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1972

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# The Effect of Specimen Geometry on $J_{Ic}$

**REFERENCE:** Landes, J.D. and Begley, J.A., "**The Effect of Specimen Geometry on $J_{Ic}$**," *Fracture Toughness, Proceedings of the 1971 National Symposium on Fracture Mechanics, Part II, ASTM STP 514,* American Society for Testing and Materials, 1972, pp. 24-39.

**ABSTRACT:** Rigid-plastic slip line field analysis shows that fully plastic flow fields and hydrostatic stress elevation are greatly influenced by geometry. This argues against a one parameter fracture criterion, such as $J_{Ic}$, which purports to work in the plastic range. Two specimen geometries, a center cracked panel and a bend bar were tested to provide a critical evaluation of the $J_{Ic}$ concept. An intermediate strength Ni-Cr-Mo-V rotor steel was used in the investigation. Results show that $J_{Ic}$ is a consistent fracture criterion for plane strain behavior for essentially elastic to fully plastic conditions. This work supports the contention that a plastic crack tip singularity is a dominant consideration for crack initiation even in fully plastic bodies.

Bend bars of two thicknesses were tested to show that nearly plane strain behavior can be achieved with conventional geometries as much as an order of magnitude smaller than sizes required for linear elastic plane strain fracture toughness tests. The method used for calculating the $J$ integral for experimental load versus deflection curves is explained in detail.

**KEY WORDS:** fracture (materials), failure, crack initiation, cracking (fracturing), geometries, plastic theory, stresses, bend tests, rotor steels

The fracture mechanics approach of using $K_{Ic}$ to predict unstable crack extension in a crack notched body has proved successful for the limited case of linear elastic plane strain fracture [*1*]. Complex structures may have stresses in some regions which exceed the elastic limit. This has created need for a fracture criterion which would also include elastic-plastic to fully plastic behavior. Begley and Landes [*2*] have demonstrated experimentally that the path independent $J$ integral proposed by Rice [*3*] apparently can be used to predict fracture in low to intermediate strength steels for plane strain conditions ranging from linear elastic to fully plastic. The value of $J$ at the onset of initial crack growth, $J_{Ic}$, was a constant over the entire range and was equal to $G_{Ic}$, the energy release rate per unit crack extension in the linear elastic range.

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For fully plastic behavior the stress state in the ligament ahead of the crack is greatly influenced by geometry. Rigid-plastic slip line theory shows changes in constraint or loading configuration cause radical changes in macroscopic flow fields with resultant changes in hydrostatic stress ahead of the crack. These changes might influence the fracture mode and the value of $J_{Ic}$. If the $J$ integral is a valid one-parameter fracture criterion, then it cannot be influenced by these differences in specimen geometry. However, the experimental verification of $J_{Ic}$ was conducted on only two specimen types, 3 point bend bars and compact tension specimens [*2*]. In both cases the loading is predominantly bending and the slip line fields are similar to those proposed by Green and Hundy [*4*]. To provide a critical evaluation of the $J_{Ic}$ concept, specimens with radically different slip line fields must be tested.

In this work a first step was taken to determine what effect specimen geometry might have on $J_{Ic}$. Center cracked panels of an intermediate strength Ni-Cr-Mo-V rotor were tested and results were compared with previously tested bend bars of the same material. There is little if any hydrostatic stress elevation in the internally cracked geometry whereas the bend bars have a moderate stress elevation [*4*]. In addition tests were made on bend bars of double thickness to determine whether conventional thickness provides proper constraint to insure a predominantly plane strain deformation mode. The method for calculating $J$ from load deflection curves is demonstrated in detail using these tests as examples and the determination of $J_{Ic}$ is discussed.

## Effect of Geometry

For fully plastic behavior rigid-plastic slip line theory shows the magnitude of stresses may change radically with changes in geometry. An internally notched specimen in tension has slip lines emanating from the notch tip at 45 deg to the applied load, Fig. 1*a*. There is no stress elevation ahead of the crack and the limit load, $P_L$, is

$$P_L = 2\tau_o B(w - 2a) \quad (1)$$

where $\tau_o$ is the shear flow stress, $B$ is the thickness, $w$ is the width, and $2a$ is the notch length.

