Applications of Fracture Mechanics

Many applications of fracture mechanics are based on the equation

$$\Gamma = \beta \frac{\sigma_c^2 a}{E}.$$

Young's modulus is usually known. Of the other four quantities, if three are known, the equation predicts the fourth. If you just read this equation, fracture mechanics sounds like a silly tautology. It is not really so silly if you think through each application. Some quantities are easy to measure. Other quantities are easy to compute. One can make real predictions.

After finding out that I am a fracture expert, people often ask me if a cracked wall is dangerous, or how much time a crack will grow across a pavement. After some embarrassment, I realize I often cannot answer their questions, because I haven't done calculations and obtained experimental data.

Application 1. Measure the fracture energy. Know β, σ_c, a. Determine Γ. The experiment follows that of Griffith.

- Start with a body of a material.
- Cut a crack of a known size a using a saw.
- Load the sample with an increasing stress, and record the stress at fracture, σ_c.
- Separately find the elasticity solution for the energy release rate, $G = \beta\sigma^2 a / E$.
- Convert the critical stress to the fracture energy, $\Gamma = \beta\sigma_c^2 a / E$

The measured fracture energy is used to (a) rank materials, (b) study the effect of various parameters (e.g., loading rate, temperature, heat treatment) on fracture resistance, (c) design a structure to avoid fracture.

Application 2. Predict critical load. Know β, a, Γ. Determine σ_c.

The body is given. The fracture energy of the material has been measured. The crack size a has been measured. Find the elasticity solution for the energy release rate $G = \beta\sigma^2 a / E$. This application requires one to determine the crack size. A large crack size is determined by visual inspection. A small crack can be determined by the x-ray or acoustic wave (Nondestructive Evaluation, or NDE). If the measurement technique cannot find any crack, simply put the smallest crack size can be detected by the technique (i.e., the resolution) into the equation, and predict a lower bound of the critical load. A crack in a structure may increase slowly over time. Inspect the structure periodically to monitor the crack size. Retire or repair the structure before the crack is too large.

Application 3. Estimate flaw size from experimentally measured breaking stress. Know β, Γ, σ_c. Determine a.

- Measure the fracture load σ_c.
- Independently measure the fracture energy of the material.
- Approximate the energy release rate by that of a Griffith crack, $G = \pi\sigma^2 a / E$.
- The flaw size a is estimated by $a = \Gamma E / \pi\sigma^2$.

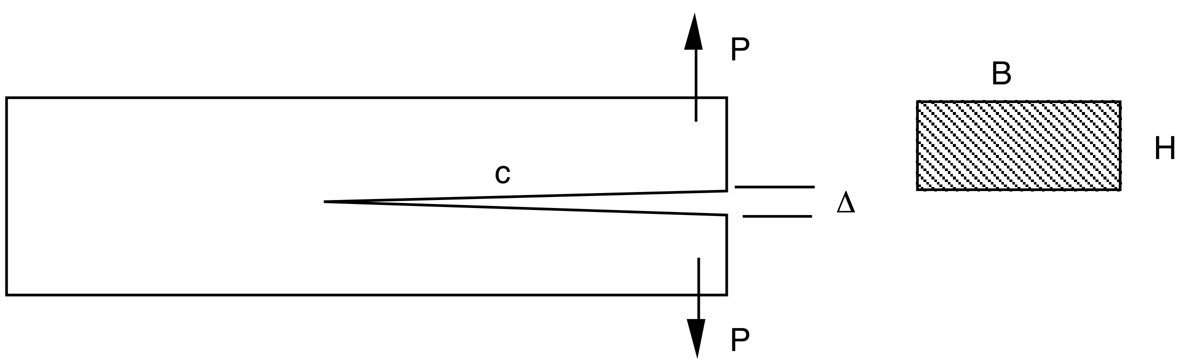
Application 4. The proof test

- Load a sample to a given load σ, and the sample dose not fracture.
- Independently measure the fracture energy of the material.
- Approximate the energy release rate by that of a Griffith crack, $G = \pi\sigma^2 a / E$.
- The flaw size a is bonded by $a < \Gamma E / \pi\sigma^2$.

Heart valve. Each proof-tested individually. “Make people feel good at heart”
Ceramic tiles on reentry vehicles. Each tile is proof-tested individually.

Application 5. Design a structure to avert fracture.

If a material is given, and the load level is prescribed, one can design a structure to avoid fracture. One also needs to know the possible flaw size.



Double-cantilever beams. To determine the energy release rate for a given cracked body, one has to solve a boundary-value problem. This requires some work. People use handbooks or finite element packages. The following example is one of a few that can be solved in the classroom.

