Research note:
An outline of the problem of special relativity, and the proper solution to it

A research note by
Ajit R. Jadhav, Ph.D.

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Abstract

This document is preliminary in nature, and its main purpose is two-fold: (a) to claim originality and independence of thoughts for the ideas being presented here, and also (b) to claim priority for any new ideas here (if these are indeed found to be new).

The text here is written in a hurried manner, and is very terse, with the purpose being just to note the main points. This *is* a research *note*.

A formal paper covering the same ideas will be written in the near future (of a few months).

Contents

1 NM ontology i.e. Galilean transformations (GT)

For a brief overview of the various ontologies implicitly used in physics, please consult Ref. [1]. Very briefly, the term “NM ontology” means that ontology which is assumed in the Newtonian mechanics of rigid bodies and particles, and the term “ED ontology” (called “EM ontology” in Ref. [1]) for that in the classical electrodynamics.

1.1 Frame 1 (F1)

Experiment

Conduct a *physical* experiment E1 involving only the NM (uncharged and massive) bodies, using some *physical* frame F1.

A source-object on an experimentally observed trajectory $\vec{\eta}_1(t)$ emits an instantaneous pulse of sound (or shoots an NM particle as the force-carrier) at an instant t_E when the source-object is at the position $\vec{r}_E$.

The aforementioned influence hits a test-object at the position $\vec{r}_D$ at time t_D with some force.

This force results in changing the trajectory of the test-object, a displacement, that can be measured by measuring the position $\vec{r}_{D_2}$ of the test-object at a later instant t_{D_2}.

Experimentally observe t_D, $\vec{r}_D$, t_{D_2}, $\vec{r}_{D_2}$.

Calculations

The most fundamental law applicable here consists of Newton's 3 laws, notably,

$$\vec{f} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = m\vec{a} \,. \tag{1}$$

Translate the experimentally measured trajectory quantities $\vec{r}(t)$, $\dot{\vec{r}}(t)$, and $\ddot{\vec{r}}(t)$ the relevant instants, and also any other relevant quantities for both the source-object and test-object (e.g. their masses m_E and m_D), into the initial and final conditions of the mathematical problem (for both the steps: when the signal reaches the test-object to force it, and when the resulting displacement of the test-object is observed).

Input the initial conditions into the general solution of the law. Predict the final conditions.

The initial and final conditions together form the datum for making comparisons later on. Identify any physical objects in the near vicinity of the objects observed in the experiments too, for this purpose.

1.2 Frame 2 (F2)

Experiment

Select or set up a second *physical* frame that is inertially moving with respect to F1. *Experimentally* observe the experiment E1 from frame F2, thus getting experimentally measured values of the initial and final trajectories of both the source- and test-objects in F2.

Denote all the quantities experimentally measured in F2 via dashes (i.e. "primes"), e.g. $\vec{r}'$, t', etc.

Calculations

Surmise that the same law viz. eq. (1) holds even while using F2.

Regard the experimentally measured quantities (now measured in F2) as the initial and final conditions.

Input the initial conditions into the same general solution. Predict the final conditions.

1.3 Comparison of results obtained using F1 and F2

It is verified that the law holds as is in both the frames.

Note: The numerical values of the spatial coordinates are different in F2 vs F1, and so are displacments, velocities. However, accelerations are the same in both the frames, and also the mass. So the momenta are different but not the forces. Since all displacements are different, so are the displacements the test-object too. But if the test-object is experimentally observed to hit a third object at t_{D_2} in F1, then it it experimentally observed to hit the same object also in F2. Nothing surprising.

Noteworthy: The test-object is also calculated to hit the same third-object at t_{D_2} in F2, even if the separation vectors are different in F1 and F2. That's because the position vector of the third object too are experimentally observed to be different in F2.

Conclusions

The conclusion (stated somewhat vaguely, i.e. assuming the full context) is this:

The physical law eq. (1) holds as is in all (inertial) frames, when the NM ontological objects are used.

BTW, throughout this document, frames — whether physical or mathematical — are inertial.

2 ED Ontology – First attempt: First thought-experiment, with GT

The most fundamental law applicable here consists of Maxwell's four vector laws of the EM fields plus Lorentz' force law; *cf.* Appendix A of this document.