In contrast the deep double edge notched specimen has a Prandtl slip line field, Fig. 1*b*, with a significant stress elevation ahead of the notch. For the fully plane strain case with a sharp crack the stress normal to the crack is $(2 + \pi)\,\tau_o$ and the limit load

$$P_L = (2 + \pi)\,B\,(w - 2a)\tau_o \quad (2)$$

This gives a limit load which is $(1 + \frac{\pi}{2})$ times that of an internally notched specimen when the two have equal net sectional areas.



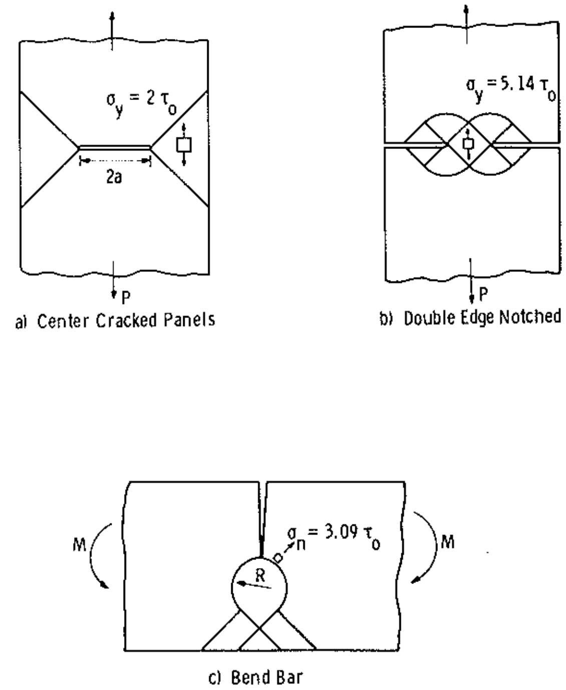


FIG. 1—*Slip line fields for three specimens.*

McClintock [*5*] has suggested that the mode of crack extension should be different in the two cases. Crack extension under high hydrostatic stress should be predominantly by initiation and growth of internal voids. Under low hydrostatic stress crack extension would be more likely to occur by slipping off on the shear planes. Consequently, no single parameter should apply as a fracture criterion for both cases. However the crack tip plastic singularity with subsequent crack tip "blunting" during loading may override the effect of different macroscopic slip line fields and make $J$ a valid one-parameter fracture criterion.

For a single edge notched bar under bending the slip line field has been described by Green and Hundy [*4*], Fig. 1*c*. For a sharp crack the stress normal to the flow lines is $(1.543)(2\tau_o)$ [*6*]. The limit moment, $M_L$ is

$$M_L = (1.261)(2\tau_o)(B)(w - a)^2/4 \quad (3)$$

This gives an elevation in limit moment which is 1.261 times that of an unnotched bar. The constraint of this geometry is intermediate to that of the center notched specimen and the deep edge notched specimen. Comparing $J_{Ic}$ for center cracked panels to that of single edge cracked bend bars gives a first step in the evaluation of $J$ as a valid one-parameter fracture criterion.

## Experimental Procedure

Tests were conducted on an intermediate strength rotor steel, Ni-Cr-Mo-V alloy heat 1196. The chemistry and mechanical properties are shown in Table 1.

TABLE 1–*Properties of the Ni-Cr-Mo-V steel, heat 1196.*

*Chemical Properties*

| C | Mn | P | S | Si | Ni | Cr | Mo | V | Sn | Sb |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.28 | 0.29 | 0.010 | 0.008 | 0.02 | 3.80 | 1.76 | 0.40 | 0.14 | 0.019 | 0.001 |

*Mechanical Properties*

| | 0.2% Offset Yield Strength | Ultimate Tensile Strength | Percent Elongation | $K_{Ic}$ |
|---|---|---|---|---|
| Room temperature | 916 $MN/m^2$ | 1020 $MN/m^2$ | 16 | 132 $MN/m^{3/2}$ |
| 394 K | 855 | 957 | 16 | 220 |

The center cracked panels, Fig. 2, were loaded in tension at a constant rate of displacement. The displacement was measured over a gage length of 57 mm where stresses are essentially uniform and entirely elastic. Crack lengths ranged from 6.8 to 15.3 mm.

