ic} behavior. Fully ductile fracture surfaces were observed for the rotor steel. Time to failure for all specimens was in the range of 1 to 2 min.

Evaluation of the J *Integral*

Values of the J integral were calculated from load-displacement curves following the interpretation outlined earlier. At a given total deflection the area under the load-displacement curve was found using a polar planimeter. This energy at constant displacement was plotted as a function of crack length. The slope of this curve is equal to $\Delta U/\Delta \ell$, the change in potential energy per unit change in crack length. J is merely $-\Delta U/\Delta \ell$, normalized to unit thickness.

Since J is experimentally evaluated in terms of an energy input to a system, the location of the displacement gage points must be such that the load displacement curve represents a well defined energy input. In general load versus crack opening curves cannot be used. The product of load and crack opening does not give a physically interpretable energy input to system with well defined boundaries.

For the bend bar tests, load versus ram travel curves were recorded. The area under these curves represents the energy absorbed between the gage points. Therefore, these curves are suitable for the experimental evaluation of the J integral. For tests of compact tension specimens, displacements at the face of the specimens were measured with a clip gage. These values were empirically adjusted to obtain displacements at the location of the loading pins. Hence,

curves of load versus pin displacement reflecting the total work done on the test specimen, were used to experimentally determine J.

One very important characteristic of the steels tested was that prior experience showed crack initiation to be generally coincident with rapid propagation for small specimens. Instances of stable crack extension under rising load has not been observed either by ultrasonic techniques or metallographic sectioning for the relative small specimens tested. Crack initiation was unambiguously defined by a drop in load. In the pressure vessel steel the load drops were generally abrupt. For limit load ductile rupture of the rotor steel, the load drops were more gradual, probably due to rigidity of the bend specimens and fixture and the ductile mode of tearing.

Results

A summary of test data is listed in Tables 1 and 2. Values of J_{Ic} and illustrations of the method of experimentally determining J are presented in the following paragraphs.

TABLE 1–*Summary of test data of a Ni-Cr-Mo-V steel (FD1196).*

Specimen Bend Bars	Maximum Load, lb	Crack Size a, in.	a/W [a]	Total Deflection δ_{max}, in.	Plastic Deflection δ_p, in.	COD, [b] in.
0.474 by 0.394 by 2.16 in.	Span = 1.58					
1196 FC1	5700	0.123	0.259	0.0235	0.0105	0.0047
1196 FC2	5920	0.119	0.251	0.024	0.0110	0.0050
1196 FC3	4470	0.168	0.354	0.0245	0.0130	0.0051
1196 FC4	4320	0.173	0.365	0.0245	0.0112	0.0042
1196 FC5	3360	0.214	0.452	0.0245	0.0120	0.0040
1196 FC6	3270	0.218	0.460	0.0222	0.0112	0.0036
1196 FC7	2275	0.265	0.559	0.0232	0.0132	0.0035
1196 FC8	2410	0.261	0.551	0.022	0.0120	0.0032
1196 FC9	860	0.349	0.736	0.0245	0.0160	0.0025
1196 FC10	1225	0.324	0.683	0.023	0.0150	0.0029
1196 FC12	725	0.356	0.751	0.028	0.0200	0.0030
0.948 by 0.788 by 4.3 in. *Series 1*	Span = 3.90					
1196 DFC3	11900	0.362	0.382	0.062	0.015	0.0045
1196 DFC6	8100	0.452	0.488	0.059	0.012	0.0033
1196 DFC8	3100	0.643	0.679	0.064	0.022	0.0034
Series 2	Span = 3.70					
1196 DFC1	8520	0.470	0.496	0.045	0.008	0.0021
1196 DFC2	4340	0.608	0.642	0.048	0.012	0.0022

[a] W = depth of bend bar

[b] Rigid body rotation assumed about mid ligament point. Only plastic deflection was considered in COD calculation.