2.1 Frame 1 (F1)

Experiment

Conduct a *physical* experiment "Expt1" with the ED-ontological objects (i.e. massive and charged) bodies, using some *physical* frame F1.

A source-charge on a prescribed trajectory is supposed as producing some EM fields when it's at a position $\vec{r}_E$ and instant t_E. The influence from this space-time event is supposed as hitting a test-charge at position $\vec{r}_D$ at time t_D with some Lorentz' force. The Lorentz' force is supposed as changing the trajectory of the test-charge.

For simplicity, consider a small time interval Δt from t_E to $t_E + \Delta t$ in which the source-charge moves on the *prescribed* trajectory. The idea is to find how the source-charge displaces of the test-charge as a result of the "signals" generated over Δt.

Assume that the test-charge is held fixed at $\vec{r}_D$ until the instant t_D, and then released so that the continuously changing Lorentz' forces acting on it (due to continuously changing position of the source-charge at and after $\vec{r}_E$ at t_E) is the only force acting on it.

The net displacement of the test-charge can be calculated from the position $\vec{r}_{D_2}$ of the test-charge at the later instant t_{D_2} and the calculated Lorentz' forces $\vec{F}$ over the interval of its displacement, viz. t_D to t_{D_2}. Note: A single frame is involved and so, $t_{D_2} - t_D$ indeed equals Δt for the emitter.

Calculations

Input the initial conditions into the general solution of the law; *cf.* Appendix A. The solution involves calculations of the retarded times. In the general case, i.e., for arbitrary trajectories, determination of the retarded instants can be done only numerically (i.e. approximately), but it's OK; the errors are small; cf. PyCharge [4]. Predict the final conditions.

The physical law has already been verified a lot. It turns out that we could have conducted even just a *thought* experiment for Expt1, and the results would be found to be valid.

Conjecture

Let's go back to 1887. The physical law of ED is the most fundamental law known to man. Technology and industry is abuzz with ED (just like computers, internet, AI of our times).

So, of course, the physical law must hold in all frames, no? Voigt (1887) chooses to put the law through the rigours of calculations from the second frame.

2.2 Mathematical Frame 2 (F2)

Assume that the same physical law holds also for observations made from a second frame F2, and so, *theoretically* describe what happens in F2.

Accordingly, select or set up a second *mathematical* frame that is inertially moving with respect to F1. *Thought*-experimentally observe the experiment Expt1 from frame F2, thus getting quantitative values of the initial and final trajectories of both the source- and test-objects as they would be *calculated* in F2.

Of course, the trajectory functions in F2 aren't the same as those in F1 — this much is known from even the simpler NM-ontological experiments (covered in the preceding section).

Assumption

Assume that in going from F1 to F2, the trajectories change the same way as in the NM ontology, i.e., as per the Galilean transformations (GT).

The Galilean transformations (GT)

Eq. (2) gives the GT for the simple case that F2 moves with a frame-speed of v with respect to F1 along the positive x-direction, and assuming that F1 and F2 coincide at $t = 0$:

$$
\begin{aligned}
x' &= x - vt\,, && \text{(2a)}\\
y' &= y\,, && \text{(2b)}\\
z' &= z\,, && \text{(2c)}\\
t' &= t\,. && \text{(2d)}
\end{aligned}
$$

Calculations in F2

Take the trajectories and other quantities of Expt1 from F1, and perform Galilean transformations (GT) on them. Thus calculate what the initial and final conditions would be in F2, starting from those prescribed in F1.

Input the initial conditions into the same general solution. Use the general solution of ED to predict the final conditions.

2.3 Comparison of results obtained using F1 and F2

Surprise!

The predicted final conditions say that (a) the Lorentz force on the test-charge in F2 isn't the same as that in F1, and (b) by the F2 calculations, the test-charge doesn't hit the *third* physical object!

Conclusions

Some thing is wrong!

3 ED Ontology – Second attempt: Second thought-experiment, now with LT

Conjecture

Suspect that GT is the culprit; spend 17 years (not hours) of intense effort to find some other coordinate transformations that would work for the ED system.

Historical tid-bits

The effort to find the correct transformations involves many noted physicists starting with Voigt (1887), Larmour (1892), and most notably, starting 1892, also Lorentz. The effort is also joined by one of the two (or few) greatest mathematicians of those times, viz. Poincaré (and not just by the equivalents of the JEE toppers and/or IMO and IPhO gold/silver/bronze medalists).

