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Extended hybrid pressure and velocity boundary conditions for D3Q27 lattice Boltzmann model

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ABSTRACT

The extended hybrid electronic-ionic, thermal, magnetic, electric and force couple fields pressure and velocity boundary conditions for D3Q27 lattice Boltzmann model is established. Then, the closed-form solutions of extended distribution functions are derived. Last, the Fukushima nuclear plant Cesium-137 penetration case is discussed.

1. Introduction

As one of the most popular fluid simulation method, lattice Boltzmann method has been widely applied in studying the fluid problem, and a lot of landmark achievements have been obtained [1–16]. On boundary condition aspect, Frisch et al. [17] developed a hexagonal lattice gas model and modeling the 2D Navier–Stokes equation. Jeremy et al. [18] present a cellular automaton model to simulate the process of seismogenesis. Maier et al. [10] proposed a supplementary rule for computing the boundary distribution, and presented 3D body-centered-cubic lattices are presented for Poiseuille flow. Noble et al. [11] developed a hydrodynamic boundary condition for lattice Boltzmann simulations. Zou and He [9] give the pressure and velocity boundary for 2D/3D lattice Boltzmann BGK model.

Extended fluid (electronic-ionic, thermal, magnetic, electric and force) flow pore-crack network problem is an interdisciplinary issues, which can help to study extended fluid flow mechanism in various porosity composites and analyze the extended fluid flow varying mechanism on coseismal slip. But there is little research about the extended pressure and velocity boundary condition lattice Boltzmann model under multiple coupled fields.

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In this paper, the extended hybrid electronic-ionic, thermal, electromagnetic (weak and strong coupled cases) and force couple fields pressure and velocity boundary conditions for the lattice Boltzmann model is established. First, the D3Q19 electronic and ionic lattice cubic, the D3Q15 thermal lattice cubic, the D3Q13 Maxwell equation lattice cubic, the D3Q13 electric lattice cubic, the D3Q7 magnetic lattice cubic and the D3Q19 force lattice cubic are established, respectively. The closed-form distribution functions for pressure and velocity condition under six flow directions (west-east, south-north, front-rear) are obtained. Then, the extended hybrid multiple coupled D3Q27 lattice cubic is created, and the relatively pressure and velocity condition for the lattice Boltzmann model is derived. Finally, a numerical model of an extended fluid flow driven pore-crack network is proposed to examine the accurate of the hybrid boundary condition. The simulation verify that the precision, stability and convergent valid is satisfied.

2. Boundary conditions for the electronic and ionic field

In lattice Boltzmann method, the Navier–Stokes equations are solved via following the evolution of a set of distribution functions $f_i^{ei}(X,t)$. It is shown in Fig. 1, a D3Q19 lattice cubic is employed for electronic and ionic fields. The lattice velocity $e_i^{mi}(i=0,18)$ at position $X(x,y,z)$ and time t for the D3Q19 model are listed in the Table 1.

The extended electronic and ionic density and the macroscopic flow velocity are defined in terms of the particle distribution function by

$$\rho_{in\_ini}^{mi} u_\alpha^{mi} = \sum_{i=0}^{18} f_i^{mi} e_{i\alpha}^{mi} \quad \rho_{in}^{mi} = \sum_{i=0}^{18} f_i^{mi}, \tag{1}$$

where $\alpha = 1 \sim 3$, the extended electronic and ionic equilibrium distribution functions for incompressible model and compressible model can be written as follows [19–21],

$$\begin{aligned} f_{i\_incom}^{eq\_mi}(x,t) &= \alpha_i^{mi}\rho_{in}^{mi} + \alpha_i^{mi}\rho_{in\_ini}^{mi}\left(\frac{e_i^{mi}u_\alpha^{mi}}{c_s^2} + \frac{\left(e_i^{mi}u_\alpha^{mi}\right)^2}{2c_s^4} - \frac{\left(u_\alpha^{mi}\right)^2}{2c_s^2}\right) \\ f_{i\_com}^{eq\_mi}(x,t) &= \alpha_i^{mi}\rho_{in}^{mi} + \alpha_i^{mi}\rho_{in}^{mi}\left(\frac{e_i^{mi}u_\alpha^{mi}}{c_s^2} + \frac{\left(e_i^{mi}u_\alpha^{mi}\right)^2}{2c_s^4} - \frac{\left(u_\alpha^{mi}\right)^2}{2c_s^2}\right), \end{aligned} \tag{2}$$