The double-cantilever beams are commonly used in fracture test. The elastic field can be determined by using the beam theory. Treat each beam as a cantilever. One end is clamped, and the other end is pulled by a force P. The opening displacement is Δ. According to the beam theory, the deflection of each beam is given by

$$\frac{\Delta}{2} = \frac{Pc^3}{3EI}.$$

The form of this expression can be understood if you remember basics of the beam theory. The displacement should be linear in the force, and should be inversely proportional to the bending rigidity *EI*, where the second moment of the cross section is $I = BH^3/12$. A dimensional consideration shows the cubic dependence on the length *c*.

The elastic energy stored in the two arms is $U = P\Delta/2$. The crack area is $A = Bc$. Express the energy as a function of the displacement Δ and the crack area *A*:

$$U(\Delta, A) = \frac{3EIB^3\Delta^2}{4A^3}.$$

The energy release rate is given by partial differentiation with respect to the crack area: $G = -\partial U(\Delta, A)/\partial A$

$$G = \frac{9EI\Delta^2}{4Bc^4}.$$

This expresses the energy release rate in terms of the opening displacement Δ. Alternatively, one can express the energy release rate in terms of the load:

$$G = \frac{12}{EH^3}\left(\frac{Pc}{B}\right)^2.$$

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Stability of a growing crack. When the double-cantilever beam is split by inserting a wedge of height Δ, the energy release rate is

$$G = \frac{9EI\Delta^2}{4Bc^4}.$$

To keep the crack growing, the energy release rate needs to be at the level of the fracture energy, namely, $G = \Gamma$. Because *G* decreases with the length of the crack *c*, the crack will arrest.

When the double-cantilever beam is split by hanging a weight *P*, the energy release rate is

$$G = \frac{12}{EH^3}\left(\frac{Pc}{B}\right)^2$$

Because G increases with the length of the crack c, the crack will not arrest once it starts to grow.

Although the two expressions of the energy release rate are equivalent, how we split the beam can make a huge difference to how the crack will grow. We will return for a fuller discussion of the stability of a growing crack later in the class.

A more accurate expression for energy release rate. The end of each arm is not really clamped, but can have some rotation. The beam theory itself is an approximation of the elasticity theory, and neglects the effect of shear. Both errors are small when the beams are long, namely when c/H is large. The above result is an exact asymptote when $c/H \to \infty$. For a finite value of c/H, numerical analysis has given a better approximation:

$$G = \frac{12}{EH^3}\left(\frac{Pc}{B}\right)^2\left(1 + 0.677\frac{H}{c}\right)^2.$$

For example, see

- G. Bao, S. Ho, Z. Suo, and B. Fan, The role of material orthotropy in fracture specimens for composites. International Journal of Solids and Structures 29, 1105-1116. (http://www.seas.harvard.edu/suo/papers/016.pdf)

Channel crack in a thin film bonded to a substrate. Compare two configurations. First consider a crack of length a, in a freestanding sheet of thickness h, is subject to a tensile stress σ remote from the crack. Assume that the displacement at the load point is fixed, so that the load does no work when the crack extends. When the crack is introduced, the stress near the crack faces is partially relieved. The volume in which the stress relaxes scales as a^2h, so that relative to the uncracked, stressed sheet, the elastic energy in the cracked sheet changes by $\Delta U \sim -a^2h\sigma^2/E$. Consequently, the energy release rate for a crack in a freestanding sheet is

$$G \sim a\sigma^2/E.$$

The energy release rate of a crack in a freestanding sheet increases with the length of the crack.

Next consider a thin elastic film bonded to an elastic substrate. When the length of the crack a is much larger than the thickness of the film h, the stress field in the wake of the crack becomes invariant as the crack extends. The volume in which the stress relaxes scales as ah^2, so that the introduction of the crack changes the elastic energy by $\Delta U \sim -ah^2\sigma^2/E$. Consequently, the energy release rate for a channel crack in a film is

$$G \sim \sigma^2 h/E.$$

Compare the above two situations.

Take $\sigma = 10^9 \mathrm{Pa}$, $E = 10^{11} \mathrm{Pa}$, and $\Gamma = 10 \mathrm{J/m^2}$. Equating the energy release rate G to the fracture energy Γ, we find a critical film thickness $h_c = 0.5 \mu\mathrm{m}$. Channel cracks can propagate in films thicker than h_c, but not in films thinner than h_c. A very thin film can sustain a very large stress without cracking.