In particular: The physicist Lorentz proposes (1895) "local time;" the mathematical physicist and mathematician Poincaré proposes his "relativity principle;" the physicist Lorentz finally arrives at the correct transformation equations that are valid to all orders in v/c, using the idea of a hypothetical aether acting as a universal frame of reference (1904); the mathematical physicist and mathematician Poincaré re-derives Lorentz' transformation equations using the principle of least action, and calls them, for the first time, "the Lorentz transformations" (09 June 1905). For more details, *cf.* the unusually excellent Wiki article [5] and also [6].

The Lorentz transformations (LT)

Set up a second *mathematical* frame that is inertially moving with respect to F1. *Thought*-experimentally observe the experiment E1 from frame F2. The LT give the new space- *and* time-coordinates in F2.

Eq. (3) gives the LT for the simple case that F2 moves with a frame-speed of v with respect to F1 along the positive x-direction, and assuming that F1 and F2 coincide at $t = 0$:

$$
\begin{aligned}
x'' &= \gamma\,(\,x - vt\,)\ , && \text{(3a)}\\
y'' &= y\ , && \text{(3b)}\\
z'' &= z\ , && \text{(3c)}\\
t'' &= \gamma\left(\,t - \frac{vx}{c^2}\,\right)\ , && \text{(3d)}
\end{aligned}
$$

where

$$
\begin{aligned}
\beta &\stackrel{\text{def}}{=} \frac{v}{c}\ , && \text{(4)}\\
\gamma &\stackrel{\text{def}}{=} \frac{1}{\sqrt{1-\beta^2}}\ . && \text{(5)}
\end{aligned}
$$

3.1 Frame 1 (F1)

This is the physical experiment. The experiment remains the same as Expt1 of the immediately preceding section.

3.2 Mathematical Frame 2 (F2)

Assumptions

Assume that

1. in going from F1 to F2, the trajectories don't change as per GT; they do as per LT.

2. the same law viz. eqs. (9) and (10) hold also for observations from F2

NB To avoid confusion, we use x' etc. for GT, but x'' etc. for LT. This notation may be changed in the subsequent revision of this note.

Calculations in F2

Calculate the quantitative values of the initial and final trajectories in F2 of all the objects: the charged source-object, the charged test-object, and also the (possibly uncharged) third-object.

Of course, the trajectories in F2 aren't the same as those in F1. (Even GT had not kept them the same; now the trajectories after LT are different from those after GT.)

For our particular problem: Calculate what the initial and final conditions would be in F2, starting from those prescribed in F1.

Thus, transform all $\vec{r}$, $\dot{\vec{x}}$, $\ddot{\vec{x}}$, and also t to their LT'ed quantities: $\vec{r}''$, $\dot{\vec{r}}''_E$, $\ddot{\vec{r}}''_E$, t'', etc. Thus, we will also have $\Delta t''_E$ (the small time interval over which we do the calculations). Note there will be separate positions, velocities, accelerations, and instants for the source-charge, the test-charge, etc. You also have to LT the other quantities.

Why “other”? Because under LT, not just the space- *and* time-coordinates change, and not just velocities and accelerations, but also: the masses m_E and m_D of the test-charge, the Lorentz force $\vec{F}$ acting on the test-charge, etc. More on this, a bit later.

Input the initial conditions into the same general solution (because the law holds). Predict the final conditions.

3.3 Comparison of results obtained using F1 and F2

Cool!

The predicted final conditions say that even in F2, the test-object *will* be hitting the *third* object — even if the space and time coordinates of all the events have changed!

Conclusions

The conclusions (stated somewhat vaguely, i.e. assuming the full context) by the physicists, mathematical physicists, and mathematicians were these:

Wow! It works! The ED law holds in all (inertial) frames, just the way Newton’s did!...

The ED law is indeed universal *because* it’s frame-independent!!

And, the ED law is more general than Newton’s, not so much because the objects now have the charge too not just the mass, but *more fundamentally* because it can be *mathematically* shown that LT reduces to GT in the limit of vanishing frame speeds.