where $\alpha_i^{mi} = \frac{1}{3}\delta_{i0} + \frac{1}{18}\delta_{im} + \frac{1}{36}\delta_{in}$ $m = 1 \sim 6$ $n = 7 \sim 18$

2.1. Front-rear flow

As shown in Fig. 1a, when the electronic and ionic flow direction is from front to rear, after streaming, the unknown distribution functions are $f_{fr:i}^{mi}$ ($i = 15,9,16,3,7$), on the contrary, after streaming, the unknown distribution functions are $f_{rf:i}^{mi}$ ($i = 17,10,18,4,8$).

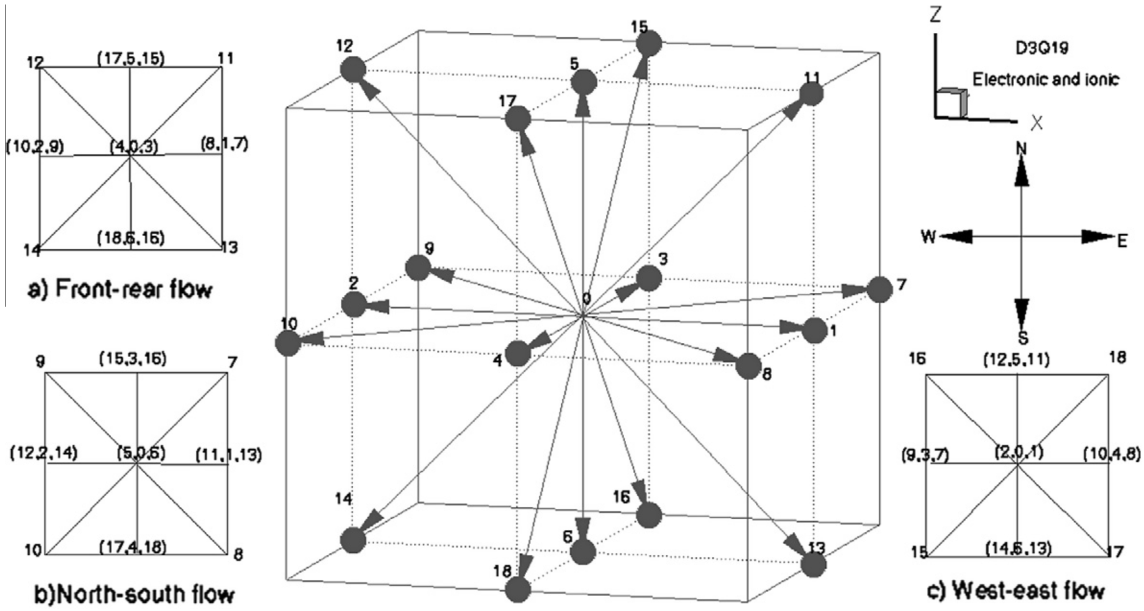

Fig. 1. Cubic lattice D3Q19 for simulating the electronic and ionic field.

Table 1
The lattice velocity $e_i^{mi}(X,t)$ ($i = 0,18$) for electronic and ionic field.

	e_0^{mi}	e_1^{mi}	e_2^{mi}	e_3^{mi}	e_4^{mi}	e_5^{mi}	e_6^{mi}	e_7^{mi}	e_8^{mi}	e_9^{mi}	e_{10}^{mi}	e_{11}^{mi}	e_{12}^{mi}	e_{13}^{mi}	e_{14}^{mi}	e_{15}^{mi}	e_{16}^{mi}	e_{17}^{mi}	e_{18}^{mi}
e_{i1}^{mi}	0	1	−1	0	0	0	0	1	1	−1	−1	1	−1	1	−1	0	0	0	0
e_{i2}^{mi}	0	0	0	1	−1	0	0	1	−1	1	−1	0	0	0	0	1	1	−1	−1
e_{i3}^{mi}	0	0	0	0	0	1	−1	0	0	0	0	1	1	−1	−1	1	−1	1	−1
	C	E	W	R	F	S	N	RE	FE	RW	FW	SE	SW	NE	NW	SR	NR	SF	SF