Calculate the energy release rate of the channel crack. The bottom of the channel is blocked by the interface. The front of the channel is curved, and extends in the film. We want to find the energy release rate at the channel front. This is a three dimensional problem. When the channel length exceeds several times the film thickness, the channel approaches a steady state. That is, the elastic energy reduction U associated with the channel extending per unit length approaches a constant, independent of the channel length. This energy reduction can be calculated as follows. The extension of the channel by a unit distance is equivalent to removing a slice of material of a unit thickness far ahead of the channel, and then appending a slice of material far behind the channel. Let $\sigma(x)$ be the stress prior to the introduction of the crack (the first slice), and $\delta(x)$ be the opening displacement after the introduction of the crack (the second slice). The two quantities, $\sigma(x)$ and $\delta(x)$, can be determined by solving the two plane strain elasticity boundary value problems. The elastic energy difference between the two slices is

$$U = \frac{1}{2} \int \sigma(x) \delta(x) dx$$

The integral extends over the length of the crack. The energy release rate at the channel front is the energy reduction associated with the crack extending per unit area. Thus, $G = U / h$, giving

$$G = \frac{1}{2h} \int \sigma(x) \delta(x) dx .$$

In summary, follow the steps below to determine the steady state channel energy release rate.

- Solve the plane strain problem without crack, and obtain $\sigma(x)$.
- Solve the plane strain problem with crack, and obtain $\delta(x)$.
- Calculate the integral to obtain the energy release rate G.

The first two steps are usually carried out by using the finite element method. The third step is carried out by numerical integration.

- J.W. Hutchinson and Z. Suo, Mixed-mode cracking in layered materials, Advances in Applied Mechanics 29, 63-191 (1992). http://www.seas.harvard.edu/suo/papers/017.pdf

Measuring fracture energy of a thin film. For a brittle solid, toughness is independent of sample size, so that one may expect to extrapolate toughness measured using bulk samples to thin films. However, thin films used in the interconnect structure are processed under very different conditions from bulk materials, or are unavailable in bulk form at all. Consequently, it is necessary to develop techniques to measure thin film toughness. Such a technique is attractive if it gives reliable results, and is compatible with the interconnect fabrication process. Ma et al. (1998) have developed such a technique, on the basis of channel cracks in a thin film bonded to a silicon substrate. The test procedure consists of two steps: (a) generating pre-cracks, and (b) propagating the cracks using controlled stress. A convenient way of generating pre-cracks is by scratching the surface using a sharp object. A gentle scratch usually generates multiple cracks on the two sides of the scratch. Care must be taken to generate cracks just in the film, but not in the substrate. Use a bending fixture to load the sample, and a digital camera to record the crack growth events. After a crack propagates some distance away from the scratch, the crack grows at a steady velocity.

By recording crack growth at slightly different bending loads, one can measure the crack velocity as a function of the stress. The steady velocity is very sensitive to the applied stress. Consequently, the critical stress is accurately measured by controlling the velocity within a certain range that is convenient for the experiment. The critical stress then determines the fracture energy according to the mechanics result. The technique has been used to measure the fracture energy of silica ($\Gamma = 16.5 \mathrm{J/m^2}$) and silicon-nitride ($\Gamma = 8.7 \mathrm{J/m^2}$) (Ma et al., 1998).

After deposition, the film has a residual stress, σ_R. The residual stress can be measured by the wafer curvature method. When the structure is bent by a moment M (per unit thickness), the film stress becomes

$$\sigma = \sigma_R + \frac{6\bar{E}_f M}{\bar{E}_s H_s^2},$$

where H_s is the thickness of the substrate, and $\bar{E}_f$ and $\bar{E}_s$ are the plane strain modulus of the film and the substrate. The bending moment also causes a tensile stress in the substrate, $\sigma_s = 6M / H_s^2$. To measure the toughness of the film, one has to propagate the channel crack in the film without fracturing the substrate. This, in turn, requires that the substrate should have very small flaw size, and that the scratch should not produce flaws in the substrate. When properly cut, the silicon substrate can sustain tensile stresses well over 1 GPa. The technique is inapplicable when the film has a very large fracture energy, large residual compressive stress, small thickness, or low modulus. In the experiment of Ma et al. (1998), a thin metal layer is deposited on the silicon substrate, and the brittle film is deposited on the metal. The metal layer serves as a barrier preventing the crack from entering the substrate. The metal layer, upon yielding, also increases the energy release rate for a given bending moment.

- Q. Ma, J. Xie, S. Chao, S. El-Mansy, R. McFadden, H. Fujimoto. Channel cracking technique for toughness measurement of brittle dielectric thin films on silicon substrates. *Mater. Res. Soc. Symp. Proc.* **516**, 331-336 (1998).