TABLE 2–*Summary of test data of A533B steel.*

Specimen	Maximum Load, lb	Crack Size a, in.	a/W^a	Total Deflection δ_{max}, in.	Plastic Deflection δ_p, in.	COD,[b] in.
ZTCT						
HS2-17-5	57 000	2.002	0.500	0.075	0.020	0.0048
HS2-17-6	52 000	2.095	0.526	0.080	0.025	0.0057
HS2-17-7	46 200	2.210	0.552	0.080	0.024	0.0053
HS2-17-8	41 500	2.295	0.576	0.080	0.025	0.0048
ITCT						
8-4-1	15 740	1.005	0.502	...	...	...
8-4-2	13 850	1.058	0.529	...	...	...
18-4-2	12 000	1.094	0.547	0.070	0.038	0.0079
18-4-3	10 800	1.146	0.573	0.073	0.038	0.0074
18-4-4	9 200	1.207	0.604	0.068	0.032	0.0056
Bend Bars 0.788 square by 4.3	Span = 3.15					
DCF 12	11 750	0.173	0.220	0.114	0.092	0.036
DCF 13	8 770	0.255	0.324	0.097	0.077	0.026
DCF 14	7 050	0.318	0.404	0.106	0.077	0.023
DCF 15	4 350	0.415	0.526	0.112	0.092	0.022
DCF 16	2 800	0.493	0.627	0.097	0.077	0.015

[a] W = width of CT specimens or depth of bend bar.

[b] Rigid body rotation assumed about mid ligament point. Only plastic deflection was considered in COD calculation.

Ni-Cr-Mo-V Rotor Steel

Figure 4 shows a plot of the work done to a given deflection versus crack length for the small Ni-Cr-Mo-V bend bars. This work is simply the area under the load deflection curve at the given deflection. The slopes of the curves in Fig. 4 are the changes in potential energy per unit thickness per unit change in crack length. As stated earlier $-\Delta U/\Delta \ell$ is equal to J. Hence the slopes of the curves in Fig. 4 enable the determination of J as a function of deflection for any crack length. Values of J were calculated for straight lines fitted to the data by the method of least squares. Except for the longest and shortest crack slopes obtained from best fit second order polynominals differed by no more than a few percent. Figure 5 shows J as a function of deflection. To a good approximation J at a given deflection does not vary over the range of crack sizes studied. From Fig. 6 it is seen that the average deflection at failure was near 0.024. This results in a critical J, that is, J_{Ic}, or 950 in./lb/in.^{2} Neglecting the

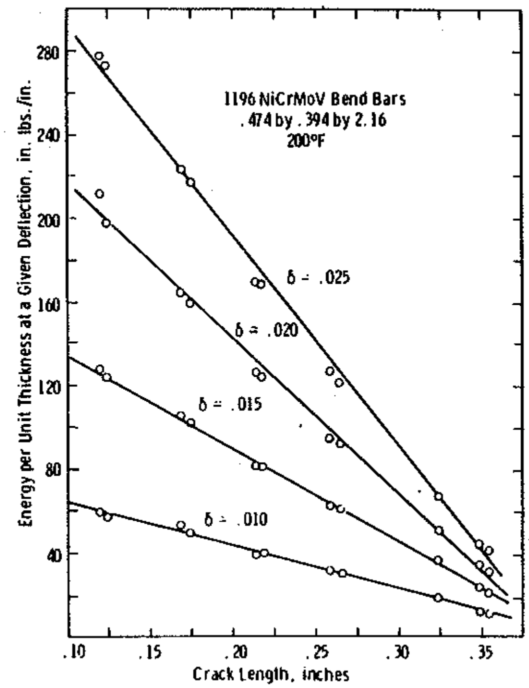

FIG. 4–*Energy absorbed at a given deflection versus crack length, NiCrMoV steel bend bars.*

extremes of crack length it is seen using Figs. 5 and 6 that J_{Ic} has a range of 850 to 1000 in./lb/in.^{2} This variation of ±15 percent in J_{Ic} corresponds to a variation of nearly ±8 percent in K_{Ic}. For rotor steels K_{Ic} can easily vary as much as ±15 percent. Hence, the experimental variation in J_{Ic} is actually better than what might be expected. Values of J_{Ic} obtained from the very limited number of tests of double size bend bars are in good agreement being 1020 in./lb/in.^{2} The next section discusses the relationship and significance of J_{Ic} values from these fully plastic bend bars and G_{Ic} values for the same material obtained from essential elastic failure of 8-in. thick compact tension (8TCT) specimens. For interest sake typical fracture surfaces are shown in Fig. 7.

