Effect of Stress Ratio on the Flexural Fatigue Behaviour of Continuous Strand Mat Reinforced Plastics

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ABSTRACT

The flexural fatigue behaviour of continuous glass-mat-reinforced thermoset-based composites was investigated using four-point bending. The tests were carried out adopting different stress ratios, R, i.e., the ratio of the minimum to the maximum applied stress. It was observed that the stress ratio has a strong influence on the fatigue life: given the maximum stress, passing from R=0.1 to R=0.7 can result in a two decades increase in the fatigue life.

Fractographic studies revealed that debond fracture was the dominant damage process. The final failure occurred uniquely in the material volume subjected to tension and was probably initiated by the coalescence of smaller cracks into a single dominant crack. A model based on the residual strength degradation, which explicitly accounts for the stress ratio is proposed in order to express analytically the strength variation during fatigue cycle evolution.

INTRODUCTION

In facing the fatigue behaviour of glass-mat-reinforced plastics, the majority of research efforts have been spent on uniaxial loading conditions /1-3/. This choice is justified when the failure phenomena taking place during fatigue are under study, because it results in a simple, uniform stress field in the sample volume. However, it must also be recognised that one of the most ambitious goals of glass-mat-reinforced plastics is to replace metals in automotive parts, such as car and truck bodies, where generally flexural loading, rather than simple tension or compression, is concerned.

Flexural fatigue tests have the benefits of being simple to perform and of providing data that are directly applicable to many engineering cases. The disadvantages are potential damage to the material surface corresponding to the load application points, a non-uniform stress field within the control volume, and a change in the position of the neutral axis during fatigue damage evolution. However, such “disadvantages” are inherent to the service environment as well, so that they motivate further study into this loading condition.

In previous works, Stupak et al. /4/ and D’Amore et al. /5/ characterised the flexural fatigue behaviour of continuous glass-mat-reinforced thermoset-based composites. Two specimen geometries were used, i.e., rectangular beams in four-point bending, resulting in a quasi-uniform uniaxial bending stress state, and simply supported, concentrically loaded circular plates, giving rise to a quasi-uniform biaxial bending stress state. The tests were carried out adopting different stress ratios, R, i.e., different ratios of the minimum (σ_{min}) to the maximum (σ_{max}) applied stress. The increased severity of loading in biaxial tension resulted in a more rapid decline in fatigue lifetime for the circular plate compared with the four-point bending geometry. Moreover, fatigue life decreased sharply with decreasing stress ratio for both circular plate and beam. The latter behaviour was interpreted in terms of mean stress (σ_{mean}) and stress amplitude Δσ. It was found that specimen lifetime increases with a) decreasing stress amplitude for a fixed mean stress, and b) decreasing mean stress for an assigned stress amplitude. It was shown that all
the fatigue data can be reduced to a single master curve, provided the product \( (\sigma_{\text{max}} \cdot \Delta \sigma) \) is plotted against the number of cycles to failure, \( N \).

Zhou et al. /6,7/ studied the flexural fatigue behaviour of an injection-moulded glass-fibre-reinforced blend of polyphenylene ether ketone and polypropylene sulfide using four-point bending with different stress ratios and different frequencies. Examination of failure surfaces for various loading conditions showed that the fatigue failure mechanisms appear to be matrix yielding at high stresses and crack growth at low stresses. Analysis of the fatigue results at various stress ratios revealed that the data at low stress levels actually superimpose to form a single curve when \( (\sigma_{\text{max}} \cdot \Delta \sigma) \) is plotted against \( N \), as suggested in /5/. In addition, it was found that the \( (\sigma_{\text{max}} \cdot \Delta \sigma) \cdot N \) trend is nearly linear on a log-log plot. However, for sufficiently high stress levels, lifetime was apparently affected by the stress amplitude alone.