The double thickness bend bars, Fig. 3, were tested in three-point loading. Crack lengths ranged from 3.5 to 9.8 mm. Displacement was measured at the center load point. In both cases the test temperature was 394 K which is in the range of upper shelf $K_{Ic}$ behavior. Specimens were initially notched and then precracked in fatigue prior to testing.

## Calculating $J$ Experimentally

$J$ can be measured experimentally by a specimen compliance method which is equally applicable from linear elastic to fully plastic behavior. The general approach is evident from the graphical presentation of Rice in Ref 7. The specific approach of the authors was outlined earlier [2] and is recounted here in detail.



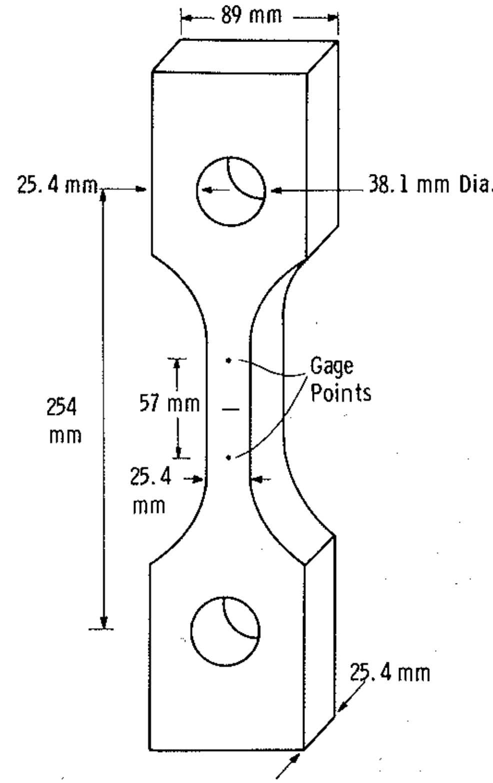


FIG. 2–*Center cracked panel.*

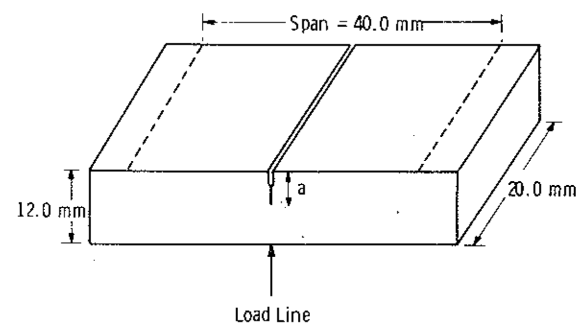


FIG. 3–*Double thickness bend bar.*

As discussed by Rice [7], the $J$ integral can be interpreted as the potential energy difference between two identically loaded specimens of unit thickness having neighboring crack sizes. That is

$$J = -\frac{dU}{da} \tag{4}$$

where $U$ is the potential energy and $a$ is the crack length. The potential energy of a body of area $A$ with a boundary $S$ is given by

$$U = \int_A W dx dy - \int_{S_T} \overline{\mathbf{T}}\,\overline{\mathbf{u}}\, ds \tag{5}$$

The first term is the integrated strain energy density or simply the work done on the body in loading to a given condition. In the second term, $\overline{\mathbf{T}}$ is the traction vector, which specifies the stresses on the boundary $S$, $\overline{\mathbf{u}}$ is the displacement vector of points on $S$, and $S_T$ refers to that portion of the boundary over which stresses are prescribed as boundary conditions.

On a plot of load versus load point displacement constructed for two specimens with crack lengths $a$ and $a + \Delta a$, the area between the curves is $J\Delta a$. This is illustrated in Fig. 4. In the case of constant load the additional triangular

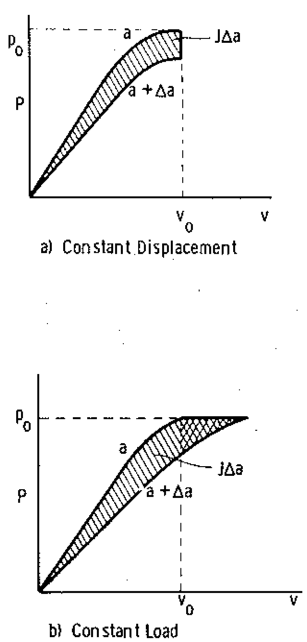


FIG. 4—*Graphical measurement of* J.



area is a second order differential and may therefore be neglected. Hence, $J$ is the same whether determined at constant load or constant deflection.