A533B Pressure Vessel Steel

The work done on 2TCT specimens of A533B steel is plotted in Fig. 8 as a function of crack length. Each curve refers to the energy absorbed at a given deflection. Again taking the slopes of the curves for various values of deflection J values were calculated. From Fig. 8 it is seen that the deflection at failure was 0.080 in. except for the specimen with the shortest crack. In this case the deflection at failure was 0.075 in. This is consistent with the indication that the

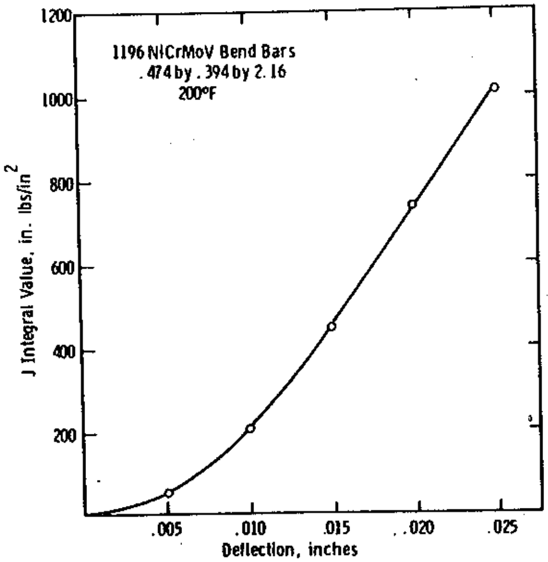

FIG. 5—*J value as a function of deflection, NiCrMoV bend bars.*

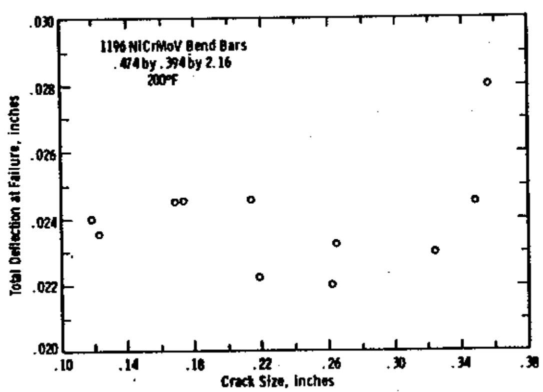

FIG. 6—*Total deflection at failure versus crack size, NiCrMoV bend bars.*

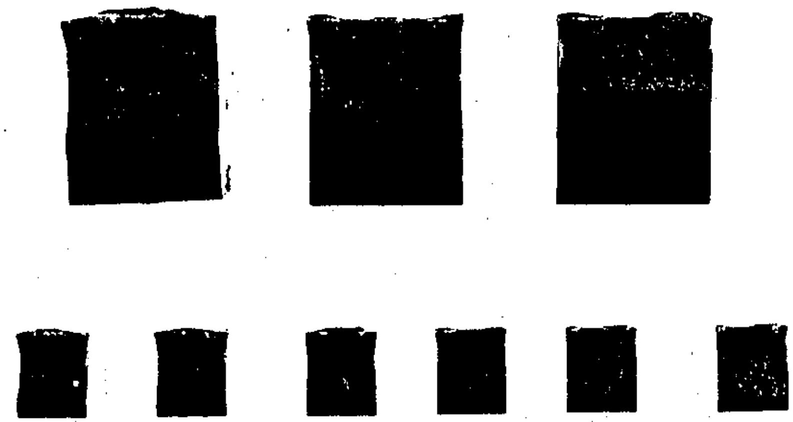

FIG. 7—*Fracture appearance of NiCrMoV steel bend bars.*

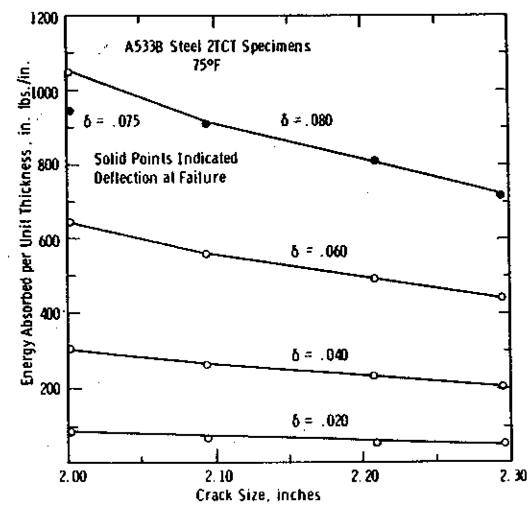

FIG. 8—*Energy absorbed at a given deflection versus crack size A533B 2TCT specimens.*

slope of the $\delta = 0.080$ curve in Fig. 8 begins to increase at the smallest crack length. Consistency of J_{Ic} then requires failure to occur at a smaller deflection. The actual value of J_{Ic} was calculated to be 945 in./lb/in.^{2} Following a similar procedure for 1TCT specimens, the experimental J_{Ic} was 1030 in./lb/in.^{2} Both of these numbers are in good agreement with results of a test on a 12-in. thick compact tension specimen of the same material and extrapolated value of the K_{Ic} versus temperature curve [16]. Converting expected values of K_{Ic} at 75 F to G_{Ic} yields a result of about 1100 in./lb/in.^{2} which should be equal to J_{Ic}. Considering the extreme temperature variation of the toughness of A533B near 75 F, the consistency of G_{Ic} and J_{Ic} is very good.