It must be recognised that the nature of the method adopted in /5-7/ is essentially empirical, because it relies on experimental observations only. Therefore, no information is gathered on the causes of fatigue damage development. In this work, a two-parameter model is presented, explicitly accounting for the effect of the fatigue parameters on the fatigue life of glass mat-reinforced plastics. A power law, formally identical to the one proposed in /8/ to express analytically the change in material stiffness with fatigue cycle evolution, is assumed to describe the strength decrease with increasing number of cycles. Supposing that the strength degradation is uniquely affected by stress amplitude, a correlation is found between the fatigue life and the stress ratio. An easy method to evaluate the unknown constants appearing in the model is presented. The experimental data, concerning continuous strand-mat reinforced plastics fatigued in four-point bending using different stress ratios, show good agreement with theoretical predictions. Some possible limitations of the model, only taking into consideration mechanical damage, are highlighted. It is suggested that the model could be inefficient when creep or static fatigue of reinforcement play a prominent role in determining fatigue behaviour.

**ANALYSIS**

In /8/, Wang and co-workers studied the problem of fatigue damage evolution and associated elastic property degradation in a random short-fibre SMC-R50 composite. The experimental tests were conducted in tension, adopting a stress ratio \( R = 0 \) and a maximum stress \( \sigma_{\text{max}} = 60\% \) of the composite static strength. Microscopic observation showed the development of microcracks uniformly dispersed within the entire material volume, consisting of matrix cracking, fibre-matrix interface debonding, and fibre-end cracking. It was noted that the microcrack density increased continuously during fatigue, and microcracks developed with preferred orientations relative to the cyclic loading direction. As a consequence, a damage-induced anisotropy in elastic properties degradation was found. In order to describe quantitatively the change in stiffness of the composite as a function of fatigue cycles \( n \), a damage parameter \( D \), indicating the relative change in material stiffness, was introduced. Based on the results of a statistical model for microcrack development, \( D \) was assumed to be dependent on \( n \) through the following power-law relationship:

\[
\frac{dD}{dn} = A \cdot n^{-B}
\]

where \( A, B \) are constants to be experimentally determined. The results reported in /8/, referring to the Young's modulus degradation during fatigue, were in excellent agreement with theoretical predictions.

In this work, a residual strength degradation model is presented to predict the fatigue life of a composite material. An equation, explicitly accounting for the stress ratio \( R \), is proposed in order to express analytically the strength variation during fatigue cycle evolution. Then the critical number of cycles for catastrophic failure is calculated, equaling the maximum applied stress during fatigue and the material residual strength.

It is assumed that the decrease in material strength as a function of the number of cycles is described by a power law formally identical to eq. (1):

\[
\frac{d\sigma}{dn} = -a \cdot n^{-b}
\]
where \( \sigma_0 \) is the residual material strength after \( n \) cycles, and \( a, b \) are positive definite constants.

To account explicitly for the well-known effect of the stress level on the fatigue behaviour, it must be supposed that at least one of the two constants appearing in eq. (2) is dependent on the stress level. In this work, the simple hypothesis is made that the constant “\( b \)” is only dependent on the material type and load conditions, whereas “\( a \)” linearly increases with stress amplitude:

\[
a = a_0 \cdot \Delta \sigma \quad (3)
\]

where \( a_0 \), similarly to \( b \), is a constant for given material and load conditions, and the stress amplitude \( \Delta \sigma \) is defined as:

\[
\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}} \quad (4)
\]

In eq. (4), \( \sigma_{\text{max}} \), \( \sigma_{\text{min}} \) are the maximum and minimum stress during fatigue cycling, respectively.

Substituting eq. (3) in eq. (2) yields:

\[
\frac{d\sigma_n}{dn} = -a_0 \cdot \Delta \sigma \cdot n^{-b} \quad (5)
\]

Integrating eq. (5), and introducing the stress ratio \( R = \sigma_{\text{max}}/\sigma_{\text{max}} \), the following equation is obtained:

\[
\sigma_n = -a_0 \cdot \sigma_{\text{max}} \cdot (1-R) \frac{n^{1-b}}{1-b} + \sigma_o \quad (6)
\]

Indicating by \( \sigma_o \) the strength of the virgin material, the constant in eq. (6) is obtained by the boundary condition \( n = 1 \rightarrow \sigma_n = \sigma_o \). Rearranging eq. (6), the following equation is easily achieved:

\[
\sigma_o - \sigma_n = \alpha \cdot \sigma_{\text{max}} \cdot (1-R) \cdot (n^\beta - 1) \quad (7)
\]

with

\[
\alpha = \frac{a_o}{1-b} \quad (8')
\]

\[
\beta = 1-b \quad (8'')
\]