Something the odd

... But, but, something is *odd* — even if it *can’t* be wrong (because it’s *mathematically* proven): Not only do the x coordinates change, t do too! Worse: Not just Δx intervals change, Δt do too!

4 Optional section: ED Ontology — during the development of LT, and after the Second thought-experiment with LT:

4.1 Special note for this section

This section covers more philosophy than physics. Strictly speaking, it’s optional. However, to understand the depth of the impact of the Special Theory of Relativity (STR), it’s recommended that the reader not skip it.

However, there is a sense of dissatisfaction that the present author must note too: This section was not written very carefully. So, this section might have characterized the various positions somewhat inaccurately, especially the more philosophic positions, even if the description does get the general direction right at a very broad level.

4.2 Various thoughts by physicists and mathematicians

Lorentz

... So, there is a special "local time" to F2. ... So, moving objects must have been contracting all along, it's just no one noticed because our experiments didn't probe the range of very high object speeds.

Poincaré

I can show that it has the most solid basis in the principles of mathematical physics — nothing but the revered Calculus of Variations. These results (of LT) have got to hold.

Seemingly NM Ontology but with the ED-properties of Light thrown in — Einstein's development

Einstein (after the fact of all of the preceding development): I can *derive* LT from two simple and most *fundamental* postulates:

1. Fundamental laws of physics hold unaltered the same way in all the inertial reference frames

2. The speed of light is the same in all inertial reference frames,

It's not objects that contract; it's the entire space that contracts and the entire time dimension that dilates.

Since space and time are the most fundamental *archetypal* concepts (Einstein doesn't explicitly mention Kant whose metaphysics and ontology assert them as being archytypal), *all* our physical laws are expressed in terms of space and time.

Therefore, *any* physical law — known *and* unknown — must undergo the same changes in space and time with changes in frames of observation. In other words, the principles of Special Relativity must hold for all laws: even to those that are not yet discovered.

But since the changes in space and time are implied by the two postulates, the *most fundamental* ***fact*** *pertaining to the physical universe* is that the speed of light is a universal constant that holds in all (inertial) reference frames.

Light!

Incidentally, Einstein doesn't explicitly allude to that incident when a Pope had said "God said, Let Newton be! and all was light." This papal pronunciation had its inspiration rooted in the Bible (where else?): "And God said, Let there be light: and there was light." But of course, the quote was famous enough that Einstein didn't have to allude to it himself.

With his new addition that the speed of light was a universal "fact," people were now free to imagine some kind of a Pope in Einstein too, a rather magnanimous Pope at that, one who extolled the virtues of not just the Christian Saints but also of Natural Philosophers.

Poincaré

(With the then French equivalent of) "Awesome!" "... And, BTW, aether is no longer necessary too, even if Lorentz had used it in deriving his LT."

4.3 Post-Einstein

Philosophers, initially

See, see, that most venerated of all sciences, physics, *itself* has proved that the *Kantian* archetypes of space and time are not absolutes, contrary to what *Newton* had *wrongly* supposed.

Instead, with progress, physics has only come to *confirm* exactly that which the Pre-eminent Academic Philosophers were always implying: If Kant's epistemology is correct, then Man's very cognitive apparatus is compromised in such a way that he cannot even perceive things as they in themselves are; he can *only* perceive them as what they *appear* to him. If so, it's little wonder that space and time are not absolutes, but only as they appear to an observer. Appearances are the final reality.

Philosophers, later

Reality itself is subjective. Even physics cannot be arrogant enough to think of being in possession of objective truths, because physics itself has demonstrated that everything is relative.

Later philosophers (now also joined by physicists themselves) True!

Still later...

Philosophers come to advocate *moral* relativity.

Note: This is actually a *correct* inference to be drawn in deduction, once you grant its starting premises. The development, roughly, goes like this, the present author's comments are included in the square brackets:

1. Kant indeed was the first to demonstrate the power of epistemology. [Objectively true.]

2. Physicists and mathematicians were crediting relativity to Einstein [ignoring his predecessor for the relatity principles themselves viz. Poincaré and both their predecessor for LT viz. Lorentz].

3. Physics had demonstrated its power in the 19th century. [Abundantly true.]

4. Do the obvious: Replace the metaphysical by the physical (because it's demonstrably powerful), and the physical by what physicists like Einstein and Poincaré, viz., the in-principle relativity of space and time. [Sloppy, but interesting.]