Bend bars of A533B steel were also tested. The test temperature was nominally 75 F, or room temperature. However, the initiation of fracture in the bend bars was ductile compared to the fully cleavage fracture of the compact tension specimens. The question of whether specimen geometry or test temperature was responsible for the difference in fracture appearance was settled by tests of identical bend bars in carefully controlled isothermal baths. Comparison of the fracture appearance of these bars with the original bend bars indicated an original test temperature of 85 to 90 F. From the work of Wessel et al [16], on the toughness of A533B it is evident that a change in test temperature from 75 to 85 F or higher should caused a substantial increase in toughness. The measured J_{Ic} values for the bend bars were in the range of 2000 in./lb/in.^{2} In terms of K_{Ic} this would be about 250 ksi $\sqrt{\text{in.}}$ This is a reasonable estimate of the start of upper shelf toughness of the HSST 02 baseplate.

Discussion

The objective of the experiments previously described was to demonstrate the potential of the J integral as a failure criterion. If J_{Ic} is a valid failure criterion, it must be independent of geometry. Also, because the J concept applied equally well to structures failing in the essentially elastic or fully plastic range, J_{Ic} must be related to K_{Ic}. As shown earlier, this relationship can be stated in terms of

$$J_{Ic} = G_{Ic} = \frac{(1 - \nu^2)}{E} \quad K_{Ic}{}^2$$

This latter requirement can be tested by comparing J_{Ic} for the fully plastic Ni-Cr-Mo-V bend bars with the valid upper shelf G_{Ic} toughness established by Begley and Toolin [18] for the same materials. A good average J_{Ic} value is 1000 in./lb/in.^{2} The plane strain fracture toughness at the same temperature is 200 ksi $\sqrt{\text{in.}}$, giving a G_{Ic} value of 1200 in./lb/in.^{2} But the agreement of J_{Ic} and G_{Ic} is even better than this indicates. The K_{Ic} value was based on 2 percent crack growth. In the 8TCT tests up to 0.160 in. of crack growth, evidenced by audible popin and load perturbations, occurred prior to the K_{Ic} load. Initiation of growth started closer to a G_I value of 1000 in./lb/in.^{2}

Additional data supporting the equivalence of experimental J_{Ic} and G_{Ic} values have been presented in the results. Measured J_{Ic} values for A533B steels at 75 and 85 F agree well with extrapolated G_{Ic} values.

The geometry independence of J_{Ic} is supported by the fact that a reasonable ±15 percent variation at most in J_{Ic} is exhibited when all the test data is compared. This includes comparison of the fully plastic single and double size bend bars of Ni-Cr-Mo-V over a range of crack lengths and the 8-in. thick essentially elastic compact tension specimens. Likewise the A533B 1 and 2-in. thick compact tension specimens have essentially the same J_{Ic} over a range of crack sizes.

Our view on the effect of radically different fully plastic slip line fields has been expressed. Preliminary results on center cracked panels bear out the original conclusion. These results will be reported shortly.

A final point which needs to be emphasized is that the J_{Ic} failure criterion has its limitations. These limitations are similar to those restricting the applicability of the linear elastic K_{Ic} criterion. As presently stated, the concept of J refers to two-dimensional problems, J_{Ic} refers to crack initiation and for the present only plane strain conditions; that is, plane strain even in the fully plastic region. The validity of the J_{Ic} failure criteria rests on the dominance of the Hutchinson-Rice-Rosengren crack tip singularity. When combinations of specimen geometry, flow properties, and other relevant factors destroy this dominance, the applicability of J_{Ic} will be limited.

Summary and Conclusions

(1) The J integral is an average measure of the near tip stress strain environment of cracked elastic plastic bodies, as such it is an attractive failure criterion.

(2) Being a field parameter J is compatible with COD and other approaches concerned with specific crack tip features.

(3) The J integral should be relatively easy to calculate compared to parameters of the near tip environment.

