Fast-Start Proposal	Contact PI's surname	Initials	Application Number	Panel
Proposal	Banerjee	B	09-IRL-012	PSE

5A. ABSTRACT OF RESEARCH PROPOSAL

The Problem: The macroscopic failure of heterogeneous materials is generally preceded by a series of crackling noises. These noises are usually attributed to micro- and meso-scale microcracking and damage and are emergent dynamical phenomena[1,2]. Numerous experimental studies suggest that the time interval between these acoustic emissions (AE) and the released energy follow power law distributions[3−7]. However, a full theoretical understanding of the scale invariance implied by the power law distributions is yet to be obtained[2,8]. Though most studies indicate that the power law scaling exponent appears to depend on microstructural parameters[9−11] there are some studies that indicate otherwise[8,12,13]. Also, it is possible to fit alternative probability distributions to apparently power law distributed data[14]. An improved theoretical understanding of the causes of the apparent scale similarity in AE is crucial if the predictive capacity of such emissions is to be harnessed.

The fracture and damage processes that cause acoustic emissions involve a large range of length and time scales[15]. Even if we restrict ourselves to micro- and meso-scale phenomena, dynamic numerical solutions of multiple interacting cracks are computationally costly and analytical solutions are unavailable[16]. Nonlocal approaches are needed when continuum damage[17] is used to simplify the representation of the weakened region around a dynamically growing crack tip[18]. A more efficient approach for computing multiple crack interactions is to use singular integral equations that arise from complex analysis[19,20]. However, these solutions are restricted to elastostatics in two dimensions and extension to three dimensions is quite involved[21]. Clifford algebras can be used to generalize the concept to complex variables to dimensions greater than two[22]. Though Clifford algebras have been used to solve a number of physical problems[23−28], they have not been applied to the problem of dynamic fracture. The application of Clifford algebras to dynamic fracture will be a significant contribution to the field of fracture.

Our Approach: We will use the Clifford algebra approach to develop numerical algorithms for dynamic fracture and apply these algorithms to the acoustic emission problem. The objectives that we will seek to meet are:

1. *To develop a Clifford algebraic framework for three-dimensional linear elasticity starting from the Helmholtz potential representation.* This formalism can also be extended to anisotropic linear elastic behavior and nonlinear mechanical behavior, particularly the anisotropy that is induced by accumulated damage[29]. Two-dimensional calculations in the quasistatic limit will be compared with known solutions for interface cracks to obtain geometric insight into imaginary stress intensity factors[30].
2. *To develop an algorithm for solving dynamic elasticity problems using singular integral representations based on Clifford algebra.* Many powerful ideas from complex analysis have Clifford algebraic higher dimensional analogues. Higher dimensional, Clifford Algebra based, singular integral equations are also amenable to solution using efficient techniques developed in the past 20 years[19,31]. The algorithms that we propose to develop will provide a new approach for tackling the problem of multiple crack interactions.
3. *To calculate the effective dynamic response of cracked, heterogeneous elastic solids to determine whether and why acoustic emissions exhibit scale invariance.* These calculations will provide some insight into whether parametrized invariant manifolds[32] for the AE problem can be discovered and whether renormalization group methods[33] can be used to replace detailed fracture calculations.

The Team: The PI has extensive experience developing and implementing massively parallel numerical algorithms for dynamic fracture and fragmentation in the context of rocket motors. The Uintah code, developed in part by the PI, has been used for calculations ranging from protective armour simulations to explosive-aided oil well drilling on some of the world's fastest supercomputing platforms. Dr. Banerjee will supervise a PhD student who will work on all the objectives, gaining experience in a range of cutting edge techniques. The AI is New Zealand's leading expert on geometric algebra and will provide the experience needed to resolve subtle issues in the application of Clifford algebras to dynamic fracture. Thus the team has the ability to make significant progress on the problem of solving dynamic fracture problems with Clifford algebras.

5B. REFERENCES

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