5. Combine the working metaphysics which is physics (the latter being taken in the sense of what physicists say), with Kant's specific epistemology. [First part is justifiable: philosophers cannot be supposed to be experts of physics too; for a large part they have to rely on what physicists tell them.]

6. Once both the objective bases of morality (viz. metaphysics and epistemology) have been covered in this manner, the only conclusion to be drawn is: Moral relativity. [Philosophers did draw it. Physicists never were the first to object, and for the same reason as just mentioned, they cannot be expected to be experts of philosophy too.]

5 An interlude

The above is the problem definition — i.e., if you can see a problem of *physics* to be solved in it, in the first place.

6 Towards our solution

6.1 Some of the preliminary considerations and comments

The two postulates of STR

Griffiths[3] gives the two postulates of the special theory of Relativity (STR) (on p. 481, Section 12.1) as:

1. **The principle of relativity.** The laws of physics apply in all inertial reference systems.

2. **The universal speed of light.** The speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source.

The nature of the second postulate

Einstein made the above two postulates without any explicit ontology for the objects or phenomena in question.

Since he claimed to be *deriving* LT by starting from an assumed basis of these two postulates, one cannot definitively identify *any* viable ED ontology for them either. After all, it's not clear whether the supposed ontology for the postulates is broader in scope than that of ED or it's of the same scope.

Our position We regard ED as the more fundamental theory and STR as an implication. Thus, we adopt the ED ontology as briefly explained in [1]. In a subsequent document (or a revision of this document) we shall show that the ED ontology does remain at the base of STR, even if STR is taken to apply to all physical phenomena (in the absence of gravity) — known *and* unknown.

The lack of any direct experimental proof for the postulate 2

The speed of light, in the sense of the directly (concretely) measured velocity of a light signal, and not in the sense of the universal constant $c \stackrel{\text{def}}{=} 1/\sqrt{\epsilon_0\mu_0}$, has always been measured in only one frame.

In particular, no experiment has ever been done such that:

- In an actual experiment conducted in some frame F1, an emitter emitted a flash of light, and a detector at a distance Δx measured its arrival after $\Delta t = \Delta x/c$, *and*

- Simultaneously, the same physical process was "observed" in a second physical frame F2 (installed with a second set of physical detectors) moving at such high (constant) speed with respect to F1 that any experimental errors would not cloud any change in the actually measured velocity of the signal from the emitter-to-detector signal velocity.

In short, there is no direct experimental evidence for the constancy of signal velocity across inertial frames.

Einstein's second postulate equates c with the experimentally measured light signal velocity v_L, and, in the absence of any direct experimental evidence and any supporting ontology, directly postulates that $c = v_L =$ constant across all inertial frames of observation.

We don't have to reject this postulate right at this stage. We only have to note its explicit nature: It's a postulate, not an experimentally measured fact.

6.2 Our alternative Postulate 2

We keep the first postulate as is; it's been implicitly used right since Newton's times.

Our postulate 2

In place of Einstein's postulate 2, we make the following postulate (words written hurriedly, subject to later revision):

> Distances Δx and durations Δt separating two events remain the same in all the physical reference frames.

By physical reference frames, we mean an appropriate group of actually existing physical objects (and in our ED ontology, physical objects remain spatially discrete) arranged in space such that they never change their locations with respect to each other.

Implications of our postulate 2

The "obvious" implications of our postulate 2 are that in physical measurements:

- The mathematical characterization for transforming the physically measured coordinates of object is given by the Galilean transformations (GT).

- Physical space is absolute in the sense that the origin of a reference frame may be chosen arbitrarily but the distances between two physical objects are the same using any reference frame — whether F2 is stationary with respect to F1 or moving inertially.

- Time is abolute in the same sense — instants may be different in different frames, but durations between two events are the same in all inertial frames.

- Thus, time is universal in the sense that its *"flow"* remains the same in all the inertial reference frames.

The question

The question then essentially becomes this:

If the GT doesn't work with ED phenomena including (but not exhausting) light, as 2 showed, then how can our Postulate 2 be made to work?

The next subsection, the final one for this document (at least for the time being) shows how.