2.1.1. Pressure boundary condition (PC)

For the front inlet and rear outlet case, using Zou and He method [9], the $f_{fr:i}^{p:mi}$ ($i = 15,9,16,3,7$) can be derived from the Eqs. (1) and (2), and the explicit expressions can be defined as

$$f_{fr:3}^{p:mi} = f_4^{mi} + \rho_{fr:in}^{p:mi} u_{fr:y}^{p:mi}/3 \quad f_{fr:7}^{p:mi} = f_{10}^{mi} + \rho_{fr:in}^{mi} u_{fr:y}^{p:mi}/6 - c_{fr:1}^{mi}/4 \quad f_{fr:9}^{p:mi} = f_8^{mi} + \rho_{fr:in}^{p:mi} u_{fr:y}^{p:mi}/6 + c_{fr:1}^{p:mi}/4, \tag{3}$$

$$f_{fr:15}^{p:mi} = f_{18}^{mi} + \rho_{fr:in}^{p:mi} u_{fr:y}^{p:mi}/6 - c_{fr:2}^{mi}/4 \quad f_{fr:16}^{p:mi} = f_{17}^{mi} + \rho_{fr:in}^{p:mi} u_{fr:y}^{p:mi}/6 + c_{fr:2}^{p:mi}/4. \tag{4}$$

For the rear inlet and front outlet case, $f_{rf:i}^{p:mi}$ ($i = 17,10,18,4,8$) can be defined as

$$f_{rf:4}^{p:mi} = f_3^{mi} - \rho_{rf:in}^{p:mi} u_{rf:y}^{p:mi}/3 \quad f_{rf:10}^{p:mi} = f_7^{mi} - \rho_{rf:in}^{mi} u_{rf:y}^{p:mi}/6 + c_{fr:1}^{mi}/4 \quad f_{rf:8}^{p:mi} = f_9^{mi} - \rho_{rf:in}^{p:mi} u_{rf:y}^{p:mi}/6 - c_{fr:1}^{mi}/4, \tag{5}$$

$$f_{rf:18}^{p:mi} = f_{15}^{mi} - \rho_{rf:in}^{p:mi} u_{rf:y}^{p:mi}/6 + c_{fr:2}^{mi}/4 \quad f_{rf:17}^{p:mi} = f_{16}^{mi} - \rho_{rf:in}^{p:mi} u_{rf:y}^{p:mi}/6 - c_{fr:2}^{p:mi}/4, \tag{6}$$

where

$$c_{fr:\alpha}^{mi} = \left(f_{14}^{mi} - f_1^{mi} + f_2^{mi} - f_{11}^{mi} + f_{12}^{mi} - f_{13}^{mi}\right)\delta_{1\alpha} + \left(f_6^{mi} - f_5^{mi} - f_{11}^{mi} - f_{12}^{mi} + f_{13}^{mi} + f_{14}^{mi}\right)\delta_{2\alpha}, \tag{7}$$

$$u_{fr:y}^{p:mi} = 1 - \left[2\left(f_4^{mi} + f_8^{mi} + f_{10}^{mi} + f_{17}^{mi} + f_{18}^{mi}\right) + f_0^{mi} + f_1^{mi} + f_{11}^{mi} + f_5^{mi} + f_{12}^{mi} + f_2^{mi} + f_{14}^{mi} + f_6^{mi} + f_{13}^{mi}\right]/\rho_{fr:in}^{p:m}, \tag{8}$$

$$u_{rf:z}^{p:mi} = -1 + \left[2\left(f_3^{mi} + f_7^{mi} + f_9^{mi} + f_{15}^{mi} + f_{16}^{mi}\right) + f_0^{mi} + f_1^{mi} + f_{11}^{mi} + f_5^{mi} + f_{12}^{mi} + f_2^{mi} + f_{14}^{mi} + f_6^{mi} + f_{13}^{mi}\right]/\rho_{rf:in}^{p:mi}. \tag{9}$$