(4) J values were determined experimentally for a Ni-Cr-Mo-V rotor steel and a A533B pressure vessel steel. Failure occurred at a critical J, termed J_{Ic} to denote a critical plane strain value. A number of specimen types and crack lengths were included.

(5) For the J_{Ic} criterion to be truly valid J_{Ic} must equal G_{Ic}. This was shown to be true for the Ni-Cr-Mo-V steel, J_{Ic} from limit load failures of small bend bars were in excellent agreement with G_{Ic} values from essentially elastic failures of 8-in. thick compact tension specimens.

(6) The potential of the J integral as a failure criterion has been demonstrated by experiment.

(7) Limitations of the J_{Ic} concept have been reviewed in a general manner and are analogous to those involved in the use of K_{Ic}.

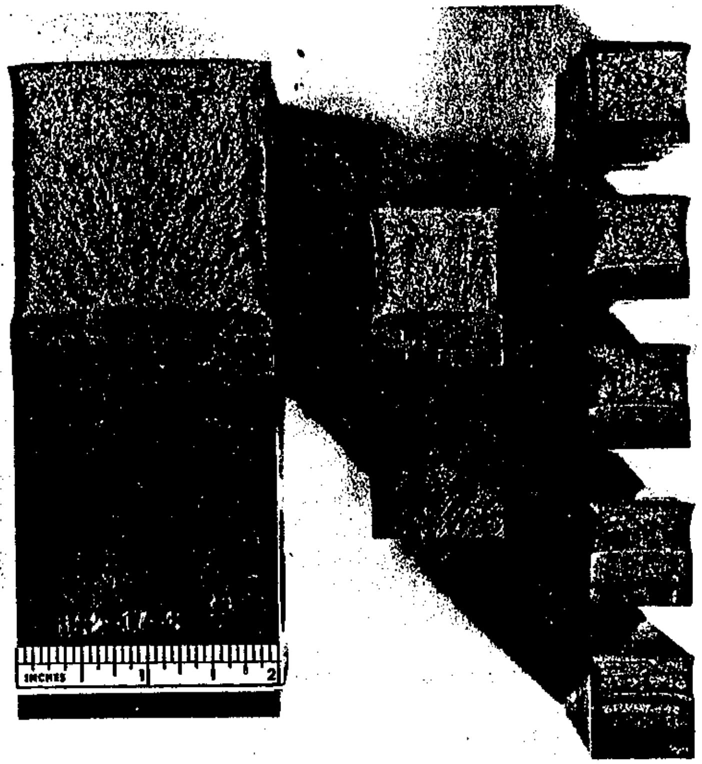

FIG. 9–*Fracture appearance of A533B steel specimens.*

(8) Radically different slip line fields may limit the applicability of J_{Ic}. But this effect will be out-weighed by the dominance of the HRR crack tip singularity.

Acknowledgments

We wish to express our appreciation to our colleagues at the Westinghouse Research Laboratories; to A.R. Petrush and F.X. Gradich who performed much of the experimental work, and especially to Dr. W.K. Wilson for many valuable discussions on the *J* integral and elastic-plastic analysis.

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DISCUSSION

P.C. Paris[1] *(written discussion)*—In applying J as a failure criterion to large scale yielding problems, including fracture under limit load conditions, the ASTM plane strain thickness index, namely,

$$B \geqslant 2.5\left(\frac{K_{Ic}}{\sigma_{yp}}\right)^2$$

must be replaced by a new criterion to define local plane strain. The region to which plane strain must be appropriate now seems only to be required to encompass the region at the crack tip where the critical separation processes are taking place.

It is noted that J/σ_{yp} is linearly related to COD where the constant of proportionality of nearly one mildly depends upon specimen configuration. Thus J/σ_{yp} is a size parameter which should represent the order of the process zone size. Consequently, it seems reasonable to examine replacing the ASTM criteria for plane strain by

$$B \geqslant \alpha \frac{J_{Ic}}{\sigma_{yp}}$$

for use of J as a plane strain fracture criterion.

An initial suggestion or guess of an appropriate α of approximately 50 to Drs. Begley and Landes has been borne out by their data. Further exploration of this thickness requirement is desirable.

I feel that this paper finally explains the reasonableness of the Rolfe-Novak-Barsom correlation of upper shelf Charpy values, CVN, with K_{Ic} numbers, that is,

$$\left(\frac{K_{Ic}}{\sigma_{yp}}\right)^2 = \frac{5}{\sigma_{yp}}\left[\text{CVN} - \frac{\sigma_{yp}}{20}\right]$$

This equation relating, CVN, a limit load related energy parameter, to K_{Ic} is not only now acceptable but is, for us, in agreement with the J failure criteria.

