NEW PERSPECTIVE OF FRACTURE MECHANICS INSPIRED BY NOVEL TEST WITH CRACK-PARALLEL COMPRESSION

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BASED ON: PNAS—IN PRESS (JUNE). EXTENSION: JAM (JULY 2020)
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Standard Fracture Test Specimens — negligible crack-parallel normal stress

- 3PB
- CT
- WS
- DCB
- SENT
- CNT
- DEN-EC
Why has the crack-parallel stress $\sigma_{xx}$ been ignored?

- All the standard notched fracture specimens have $\sigma_{xx} \approx 0$
- Generally considered: line cracks, as in
  - linear elastic fracture mechanics, LEFM (Griffith 1921)
  - cohesive crack model, CCM (Barenblatt 1959)

- A line cut in $x$-direction in a field of homogeneous uniaxial stress $\sigma_{xx}$ causes no stress change in a continuum model

What matters? Fracture process, not the visible final crack
Quasibrittle Materials

— brittle constituents, but *inhomogenity size* and the RVE or FPZ are not $\ll$ structure size $D$.

**Concrete** (archetypical, 1970s) • tough ceramics
- fiber composites • rocks • bones • sea ice
- rigid foams • dental cements • dentine
- cartilage • wood • consolidated snow
- particle board • paper • carton • cast iron
- thin films • carbon nanotubes • cemented sand
- printed materials • fiber-reinforced concrete
- cold asphalt concrete • mortars • masonry
- stiff clay • silt • grouted soil • refractories
- coal • oil and gas shales • various printed or architectured materials
- nacre • biological shells • and all brittle materials on micro- and nano-scales.

At increasing size $D$, they all transition from ductile to brittle.

All: non-negligible material characteristic length.
Fracture Process Zone (FPZ) Size

10 km – Arctic Ocean ice cover as a 2D heterogenous medium consisting of thick floes of approx. 3 km size, embedded in thin ice matrix

5 m – Sea ice pushing on oil platform legs

0.5 m – Normal concrete

2 cm – Textile composites

2 mm – Gas or oil shale

(10^{-6} m – MEMS, polysilicone, embrittled metals—We omit)

– Quasibrittleness is a relative concept
Brittle Fracture Mechanics Founding in 1921 and Its Evolution into Ductile, Cohesive and Quasibrittle

A A Griffith 1921

Nonlinear fracture mechanics

FPZ is long, wide, softening and tensorial

Brittle

Ductile

Damage mechanics

Brittle (dynamic)

ε or w

FPZ = a point

also a point

large

Brittle or cohesive
New finding: Effect of crack parallel stress $\sigma_{xx}$ on fracture energy $G_f$ of concrete is strong.

Data from regression of $3 \times 9 = 27$ tests.

M7 microplane prediction is calibrated only by $f_c$ and $G_{f,0}$.
I. Gap Test: New, yet simple and unambiguous, test of fracture at crack-parallel compression
Basic Idea of the Gap Test
– Achieve superposition in sequence (in statically determinate way)

1) \( \sigma_{xx} \) \( K_I = 0 \)

2) \( \sigma_{xx} \) \( K_I > 0 \)

\[\begin{align*}
\text{same } \sigma_{xx} & \text{ same } K_I \\
\end{align*}\]

And keep \( \sigma_{xx} \) constant as \( K_I \) is applied
Novel Experiment: GAP TEST

Four advantageous key features:

1. At crack mouth, **plastic pads** to apply crack-parallel compression \( C \).

2. A **gap** above end supports gets in contact only **after** the pads **yield**, to apply bending moment.

3. This way the test beam passes from one **statically determinate** system to another → simple and **unambiguous** evaluation.

4. The static determinacy and \( C \) constancy enables the **size effect method** – a robust and unambiguous way to measure fracture energy \( G_f \).