It is ridiculous, of course, to attempt to measure $J$ by testing two specimens with slightly different crack lengths. By testing specimens of a range of crack length the potential energy $U$ at a given load or displacement can be plotted as a function of crack length. The slope of this curve is $dU/da$ which is $-J$. In essence, this is the procedure which was followed. It is important to note that when displacements are the prescribed boundary condition, that is, when $J$ is evaluated as a function of displacement, the potential energy, $U$, reduces to the area under the load deflection record. If $J$ is evaluated in terms of load, $U$ is the negative of the area above the load deflection record. Because of the leveling off of load deflection records and the eventual attainment of a limit load, it is more definitive to evaluate $J$ in terms of displacement when there is large scale plasticity. In the latter case, the second term of Eq 5 drops out and the potential energy $U$ is the work done on the body (boundary $S$) that is, the area under the load deflection curve. In the following sections the thickness, $B$, appears in some equations since the actual load deflection curves were not normalized to unit thickness.

## Bend Bars

To calculate $J$ experimentally load displacement curves are generated for specimens with different crack lengths. For the bend bars some examples are shown in Fig. 5. The curve is integrated graphically to determine the work done in loading to a given displacement. (Areas were measured with a compensating polar planimeter.) Figure 6 shows an example of this; the work done in loading

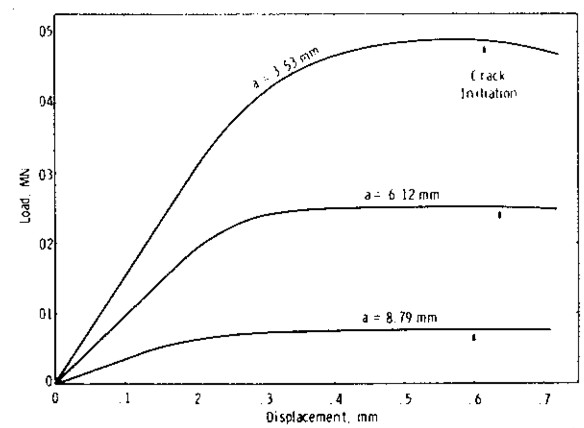


FIG. 5—*Load-displacement curves for double thickness bend bars.*

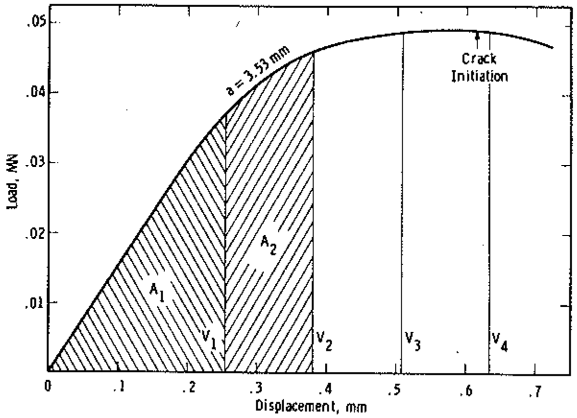


FIG. 6—*Illustration of work to a given displacement.*

to $v_1$ is $A_1$, the work done in loading to $v_2$ is $A_1$ plus $A_2$. For each specimen the work was determined at four different displacements. Work was measured to the same four displacement values for each specimen so that a plot of work versus crack length could be made for each of the displacements, Fig. 7. $J$ at constant displacement for a specimen of thickness $B$,

$$J = -\frac{1}{B} \left. \frac{\partial U}{\partial a} \right|_{v\,=\,\text{constant}} \tag{6}$$

is measured by taking the negative of the slopes of the curves in Fig. 7 and dividing by $B$. $J$ is then a function of crack length and displacement. The curves in Fig. 7 can be fitted numerically to a polynomial so that $J$ can be determined by differentiating the polynominal. A plot of $J$ versus displacement can then be made, Fig. 8. To determine the critical value of $J$ for crack initiation, $J_{Ic}$, a critical value of displacement must be determined. From previous work [*2*] it was determined that crack initiation corresponds to the point where the maximum load first begins to drop off, Fig. 5. Therefore, from each load-displacement curve a critical value of displacement is determined and plotted as a function of crack length, Fig. 9. $J_{Ic}$ can then be determined by taking the critical displacement from Fig. 9 and determining its corresponding value of $J$ in Fig. 8.