[1] Del Research Corp., Hellertown, Pa.

Moreover, the loss of correlation by this equation at σ_{yp} less than 120 ksi (or better CVN greater than one hundred ft·lb) may be due to the violation of

$$B \geqslant 50 \frac{J_{Ic}}{\sigma_{yp}}$$

Again, all these thoughts are speculative, but so promising as to warrant requesting further comments on them by Drs. Begley and Landes.

G.R. Irwin[2] *(written discussion)*—When a critical load for crack extension is measured using a compact tension or notched-bend specimen, the only characterization parameter which is determined without ambiguity is the thickness-average value of G. Careful measurements of the change of compliance with crack size, dC/da, have agreed within a few percent with two-dimensional numerical computations. At NRL, compliance observations using notched-bend specimens showed no influence of the depth to thickness ratio across the range $W/B = 8$ to $W/B = 4/3$. From this one would expect the relationship of K^2 to the thickness-average G is given by

$$E\bar{G} = \bar{K}^2$$

From this viewpoint, the K calculations of the ASTM K_{Ic} testing method correspond nearly to assuming $K_{Ic} = \sqrt{E\bar{G}}$

Begley and Landes compare values termed J_{Ic} from specimens which undergo general yielding prior to crack extension to values of G_{Ic} from the equation

$$EG_{Ic} = (1 - \nu^2)\, K_{Ic}^2$$

where K_{Ic} is provided by ASTM K_{Ic} tests using specimens of adequate size. However, in the case of the ASTM K_{Ic} test as well as in the case of the present J_{Ic} tests, values of the characterization factor for mid-thickness regions of the initial crack are not directly determined. In both cases it is the thickness-average values of G and J which are measured.

Some readers may feel that the value of the thickness-average G (from $EG = K_{Ic}^2$) should have been compared to the thickness average value of J (for initiation of crack extension). The G/J ratio found by the authors was well above unity and the alternative comparison method suggested above would increase this ratio by about 9 percent. This does not lessen interest in the findings. Clarification of the situation will, in fact, be assisted if we keep in mind that the comparison which is of primary theoretical interest concerns the G and J values locally applicable to central regions of the leading edge of the initial crack rather than the thickness averages which are directly observed. For example, the plastic thickness contraction in tensile regions of the J determina-

[2] Lehigh University, Bethlehem, Pa.

tion specimen may elevate the tensile stress parallel to the leading edge of the crack, σ_z, in mid-thickness regions to a greater extent than would occur in the larger specimens used for K_{Ic} testing. Elevation of σ_z would elevate the resistance to plastic yielding and thus might elevate the J value local to central regions of the leading edge of the initial crack. The explanation by the authors of the G to J discrepancy in terms of the larger initial crack extension which accompanies a K_{Ic} observation remains applicable, but need not be regarded as the only contributing factor. This paper, along with the companion paper by the same authors, provides a new viewpoint on fracture measurements of unusual novelty and importance.

J.A. Begley and J.D. Landes (authors' closure)—We thank Dr. Paris and Dr. Irwin for their comments.

Dr. Paris' suggestion of developing limits to the applicability of J based on the thickness relative to J_{Ic}/σ_{yp} is certainly attractive. It is consistent with our premise that the logic of J as a fracture criterion rests on a crack tip plastic singularity of the HRR type. It is reasonable that this dominance should break down when the thickness and other pertinent dimensions, such as crack length and remaining ligament, become comparable with a crack tip uncertainty region characterized by J_{Ic}/σ_{yp}. Thanks to Dr. Paris our current experimental work is proceeding on this tack.

In reply to Dr. Irwin, we agree there are contributions to the difference between G_{Ic}, obtained from K_{Ic} tests, and J_{Ic} other than a large difference in the extent of crack growth at the measurement points. There are a number of viewpoints on the relationship of K_{Ic} to the thickness averaged, measured values of G_{Ic} and J_{Ic}. Our present thinking is to retain the $1-\nu^2$ term. Relating G to K in the linear elastic case by means of the stress field equations and a virtual crack advance would seem to require the $1-\nu^2$ term for plane strain. On a macroscopic basis, the problem is one of generalized plane stress, hence elastic compliance measurements might not reveal a significant thickness dependence.

To Zhigang Suo
I am happy to finally meet you
John D. Landes

J. A. Begley and J. D. Landes

The J Integral as a Fracture Criterion

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