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Typical Measured Load Evolution

Test Setup

DIC used to verify strain field for calibration

$\sigma_{\text{pad}}$ (MPa)

0 10 20 30 40 50 60

0 0.05 0.1 0.15

Nominal strain

0.39% rise

0.45% rise

Thickness b

Steel

no lateral slip

Extensometer

Plastic block (polypropylene)

Gap

2L

F

σ

pad (MPa)
Test Setup in More Detail

Plastic pads (polypropylene)

$F, \delta$

Extensometer

Steel

Gap

Pads

$a$
Distinguish:

\( \sigma_{pad} = \) normal stress under the plastic pad, as measured

\( \sigma_{xx} = \) inferred (by FE or DIC) crack-parallel stress at the notch tip FPZ, which is what matters for damage constitutive model.

Typically: \( \frac{\sigma_{xx}}{\sigma_{pad}} \approx 0.96 \)

generally \( \in (0.94, 0.97) \)
Gap Test for Tension

**STAGE 1**

\[ F = -2T \]

Gap test for tension with a steel strap glued and pads yielding in tension.

**STAGE 2**

\[ F = 2R - 2T \]

Again statically determinate with the gap closed and pads yielding in tension.

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Optimum Design of Polypropylene Pads

\[ H = \frac{\mu}{4} \frac{l^3 L}{h^3} = \min \quad \text{(strip)} \]

\[ H = \frac{3\mu}{8\pi} \frac{A^2}{h^3} = \min \quad \text{(circle)} \]

\( \mu, H \) = tangential stiffnesses of polypropylene and of pads

\( h, L, A \) = height, strip width and circle area of pads

Possible alternative to plastic pad: Tin (St)

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
II. 
Size effect method — essential part of gap test to measure material fracture energy $G_f$

Standard RILEM Recommendation, 1990. ACI-446 endorsed it and recommended it to ASTM C-09
For quasibrittle materials, distinguish the initial and total fracture energy, $G_f$ and $G_F$.

We consider only $G_f$.

$G_F$ is to be calculated from the ratio $G_F/G_f$ determined separately.
For Size Effect Test Method of $G_f$:
Specimens Geometrically Scaled (as 1 : 2 : 4)

Depths:
$D = 4, 8, 16$ in.

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Required 1\textsuperscript{st} and 2\textsuperscript{nd} Order Small- and Large-Size Asymptotic Properties of Type 2 Size Effect on Fracture

For $D \to 0$:

\[ \sigma_N \propto 1 - \frac{D}{D_s} - \ldots \]

For $D \to \infty$:

\[ \sigma_N \propto \frac{1}{\sqrt{D}} \left( 1 - \frac{D_0}{2D} + \ldots \right) \]

Asymptotic Matching yields the Size Effect Law (SEL)


\[ \sigma_N = \frac{\sigma_0}{\sqrt{1 + \frac{D}{D_0}}} \]

In detail: ZP Bažant (2004), PNAS, p. 13400
Size Effect Method to Measure Fracture Energy $G_f$ and Material Characteristic Length $c_f$

Transformation to linear regression:

$$Y = AX + C, \quad \text{with} \quad X = D, Y = \frac{E'}{g_0 \sigma_N^2}$$

$$G_f = \frac{1}{A}, \quad c_f = \frac{g_0}{g_0'} C$$

$$\sigma_N = B f_t' \left(1 + \frac{D}{D_0}\right)^{-1/2}$$

$$= \sqrt{\frac{E'G_f}{g'(\alpha_0)c_f + g(\alpha_0)D}}$$

$g(\alpha) = \text{dimensionless energy release function of LEFM}$

RILEM Standard Recommendation 1990
ACI-446 Recommendation

Size effect regression of 9 Gap Tests to get Gf for, e.g., the medium crack-parallel compression (the means and the scatter width)

Note the closeness of fit

H. Nguyen, Z.P. Bažant, JAM, in press
Classical Work-of-Fracture Method of Measuring Total Fracture Energy $G_F$

Why was it not used?