We do not mean to imply that crack initiation in general, occurs at point of drop off of maximum load. This was found to be true for the particular material tested and only with the relatively small fully plastic test specimens employed. Defining crack initiation and determining its onset is perhaps the most difficult part of measuring $J_{Ic}$.



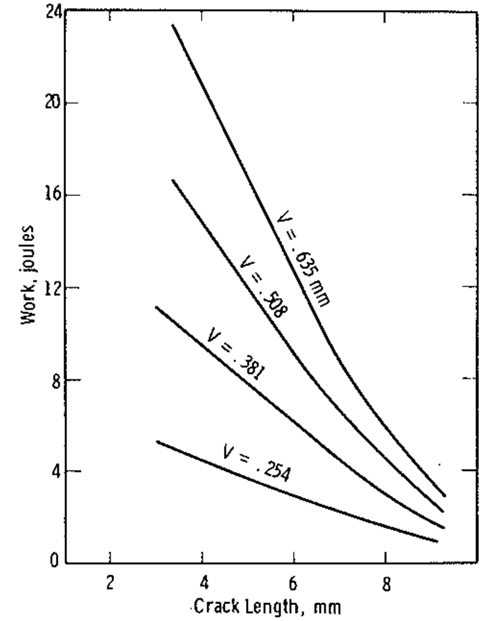


FIG. 7—*Work to a fixed displacement versus crack length for double thickness bend bars.*

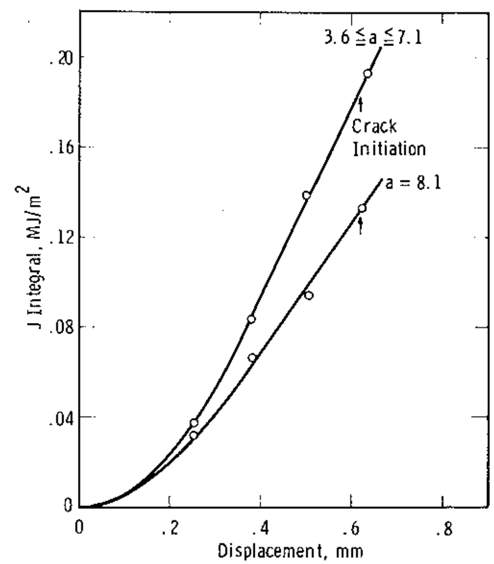


FIG. 8—J *versus crack length for double thickness bend bars.*

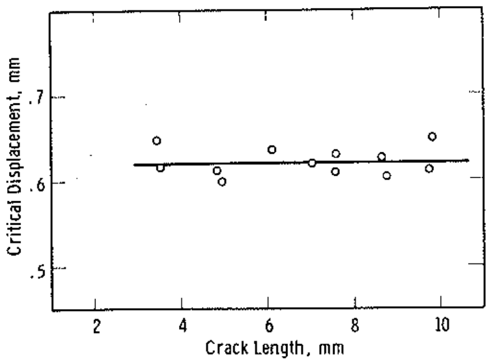


FIG. 9—*Critical displacement for crack initiation versus crack length double thickness bend bars.*

A check can be made on the constraint caused by the specimen geometry, that is, the degree of plane strain, by plotting limit moment versus crack length, Fig. 10. Using the limit solution from Green and Hundy [*4*] and the tensile properties, an estimation of the limit moment for a properly constrained plane strain specimen can be made, Fig. 10. This is done for the specimens used in this experiment as well as conventional thickness bend bars reported in a previous