7 Our solution to the problem

We shall expand this document later on. For the time being, we shall note our solution in the form of a "scheme" of calculations or an "algorithm." The description and terminology introduced in Sec. 2 and Sec. 3 is taken for granted, as also our postulate introduced in Sec. 6.

Alright, so, here is the algorithm:

Denote the quantities as measured in F1

The trajectories of the source-charge, test-charge, and the "third"-object would all be denoted without any dashes: t, $\vec{r}$, $\dot{\vec{r}}$, $\ddot{\vec{r}}$. Attach the suitable subscripts for the emitter ($_E$) and detector $_D$. Similarly for all other properties like mass (m_E and m_D), the Lorentz forces $\vec{F}$, etc.

Denote the quantities as measured in F2

Suppose that *physical* measurements were actually made in Frame 2 (F2).

Since we wish to have *GT* working at an overall level, we would make sure that the final coordinates and other quantities measured in F2 turn out to be GT-compatible. Therefore, right from this stage (in anticipation of the later development), we choose to denote them with a single dash.

Thus, the trajectories of the source-charge, test-charge, and the "third"-object would all be denoted with single dashes, as t', $\vec{r}'_E$, $\dot{\vec{r}}'_E$, $\ddot{\vec{r}}'_E$, etc., and similarly for all other quantities (including masses m'_E, m'_D, the Lorentz forces F', etc.)

Notation about the forward and inverse transforms

We having been noting the relative motion between the two frames by specifiying the velocity of *F2* with respect to *F1*.

Therefore, Forward-GT means GTing from F1 to F2. Inverse-GT means from GTing F2 to F1. Similarly, Forward-LT means LTing from F1 to F2. Inverse-LT means LTing from F2 to F1

Inverse-transformations require flipping signs; we won't go into it right now; just use common sense: Figure out which frame is moving w.r.t. which frame, how the interframe velocity was defined in the first place (it was $+v$ as the velocity of F2 w.r.t. F1), and then use the correct algebraic sign for the velocity of the *other* frame now to be used, and *then* use the transformation equations "as is". Ditto for other inputs like velocities and accelerations of *objects*, their masses, forces, momenta, etc.

Step 1: Assume that all the experimental observations have been made in both physical frame F1 and physical frame F2

This is just as in Sec. 3 i.e. in Sec. 2. Thus, in F1, you will have a set of unprimed quantities.

By adopting our convention, in F2, you will have a set of single-primed quantities.

Our task it to find that transform which converts the first set to the second or vice-versa, with the spatial and temporal intervals the same in both frames.

Step 2: Find the parametric representation of the space-time coordinates and masses

Don't put the F2-quantities directly into the ED laws. Instead, follow this procedure:

Inverse-GT the trajectories and masses from F2 to F1. This will get you the same set as that measured from F1 — the unprimed set.

Forward-LT the trajectories and masses from F1 to F2. This will get you the double-primed quantities. These are the merely mathematical / parametrized / hypothetical / abstract coordinates.

Now plug these abstrac (double-primed) quantities into the general solution eqs. (14) and (15), and find the $\vec{E}''$ and $\vec{B}''$ in the frame F2. Also find the Lorentz force $\vec{F}''$ using eq. (10).

Note, these fields and forces are valid *only* in F2, i.e. they do assume the contracted positions and dilated times. Just the way we found a parametric or hypothetical representation for the actual coordinates and masses, similarly, these fields and forces too are an abstract representation of the actual forces in F2.

Step 3: Verify that the Lorentz force is correct

Inverse-LT the Lorentz force as calculated in *F2*, $\vec{F}''$, to that in F1, $\vec{F}$. Verify that the result is correct with experimental observations in F1.

BTW, the transformation equations for LT-ing any forces, including the Lorentz forces (in the simple case that the frame-velocity is parallel to the x-axis) are:

$$F''_x = \frac{F_x - \eta\left(\vec{U}\cdot\vec{F}\right)}{1-\eta U_x}\,, \tag{6a}$$

$$F''_y = \frac{F_y}{\gamma\left[\,1-\eta U_x\,\right]}\,, \tag{6b}$$

$$F''_z = \frac{F_z}{\gamma\left[\,1-\eta U_x\,\right]}\,, \tag{6c}$$

where

$$\eta \stackrel{\text{def}}{=} \frac{\beta}{c} = \frac{v}{c^2}\,, \tag{7}$$

and $\vec{U}$ is the velocity of the *object* in the F1 frame, and U_x is its x-component. Note: For frame-speeds, we use $\vec{v}$ or v, not $\vec{U}$.