According to eq. (7), the evolution of strength degradation with fatigue cycling can be calculated, provided the constants \( \alpha, \beta \), only dependent on the material and loading conditions, are known. Of course, it is natural to think that failure will happen when the residual material strength equals the maximum applied stress during fatigue, so that the critical number of cycles for failure, \( N \), can be calculated putting \( \sigma_n = \sigma_{\text{max}} \) in eq. (7). Solving for \( N \), we obtain:

\[
N = \left[ 1 + \frac{1}{\alpha \cdot (1-R) \cdot (\sigma_{\text{max}}/\sigma_o - 1)} \right]^{1/\beta} \quad (9)
\]

From eq. (9), it is important to observe that, although the strength degradation is assumed to be uniquely governed by the stress amplitude, the number of cycles to failure is affected separately by the stress ratio and the maximum applied stress.

A useful form for eq. (9) is the following:

\[
\left( \frac{\sigma_o}{\sigma_{\text{max}}} - 1 \right) \cdot \frac{1}{1-R} = \alpha \cdot (N^\beta - 1) \quad (10)
\]

from which it is seen that, when the quantity on the left side of eq. (10) is plotted against \( N \), all the fatigue data should converge to a single curve, irrespective of the stress ratio adopted.

MATERIALS AND EXPERIMENTAL PROCEDURES

Square plates of glass-fibre-reinforced plastics (GFRP), approximately 200 mm in side and 4 mm in thickness, were fabricated by reaction injection moulding. The reinforcing fibres were in the form of a continuous strand mat; the resin was a polyester/polyurethane interpenetrated network. The measured fibre content by volume was \( V_f = 35\% \).

From the plates, rectangular specimens, 80 mm in length and 20 mm in width, were cut by a diamond saw. The samples were subjected to fatigue testing in four-point bending at room temperature, adopting an outer span \( l = 66 \) mm, an inner span \( a = 22 \) mm, and a sinusoidal waveform. The testing machine was an Instron 8501 servo-hydraulic system in load control. To avoid heating of the material during fatigue, particularly low frequencies, in the range \( f = 0.8 - 2 \) Hz, were used. In particular, the actual frequency was the higher the lower the maximum stress level, to maintain an approximately constant stress rate. Four stress ratios, namely \( R = 0.1, 0.3, 0.5, 0.7 \), were used. Each fatigue test was stopped at complete failure of the
specimen. The stress levels during fatigue were selected in the range 0.4 - 0.9 of the static flexural strength of the material. The latter was measured as the mean value of six tests, carried out in displacement control under the same load conditions specified for fatigue tests, at a crosshead speed of 100 mm/min. Such a high crosshead speed was chosen to provide a stress rate similar to that accomplished in fatigue. In all, thirty samples were tested.

After the tests, the external surfaces of all the specimens were observed at 32× magnification by optical microscopy to determine failure modes.

RESULTS AND DISCUSSION

In Fig. 1, a typical load-displacement curve recorded during the static tests of GFRP is shown. By the well-known strength of materials formulae, the flexural strength \( \sigma_0 = 290 \) MPa was calculated. The material behaviour is clearly linear up to about 75% of ultimate failure, but progressively departs from linearity beyond this limit, probably due to the initiation of microcracking. It is interesting to recall that in /1/ some experimental results were discussed, concerning the static tensile behaviour of SMC having various fibre contents. It was shown that a knee is clearly visible in the \( \sigma-e \) curve of SMC, corresponding to the initiation of fibre/matrix debonding and matrix fracture. The ratio of stress for knee occurrence, \( \sigma_k \), to ultimate strength, \( \sigma_u \), was affected by the fibre content, decreasing with increasing the latter. In particular, for \( V_f = 35\% \), a value \( \sigma_k/\sigma_u \approx 0.4 \) was found. Comparing this value with that resulting from Fig. 1, it can be concluded that the GFRP examined in this work tends to undergo microcracking at ratios \( \sigma_k/\sigma_u \) considerably higher than a SMC. It is hard to say whether the load condition (flexure against tension), form of reinforcement (continuous against chopped), or matrix type (interpenetrating network against pure thermosetting) plays the most important role in determining this behaviour.