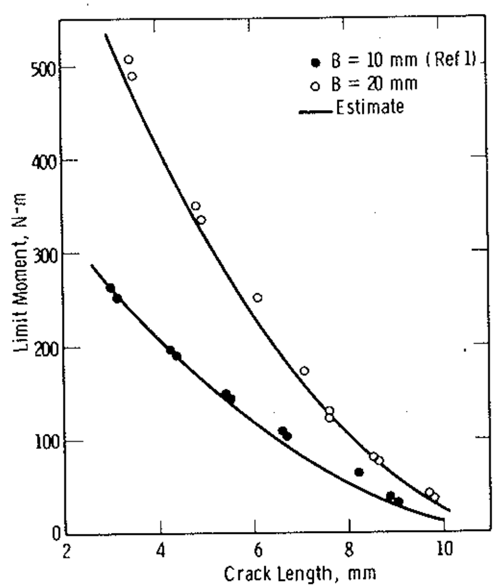


FIG. 10—*Limit moment versus crack length for single and double thickness bend bars.*



work [*2*]. The value used for shear flow stress, $\tau_o$, was determined from the uniaxial ultimate stress to account for strain hardening, the constant, $\sqrt{3}$, is a consequence of using the von Mises yield criterion.

$$\tau_o = \sigma_{uts} / \sqrt{3} \tag{7}$$

*Center Cracked Panels*

$J$ is calculated for the center cracked panels by using the same technique. The load displacement curves, Fig. 11, are integrated graphically and the work to a given displacement is plotted as a function of crack length for several displacements, Fig. 12. $J$ is the negative of these slopes, that is the slope/2 because of the presence of two crack tips. $J$ is plotted as a function of displacement for several crack lengths, Fig. 13. The critical displacement for crack initiation is determined as a function of crack length, Fig. 14. In contrast to the bend bars, this is not a constant. To find $J_{Ic}$ the critical displacement must be taken for a specific crack length, Fig. 14. When $J_{Ic}$ is determined for several crack lengths the results show a nearly constant value.

The limit load was plotted as a function of the crack length and compared with an estimate using $\tau_o = \sigma_{uts}/\sqrt{3}$, Fig. 15.

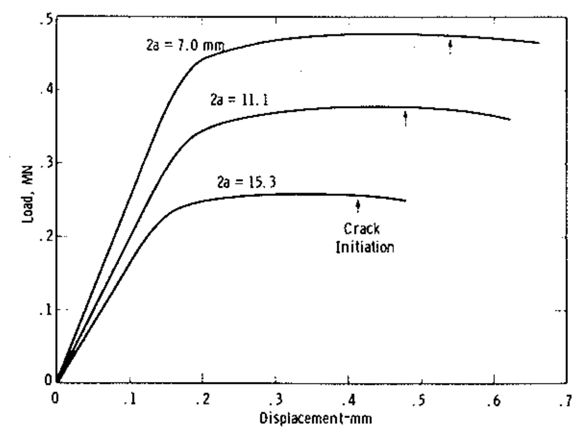


FIG. 11—*Load-displacement curves for center cracked panels.*

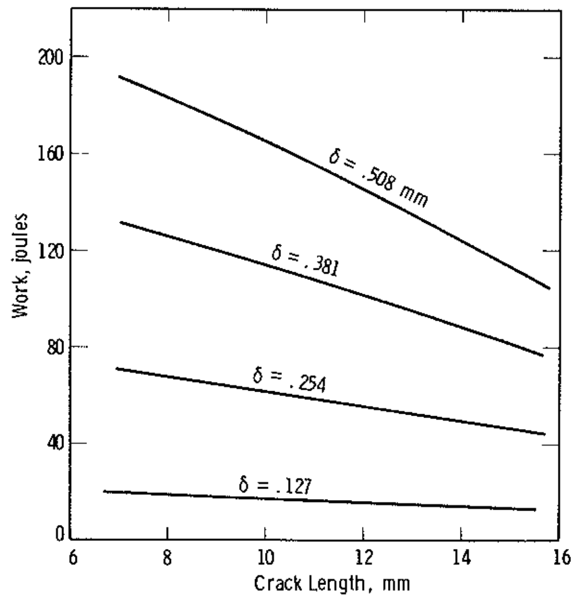


FIG. 12—*Work to a fixed displacement versus crack length for center cracked panels.*