Step 4: A trick to change the electric and magnetic fields

If we directly inverse-LT the fields from F2 to F1, then we will get the same fields as those measured in F1 — even if these were obtained after makikng calculations using trajectories etc. in F2 as the intermediate step, as demonstrated in [2].

So, at present, we have reached two representations for the fields:

- $\vec{E}$ and $\vec{B}$ are in F1, the frame in which the experiment was physically conducted. Therefore, these fields don't assume LT-ed space- and time-intervals. So, they can work with the physical coordinates — even with *any* GT'ed physical coordinates.

- $\vec{E}''$ and $\vec{B}''$ are the values that do reflect the relative changes within the fields (from electric-to-magnetic and magnetic-to-electric fields as occurring due to the changes in the velocities and accelerations of the source-charge). But since they are calculated in the F2 frame, they can work only over the space-contracted and time-dilated coordinates, not in the physical coordinates (i.e., not the GT-ed physical coordinates)

The trick for $\vec{E}''$ and $\vec{B}''$ to work with the GT-ed coords specific to F2 i.e. (to make them work with the single-primed coordinates) is this:

The Lorentz force is given very simply by

$$\vec{F} = q_T \left(\vec{E} + \vec{U}_T \times \vec{B} \right) , \tag{8}$$

where $\vec{U}_T$ is the velocity of the *test* charge.

Rearrange the steps and the entire notation to include GT-ing and IGT-ing to a frame to the simpler experession for the LT of the Lorentz force. Also consider: Just giving the schematics using frames, not the detailed notation. **TBD**

The trick, in essence

The trick, in essence, consists of two parts:

- Avoid the temptation to use either GT or LT but put to use *both* of these *mathematical* transformations in such a way that in the actual physical measurements, the net effect is as if GT had been observed.

- Change the meaning of the law in the process — i.e. *reformulate* the mathematical law of the physics (the system of PDEs).

All the physicists took the law i.e. eqs. (9) and (10) as if it were some kind of an intrinsic truth, not a law of physics arrived at through observation, ideas, controlled experimentation, appropriate mathematical methods, and overall, inductive generalization. That was their error. But let's not get into polemics. Let's turn to the solution.

References

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8 Appendix A: Equations of classical electrodynamics

This Appendix is copy-pasted from the present author's forth-coming paper[2].

8.1 Governing differential equations

The governing equations of classical electrodynamics consist of Maxwell's laws and the Lorentz force law.

Maxwell's equations form a system of four coupled, first-order, partial differential equations. Following Griffiths[3] but with a small change, these equations can be given as:

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0}\rho \,, \tag{9a}$$

$$\nabla \cdot \vec{B} = 0 \,, \tag{9b}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \,, \tag{9c}$$

$$\nabla \times \vec{B} = \mu_0\, \rho\vec{U} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t} \,. \tag{9d}$$

The small change we made in writing down Eqs. (9) is that while Griffiths uses the volume-based current-density $\vec{J}$ in the first term on the RHS of Eq. (9d), we have used $\rho\vec{U}$, where ρ is the (mobile) volume-based charge-density and $\vec{U}$ is the latter's velocity.

We made the preceding change mainly because we wish to highlight the fact that while the system of Newton's three laws does not show a dependence on the velocity of the object, the system of Maxwell's equations does. Further, while Newton's second law has a single (mechanical) force, Maxwell's equations have two force-fields coupled to each other. This difference leads to the difference between the Galilean transformations (GT) and the Lorentz transformations (LT).

Lorentz's force law is given as:

$$\vec{F} \;=\; q_T\left(\vec{E} \,+\, \vec{U}_T \times \vec{B}\right)\,, \tag{10}$$

where q_T and $\vec{U}_T$ respectively denote the charge and the velocity of the *test* charge, whereas the fields $\vec{E}$ and $\vec{B}$ themselves are supposed as having been produced by some unspecified source charge(s).