1. J-integral varies with crack length $x$
2. Ambiguity of tail—ratios $G_F/G_f, f_t/f_1$ matter but vary
3. Tail hard to measure; stable postpeak required

Nakayama 1965
Hillerborg 1976
III. Gap Test Results and Mesoscale Physical Mechanism
Found: Effect of crack parallel stress $\sigma_{xx}$ on fracture energy $G_f$ of concrete is strong.

\[ \frac{G_f}{G_{f,0}} \]

3 data points – based on regression of $3 \times 9 = 27$ experiments. M7 microplane prediction is calibrated by only $f_c$ and $G_{f,0}$. 

\[ \frac{\sigma_{xx}}{\sigma_{xx,0}} \approx 0.96 \]

LEFM, CCM, basic XFEM

M7 + crack band model

Tensile strength

Adjustment: $\frac{\sigma_{xx}}{\sigma_{pad}}$

Strengthening $G_f$

Weakening $G_f$

9 tests

σ measured

σ pad

9 tests

9 tests

σ pad measured
Material characteristic length $c_f$ dependence on crack-parallel stress $\sigma_{xx}$

Data from regression of $3 \times 9 = 27$ tests

Note: Approx. $c_f = 0.4$ FPZ length

Meso-Scale Mechanisms of $\sigma_{xx}$ Effect on $G_f$

Regimes of:

1) strengthening $G_f$
\[0 < \sigma_{xx} < 0.75\sigma_c\]

2) weakening $G_f$
\[0.75\sigma_c < \sigma_{xx} < \sigma_c\]

Static friction without slip enhanced by compression

Transverse widening of FPZ caused by inclined slips and splitting
IV.
FE Extrapolations and Predictions for Fiber Concrete and Shale
The crack band model M7 with its default parameters was scaled to fit only:
1) uniaxial compression strength of the concrete, and
2) the measured basic fracture energy (at $\sigma_{xx} = 0$).

The predictions for the 2\textsuperscript{nd} and 3\textsuperscript{rd} data points were acceptable (10% error) and excellent (1% error).

Overall $7\%$ error (which is normal for concrete)

So it makes sense to use microplane model M7 to make predictions for other situations and materials.
Effect of combined **in-plane** and **out-of-plane** stresses $\sigma_{xx}$ and $\sigma_{zz}$ on $G_f$ and $c_f$

Calculated for Gap Test Geometry

Fracture Energy

Characteristic Length

$\sigma_{xx} / \sigma_c$

**crack–parallel stress**

Note the non-monotonic effect of $\sigma_{zz}$

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Comparisons with classical tensorial strength or damage models based on invariants

\[ G_f / G_{f,0} \]

CDPM2 concrete damage model (Grassl, Abaqus)

Drucker-Prager \( I_1 \) and \( J_2 \) criterion (Abaqus)

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Fiber Reinforced Concrete (FRC) with 3% Dramix Steel Fibers vs. Plain Concrete

Fracture Energy

\[ G_{f,0} = 143.1 \text{ N/m} \]

\[ G_{f,0} = 86.7 \text{ N/m} \]

Characteristic Length

\[ c_{f,0} = 11.2 \text{ mm} \]

Crack-parallel compression \( \sigma_{xx} / \sigma_{xx,c} \)

Calculated with M7 for concrete and M7f for FRC

– tests yet to be made!

Gap Test for Gas Shale Predicted with Spherocylindrical Anisotropic Microplane Model

Fracture Energy

Characteristic Length

Crack–parallel compression $\sigma_{xx} / \sigma_{xx,c}$

*Li, Cunbao,...Bažant, JMPS 103, 155-178 (2017)

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press

NOTE: After the Virus, collaborative tests will resume with LANL (Luke Fray, Hari Viswanathan, Bill Carey and Esteban Rougier)
Strong effect of history (or path) of loading, calculated for concrete

\[ \sigma_N \propto K_I \]

First \( \sigma_N^* \)

1.81\( \sigma_N^* \)

Reversed: first

0.62 \( \sigma_N^* \)

Hence, no universal formula for \( G_f \) as function of \( \sigma_{xx} \) exists!

\[ G_f \text{ strengthening} \]

\[ G_f \text{ weakening} \]

Gap Test

First \( \sigma_N \) then \( \sigma_{xx} \)

First \( \sigma_{xx} \) then \( \sigma_N \)

Proportional

H. Nguyen,.... Z.P. Bažant, PNAS, in press; JAM July 2020
V. Alternative Tests Considered and Retrospective
Alternative setup – proportional loading