Since the stress ratio \( R \) adopted in this work was always positive, the lower surface of the specimens loaded in four-point bending was uniquely subjected to tension, whereas the upper surface was stressed in compression. The microscopic analysis showed that, in general, no cracks were present on the upper surface, where only occasionally buckling failures were revealed. Apparently, buckling was provoked by insufficient support offered to reinforcing fibres by the matrix, where high fibre density was locally reached within strands located near the surface of the specimens.

On the lower surface, subjected to tension, damage mainly consisted of cracks due to fibre/matrix debonding, uniformly distributed within the inner span of the beam. The cracks were preferentially generated where strands oriented at high angles with respect to the stress direction were present, and nearly always followed the strand direction.

The final collapse was due to a dominant crack, extending along the entire sample width and approximately orthogonal to the stress direction. The main crack, starting from the tensile surface, propagated through the thickness fracturing the fibre strands encountered (Fig. 2). Near the neutral axis of the specimen, the dominant crack deviated, resulting in extensive delamination (Fig. 3). A higher density of superficial cracks due to fibre/matrix debonding was noted in the proximity of the main crack. Probably, the final failure was initiated by coalescence of smaller cracks, and subsequent fast growth after a critical dimension was reached. In fact, the fast propagation rate is inferred from the sudden increase in compliance, recorded just before sample failure.
Fig. 2: Dominant crack leading to final collapse on the lower surface of the specimens.

Fig. 3: Deviation of the dominant crack to form extensive delamination. Bottom: surface subjected to tension during flexural fatigue.
The observed failure modes are substantially in agreement with those reported by other researchers /1, 9,10/. They support the hypothesis that the damage mechanisms during fatigue are correlated to matrix and interface properties. The only characteristic feature revealed here is the presence of extensive fibre failure after final collapse, which has been rarely evidenced in previous works /11, 12/. Probably this depends on the fact that, in this case, continuous fibres were used instead of short fibres. An important conclusion from the previous observations is that collapse was uniquely provoked by phenomena occurring in the material volume subjected to tension. No contribution to failure of the compressed part of the specimen was noted in fatigue or in static tests. This indicates that the \( \sigma_e \) value calculated from the static tests is a correct reference value for fatigue data evaluation.

The results of the experimental program are reported in Fig. 4, where the maximum non-dimensional applied stress, \( \sigma_{\text{max}}/\sigma_n \), is plotted against the critical number of cycles, \( N \), on a semi-log scale. It is seen that the stress ratio has a strong effect on the fatigue life: for a given maximum stress, the higher \( R \), the higher the number of cycles to failure. Passing from \( R = 0.1 \) to \( R = 0.7 \) can result in a two-decade increase in fatigue life. This highlights the necessity to model the stress ratio effect correctly, if an efficient description of fatigue behaviour is desired. Similar results have been reported by other researchers /5,13,14/.

As previously noted, a powerful tool to verify the validity of the proposed model is provided by eq. (10): according to this relationship, all the fatigue points should converge to a single curve, if the term on the left side is plotted against \( N \). This procedure was applied in the semi-log plot shown in Fig. 5, where \( K \) indicates the quantity \( (\sigma_o/\sigma_{\text{max}}-1)/(1-R) \). As predicted by theory, all the experimental points sensibly fall on a single curve, irrespective of the actual \( R \) value.

In order to find the values of the constants \( \alpha, \beta \) appearing in eq. (9), the following method was adopted.

From eq. (10), plotting \( K \) against \( (N^p-1) \) should result in a straight line of slope \( \alpha \), passing through the origin. The experimental data concerning \( R = 0.7 \) were plotted on a \( K - (N^p-1) \) diagram, using a trial value for \( \beta \), and were fitted by a straight line, using least square fit. Then \( \beta \) was varied until the best fit straight line passed through the origin. The results are shown in Fig. 6, where \( \beta = 0.200 \) was adopted. From the slope of the straight line (continuous line in the Figure), the value \( \alpha = 0.184 \) was calculated. It can be noted that the experimental points follow a linear trend with sufficient accuracy, adding to confidence in the proposed model.