Although Lorentz's force law is not directly coupled with Maxwell's four equations, the test charge has, in the most general scenario, its own fields too, and these latter fields exert their own forces on the source charges too, which in turn change the fields of the source charges noted in Eq. (9). Thus, even if indirectly, Lorentz's law does ultimately enter the coupling given by Maxwell's four equations. Thus, Eqs. (9) and (10) together form a *nonlinearly* coupled system.

These five equations *completely* specify the *entire* theoretical content of the classical electrodynamics. A word about terminology: We use the term "electromagnetism" (EM) to denote Maxwell's four equations, and "electrodynamics" (ED) to denote all the five equations. Thus, we take ED as having a more general scope than EM.

Notice that the entire system of ED equations was arrived at using the empirical method of science, and it may be regarded as having been formulated using a *single* frame of reference (which, in turn, may conveniently be called the "lab" or "earth" or even "fixed" frame). In particular, in formulating the Eqs. (9) and (10), no attention was paid for their frame-independence. This point acquires a certain significance in our later discussion.

8.2 General solution for fields of a moving point charge

In this paper, all our modelling involves only point-charges. The general solution for fields of an arbitrarily moving point-charge is obtained by using the Liénard-Wiechert potentials[3].

Suppose that the fields detected by a *test* charge situated at a space-time point $(\vec{r}, t)$ are to be *calculated*. As to the *source* charge q, denote its time dependent position at the instant t as being given by a function of time, say $\vec{w}(t)$.

The *retarded instant* t_r, which is used in the calculations, is determined *implicitly* via the equation:

$$|\vec{r} - \vec{w}(t_r)| \;=\; c(t - t_r)\,, \tag{11}$$

where c is the speed of light. Notice that, in contrast, t is called the *present* instant.

The *retarded position* is then obtained as $\vec{w}(t_r)$.

The *separation vector* from the *retarded* position of the source charge to the *present*-time field point $\vec{r}$ is denoted as $\vec{R}$. Thus,

$$\vec{R} \;\equiv\; \vec{r} - \vec{w}(t_r)\,. \tag{12}$$

Next, to reduce clutter, it's convenient to define an abstract velocity denoted as $\vec{u}$ and defined as:

$$\vec{u} \;\equiv\; c\widehat{R} - \vec{U}(t_r)\,, \tag{13}$$

where $\widehat{R} \equiv \vec{R}/R$, $R \equiv |\vec{R}|$, and $\vec{U}(t_r)$ is the velocity of the source charge at the *retarded* instant t_r. Thus, $\vec{u}$ represents the difference that the light ray's velocity vector has from the source charge's velocity vector, with both of them directly or indirectly referring to the *retarded* instant. Notice that $\vec{r}$ always corresponds to the *present* instant.

For a point-charge (as in contrast to a continuous charge distribution), the general solution to the system of Eqs. (9) is then given[3] as:

$$\vec{E}(\vec{r}, t) \;=\; \frac{q}{4\pi\epsilon_0} \frac{R}{(\vec{R} \cdot \vec{u})^3} \left[(c^2 - U^2)\vec{u} \;+\; \vec{R} \times (\vec{u} \times \vec{a}) \right] , \tag{14}$$

and

$$\vec{B}(\vec{r}, t) \;=\; \frac{1}{c} \widehat{R} \times \vec{E}(\vec{r}, t) \,. \tag{15}$$

Notice that both the electric and magnetic fields depend on the position $\vec{w}(t_r)$, velocity $\vec{U}(t_r)$, as well as acceleration $\vec{a} \equiv \mathrm{d}\vec{U}(t_r)/\mathrm{d}t$ of the source charge, and all these three quantities are defined at the *retarded* instant.

However, as Eq. (11) shows, the retarded instant itself is only defined *implicitly*. Therefore, there is no generally applicable analytical solution for calculating retarded instants. Consequently, even if we do have the exact *form* available for the general solution, *approximate* numerical methods still must be used in the general case.

After expanding the square-bracket on the RHS of Eq. (14), the RHS comes to have two terms. The first term is called the *velocity field*, and the second term the *acceleration field*. When a point-charge does not accelerate, the second term drops out. Further, when a point-charge is fixed, the velocity term reduces to the same inverse-square field as is given by Coulomb's law of electrostatics.