Problem: \( \sigma_{xx} \) is not constant

Evaluating \( G_f \) would require optimal FE fitting with crack-band damage model
Tschegg’s 1995 wedge-splitting test with crack-parallel compression

— pioneering idea!* Tests showed some effect but were ambiguous because:

- work-of-fracture method was used
- no size effect was tested
- non-uniform stress field
- bending and friction from weight of hydraulic jacks

*Ignored by fracture community due to lack of simplicity and high scatter
Retrospective

• Elasto-plastic metals: Stress triaxiality in an annulus around crack tip, led to extra parameter $Q$ accounting for compressive $T$-stress ($= \sigma_{xx}$), and an increase of $J$-integral based on HRR field for ductile fracture (Shih, Hancock, Tvergaard,… 1990s).

• Crack path deflection due to $T$-stress in LEFM was solved by Cotterell and Rice (1980)

*But both problems are different.*
VI.

New Perspective of Fracture Mechanics of Quasibrittle Materials
Can the existing FE programs for LEFM, CCM, XFEM and phase-field model be used?  

Hardly, because:

— **Path dependence of damage** is even stronger than it is in plasticity

— **Different** formulae would be needed for **different** materials, stress ratios $\sigma_{zz}/\sigma_{xx}$, plane strain, ...
• To adjust $G_f$ of the cohesive crack model is **ambiguous** since many different combinations of tensile strength $f_t$ and softening slope are possible, and adjusting ratio $G_F/G_f$ is unclear.

• Expected: A strong $\sigma_{xx}$ effect in **anisotropic fiber-polymer composites** – e.g., a pressurized **fuselage** is under high **biaxial tension**.

• Delamination fracture is sure to depend on $\sigma_{xx}, \sigma_{zz}$.
Paramount Problem: **Realistic Tensorial Damage Model**

for concrete, fit 22 different benchmark triaxial tests (microplane model does)*

- uni-, bi-, tri-axial
- splitting strength
- proportional or nonproportional
- post-peak softening at FPZ scale

- **Plus Vertex Effect:**
  - Tangential stiffness at start of torsion vs. axial strain of compressed cylinder rotating principal stress axes

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<th>Tangential Shear Stiffness</th>
<th>Tensorial invariant-based models</th>
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<td>Experiment: Caner-Bažant 2002</td>
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Initial Compressive Strain

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<th>Compressive Strain at Start of Torsion</th>
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**FC Caner, ZP Bažant (2013), JEM, p. 1714**

* FC Caner, ZP Bažant, J Cervenka, J. (2002). JEM, 24-33; ZP Bažant (!982), J Eng Mech ASCE 109, 849--865

**FC Caner, ZP Bažant, J Cervenka, J. (2002). JEM, 24-33; ZP Bažant (!982), J Eng Mech ASCE 109, 849--865**
A hundred different damage constitutive models for concrete exist!

- All can work for one crack:
  - All can fit Mode I test of $F(u)$ softening curve, using:
    - scalar softening $\sigma = f(w)$
    - $I_1^\varepsilon$ volumetric softening (Mazars)
    - $J_2^\varepsilon$ deviatoric softening (~von Mises)
    - etc.

But also should fit:

- basic material test data and benchmark triaxial tests (at approx. FPZ size)
- multiple interacting cracks
- gap tests
Both the tensorial nature of the FPZ and the minimum possible spacing of parallel cracks can be effectively captured by the crack-band model.

Eigenstrain $\varepsilon^0$

Localizing Not
Effects of $\sigma_{xx}$, $\sigma_{zz}$ in fracking of shale – probably great

3 km deep, vertical crack faces are under high tectonic and overburden crack-parallel stresses $\sigma_{xx}$, $\sigma_{zz}$, combined by body forces applied by diffusion pressure gradients.

Simulated vertical hydraulic cracks in shale with preexisting weak layers if $G_f$ is constant — no localization!

(Rahimi,...Bažant, PNAS 2019, p. 1532)
Continue using Mohr failure envelope for large crack-parallel stress $\sigma_{xx}$? — in doubt

—widely used in geophysics and in fracking studies.