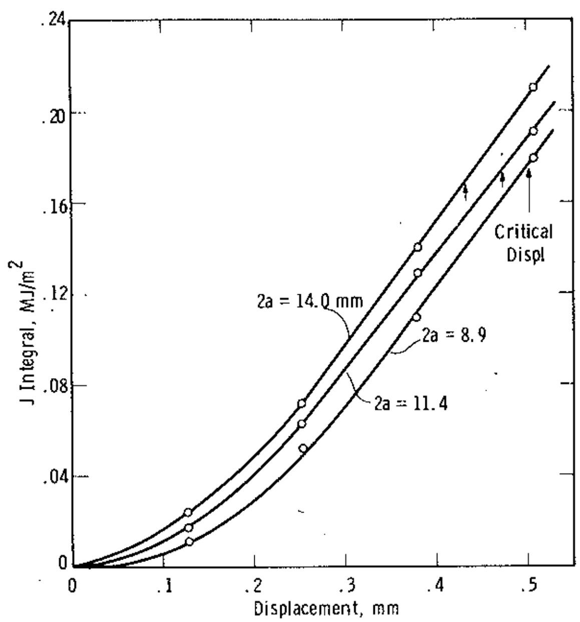


FIG. 13—*J integral versus displacement for center cracked panels showing* $J_{Ic}$.



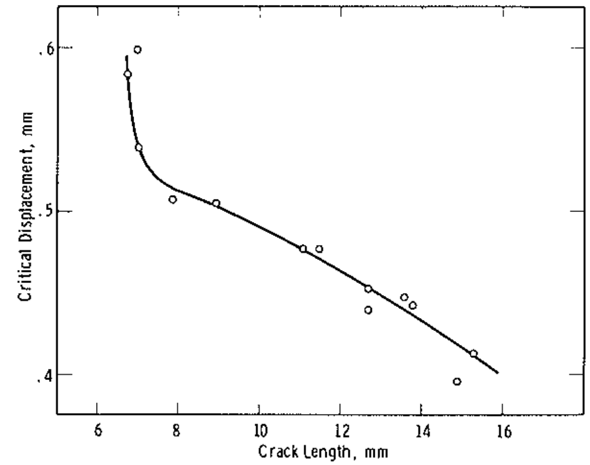


FIG. 14—*Critical displacement for crack initiation versus crack length, center cracked panels.*

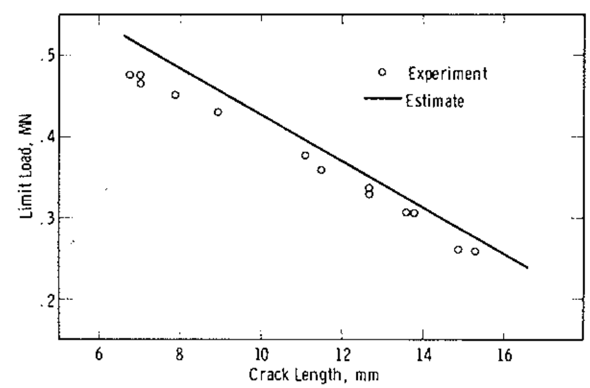


FIG. 15—*Limit load versus crack length for center cracked panels.*

## Results

$J_{Ic}$ for the center cracked panels averaged 0.172 MJ/m$^2$ for crack lengths varying from 8.9 to 14.0 mm, Fig. 13. Considering the variation in critical displacement from Fig. 14 the values of $J_{Ic}$ could vary from 0.158 to 0.184 MJ/m$^2$. This is a scatter of less than ± 8 percent. This variation is insignificant considering that the variation in $G_{Ic}$ from $K_{Ic}$ testing is typically ± 30 percent for rotor steels. Comparing the actual limit loads with the estimated ones shows that there is no stress elevation in the net section due to the presence of a crack, Fig. 15.

The double thickness bend bars show an average $J_{Ic}$ of 0.187 MJ/m$^2$ for cracks ranging from 3.6 to 7.1 mm, Fig. 8. The variation in $J_{Ic}$ was from 0.175 to 0.200 MJ/m$^2$, about ± 7 percent. From previous tests on standard thickness bend bars the values of $J_{Ic}$ ranged from 0.15 to 0.19 MJ/m$^2$ [*2*].