Substituting in eqs. (9), (10) the values of \( \alpha, \beta \) previously found, the continuous lines reported in Figs.
4 and 5, representing the predictions of the theoretical model, were obtained. Looking at Fig. 4, it is seen that eq. (9) is sufficiently accurate in describing the effect of the stress ratio.

Since the approach followed here is based on the assumption of a strength degradation trend analytically expressed by eq. (7), a more direct method to assess the present model could be based on the measurement of the residual strength after a predetermined number of cycles, n. According to eq. (7), the strength degradation for a fixed n should uniquely depend on the stress amplitude \( \Delta \sigma = \sigma_{\text{max}} (1-R) \).

Eq. (7) was used, in conjunction with the values of the constants \( \alpha, \beta \) previously evaluated, to predict the residual strength decay as a function of the number of cycles for different stress amplitudes (Fig. 7). Of course, the strength decreases more rapidly for higher stress amplitudes. Unfortunately, for the time being no experimental data are available to support the theoretical trends reported in Fig. 7. However, if this would happen, it should be concluded that the material behaviour is markedly different from that of classical laminates, where a sudden drop in residual strength is observed only during the last cycles of fatigue life /15-18/.

From eq. (9), the critical number of cycles N becomes infinite when R = 1 (static fatigue). This prediction is apparently in contrast with experimental evidence: it has been demonstrated that a GFRP suffers a strength reduction quoted as about 5% per decade of time under static fatigue /1/. The discrepancy between the present theory and real behaviour depends on the fact that the model adopted here only accounts for the damage produced by the stress variation. Some interesting conclusions can be drawn assuming that the two damage modes (due to stress amplitude and static fatigue, respectively) develop independently from each other, so that the actual fatigue curve can be simply found by superposing the curves for the two phenomena. This procedure is graphically illustrated in Fig. 7, where the dashed straight line represents the static fatigue curve. The latter was drawn assuming a 5% life reduction per decade of time, and a frequency f = 2 Hz. It is seen that, when a sufficiently high stress amplitude is adopted, fatigue failure is dominated by \( \Delta \sigma \). On the contrary, for quite low stress amplitudes, static fatigue predominates in reducing residual strength at low cycles, so that it determines fatigue failure when high maximum stress values are adopted.

Finally, it must be recognised that creep phenomena are not accounted for in the proposed model. Therefore, eq. (9) is expected to hold only for materials where creep does not play a major role in precipitating fatigue failure. Although at this time no experimental data are available to support this statement, this limitation should render questionable the use of eq. (9) for
unidirectional laminae loaded off-axis, as well as for particulate and thermoplastic matrix composites.

CONCLUSIONS

A two-parameter model, accounting for the effect of stress ratio on the fatigue life of glass-mat-reinforced plastics, has been presented. The model relies on the assumptions that the strength decrease with increasing number of cycles only depends on the stress amplitude, and that the final collapse happens when the maximum applied stress equals the material strength. These hypotheses lead to the conclusion that the stress ratio governs fatigue behaviour.

The experimental results, carried out on beam specimens, made of continuous strand-glass mat embedded in a polyester/polyurethane interpenetrated network and loaded in four-point bending, show good agreement with theoretical predictions. In particular, as predicted by theory, all the data actually converge to a single master curve, irrespective of stress ratio, provided a suitable stress parameter is adopted.

From the analysis of the failure modes, the final failure was due to the slow growth of small cracks evenly distributed in the material volume subjected to tension. Only in a very late stage of the fatigue life, these cracks coalesced to form a dominant crack, rapidly leading to collapse. No clear evidence of changes in failure mechanisms was observed when going from low to high stress amplitudes. However, it is suggested that, when low stress amplitudes are adopted, the fatigue life could be precipitated by static fatigue of glass fibres, or creep in the matrix. In this case, the proposed model, based on the hypothesis of mechanical damage growing within the material, could be unable to predict fatigue life.

REFERENCES