Envelopes from gap tests, extended by FE

$\sigma_I =$ transverse tensile stress in FPZ, via FE

$\sigma_{III} = \sigma_{xx} =$ crack-parallel compressive stress at FPZ

H. Nguyen,….. Z.P. Bažant, PNAS, in press
Gap Test proves that shear fracture of beams and slabs must be analyzed taking into account a large tensorial FPZ.

Despite 40 years of studies, CCM, LEFM never succeeded; $K_I \to 0$ at $V_{\text{max}}$. This was one experience that motivated the gap test.
Promise of Multiscale Approach

- **Tensorial damage band shrunken into a line** (Remmers, de Borst, Needleman, IJF 2013). Yields a cohesive crack with an embedded **subscale tensorial** damage band.

- But tensorial damage law remains a big question.

- Calibration by data fits not yet demonstrated.

- Conceptual problem: **Minimum crack spacing will not be enforced automatically.** Parallel cracks?
Promise of Phase-Field Model

– computationally enticing, alternative. But:

• Single parameter damage—inadequate.

• Needs a realistic tensorial model for softening damage, such as microplane, validated and calibrated by existing triaxial material tests of diverse types, esp. of FPZ size

• Transition from distributed damage to isolated bands? Minimum spacing of parallel cracks?
In 1980s, forces $F$ measured on legs of oil platforms in Prudhoe Bay were an order-of-magnitude less than FE predictions with buckling.

— **One (incomplete) explanation**: Size effect

— **Now 2nd explanation**: Crack-parallel compression in large FPZ reduced $G_f$ (or $K_{Ic}$) near 0.

In glaciers — $\sigma_{xx}$ in vertical cracks, depth $>1$ km.

12

Expected: Effect of $\sigma_{xx}$, $\sigma_{zz}$ on fracture of bone and biomaterials

Why? Observed size effect implies large FPZ

Hip fracture, tooth fracture, ... often occurs at high $\sigma_{xx}$

From size effect: Cohesive softening law of bone has a long tail

Bone tests by Bažant, Kim, Yu (IJF 2013)
Tests of PEEK at Northwestern (Bažant-Kim,..., IJF 1999)

- Kink-bands in fiber composites are a fracture propagation problem with strong size effect.
- $\sigma_{xx}$ effect — unknown but likely.

Kink band micro-buckling with shear fractures leads to size effect.
Large FPZ suggests:

effect of crack-parallel $\sigma_{xx}, \sigma_{zz}$ on subcritical crack growth

1) **Charles-Evans law** for static fatigue crack growth in rocks or ceramics, and

1) **Paris law** for cyclic fatigue crack growth in quasibrittle materials.

Bažant & Xu tests (ACI J 1991), for 3 specimen sizes

Size effect proves large FPZ

In concrete, compression changes a planar shear crack to an inclined conical shear crack.

Inexplicable if constant $G_f$ or $K_c$ govern.

Torsional shear tests

Bažant, Prat, Tabbara, ACI J. 1990, 12-19
• Seismically sliding geological faults are very thin (< 1 mm), but if the FPZ at front is not, then $\sigma_{xx}$, $\sigma_{zz}$ should matter for their propagation.

• Cracks growing in prestressed concrete have a different $G_f$.

• The $G_f$ variation is generally not expected in metals, fine-grained ceramics or polysilicon, since their FPZ is of micrometer scale. But it could matter for MEMS, or devices < $10^{-4}$ m.
Conclusions

• **Gap Test** – simple, unambiguous interpretation – one *statically determinate* system transits to another – size effect test of $G_f$ possible

• **FPZ** – Tensorial!
  Effective simple approach – FE crack band model.

• The **CCM and LEFM (and XFEM)** are approximations of rather limited applicability.

• LEFM and CCM remain *essential* for understanding and teaching fracture mechanics, and for providing accurate benchmark solutions of special cases which tensorial damage mechanics must match.

• The evidence from the gap test looks compelling but so far is *scant* and deserves broad scrutiny. *Extensive testing and calibration* will be needed – for most quasibrittle materials.
Afterthought

Many hot research subjects become closed in a few decades. But, like turbulence, fracture mechanics is different. This formidable subject has been researched for a century, and probably will for another century.

Thanks for listening!