For longer cracks $J_{Ic}$ begins to deviate significantly. For example at a crack length of 8.1, where the remaining ligament is only 3.9 mm, $J_{Ic}$ is about 0.13 MJ/m$^2$. This value is below the scatter band of previous results. As the crack length increases farther, the value of $J_{Ic}$ appears to become even lower. It appears that $J_{Ic}$ is not valid for these longer cracks due probably to a minimum size limitation being reached in the uncracked ligament of the specimen.

The limit moments for the bend bars agree very well with the estimate from Green and Hundy [*4*], Fig. 10, for both thicknesses. This indicates that a sufficient degree of constraint is achieved in both thicknesses.

## Discussion

The values of $J_{Ic}$ for the two geometries, center cracked panels and bend bars, show no effect of the radically different slip line fields. This gives support to the argument that the local plastic crack tip singularity and subsequent crack tip "blunting" overrides the effect of slip line fields in determining $J_{Ic}$. Hence the $J$ integral is a valid fracture criterion for elastic-plastic behavior. A more severe test of the applicability of $J$ to cracked bodies would be the testing of a highly constrained specimen such as the deep double edge notched specimen. These tests are being conducted presently.

Comparison of the results from standard thickness bend bars with those of the double thickness specimens shows good agreement in the value of $J_{Ic}$. This indicates that nearly plane strain results can be obtained from the standard thickness bend bars, that is, ones with a nearly square cross section.

The results of most of the $J_{Ic}$ tests to date are shown in Table 2. These results show that the $J$ integral fracture criterion has worked successfully for two materials, an A533B pressure vessel steel and a Ni-Cr-Mo-V rotor steel for several different specimen geometries.

Some limitations on the application of $J$ as a fracture criterion must be explored before it can be used as an engineering tool. The effect of stress state, that is, plane strain versus plane stress, is not known. All test results to date have been for plane strain behavior. Also, there should be a size limitation for the use of the $J$ fracture criterion. When the thickness or uncracked ligament is small compared with some theoretical fracture zone the results for $J_{Ic}$ would not be valid. It has been proposed [*8*] that this fracture zone should be a function of $J_{Ic}$ divided by the yield stress of the material. The results from the bend bars show that $J_{Ic}$ begins to deviate when the uncracked ligament is less than 5 mm. This gives a ratio of size limitation to $J_{Ic}/\sigma_{yp}$ of about 25 for this material. The possible size limitation is very important to the development of $J$ as a fracture criterion and should be explored fully.



TABLE 2–*Results of $J_{Ic}$ tests to date.*

| Specimen Type | Dimensions, mm | Temperature, deg K | $J_{Ic}$ MJ/m$^2$ |
|---|---|---|---|
| | *Ni-Cr-Mo-V* | | |
| Center cracked panels | 25.4 x 25.4 x 57 | 394 | 0.172 |
| Double thickness bend bars | 12 x 20 x 40 | 394 | 0.187 |
| Single thickness bend bars [*2*] | 12 x 10 x 40 | 366 | 0.167 |
| Compact tension (8TCT) [*2*] | 406 x 203 x 488 | 394 | 0.175 |
| Bend bars double size [*2*] | 24 x 20 x 96 | 366 | 0.179 |
| | *A533B Class II* | | |
| 2TCT [*2*] | 102 x 51 x 122 | 298 | 0.165 |
| 1TCT [*2*] | 51 x 25.4 x 61 | 298 | 0.180 |

Until these limitations can be understood and regions of validity established for $J_{Ic}$ its status should be that of an experimental quantity rather than a usable engineering tool.

## Conclusions

(1) The $J$ integral has proved to be a successful fracture criterion for behavior ranging from linear elastic to fully plastic in low to intermediate strength steels.

(2) $J_{Ic}$ is not influenced by radical differences in fully plastic slip line fields for the two geometries center notched panels and single edge notched bend bars.

(3) Bend bars of conventional thickness, namely, a square cross section, provide sufficient constraint to achieve nearly plane strain results with respect to fracture properties, in the fully plastic state.

### *Acknowledgments*

We would like to express our appreciation to our colleagues at the Westinghouse Research Laboratories who contributed to various phases of this investigation; to Dr. W.K. Wilson who contributed valuable discussions on the $J$ integral and to R.B. Stouffer, A.R. Petrush, and F.X. Gradich who assisted with the experimental work.

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