2.1.2. Velocity boundary condition (VC)

At the situation, the macroscopic flow velocity should be defined as

$$u_\alpha^m = \sum_{i=0}^{18} f_i e_{i\alpha}^m. \tag{10}$$

After instead the Eq. (1) with above equation, through the similar derivation of pressure boundary condition, we can obtain the unknown distribution functions for velocity boundary condition. For the front inlet and rear outlet case, $f_{fr:i}^{v:mi}$ ($i = 15,9,16,3,7$), can be defined as

$$f_{fr:3}^{v:mi} = f_4^{mi} + \frac{\rho_{fr:in}^{v:mi} u_{fr:y}^{v:mi}}{3} \quad f_{fr:7}^{v:mi} = f_{10}^{mi} + \frac{\rho_{fr:in}^{v:mi} u_{fr:y}^{v:mi}}{6} - \frac{\rho_{fr:in}^{v:mi} c_{fr:1}^{mi}}{2(\rho_{fr:in}^{v:mi} - 3)} \quad f_{fr:9}^{v:mi} = f_8^{mi} + \frac{\rho_{fr:in}^{v:mi} u_{fr:y}^{v:mi}}{6} + \frac{\rho_{fr:in}^{v:mi} c_{sn:1}^{mi}}{2\left(\rho_{fr:in}^{v:mi} - 3\right)}, \tag{11}$$

$$f_{fr:15}^{v:mi} = f_{18}^{mi} + \frac{\rho_{fr:in}^{v:mi} u_{fr:y}^{v:mi}}{6} - \frac{\rho_{fr:in}^{v:mi} c_{fr:2}^{mi}}{2\left(\rho_{fr:in}^{v:mi} - 3\right)} \quad f_{fr:16}^{v:mi} = f_{17}^{mi} + \frac{\rho_{fr:in}^{v:mi} u_{fr:y}^{v:mi}}{6} + \frac{\rho_{fr:in}^{v:mi} c_{fr:2}^{mi}}{2\left(\rho_{fr:in}^{v:mi} - 3\right)}. \tag{12}$$

For the rear inlet and front outlet case, $f_{rf:i}^{v:mi}$ ($i = 17,10,18,4,8$) can be defined as

$$f_{rf:4}^{v:mi} = f_3^{mi} - \frac{\rho_{rf:in}^{v:mi} u_{rf:y}^{v:mi}}{3} \quad f_{rf:10}^{v:mi} = f_7^{mi} - \frac{\rho_{in}^{v:mi} u_{rf:y}^{v:mi}}{6} + \frac{\rho_{rf:in}^{v:mi} c_{fr\_1}^{mi}}{2\left(\rho_{rf:in}^{v:mi} - 3\right)} \quad f_{rf:8}^{v:mi} = f_9^{mi} - \frac{\rho_{rf:in}^{v:mi} u_{rf:y}^{v:mi}}{6} - \frac{\rho_{rf:in}^{v:mi} c_{fr:1}^{mi}}{2\left(\rho_{rf:in}^{v:mi} - 3\right)}, \tag{13}$$

$$f_{rf:18}^{v:mi} = f_{15}^{mi} - \frac{\rho_{rf:in}^{v:mi} u_{rf:y}^{v:mi}}{6} + \frac{\rho_{rf:in}^{v:mi} c_{fr:2}^{mi}}{2\left(\rho_{rf:in}^{v:mi} - 3\right)} \quad f_{rf:17}^{v:mi} = f_{16}^{mi} - \frac{\rho_{rf:in}^{v:mi} u_{rf:y}^{v:mi}}{6} - \frac{\rho_{rf:in}^{v:mi} c_{fr:2}^{v:mi}}{2(\rho_{rf:in}^{v:mi} - 3)}, \tag{14}$$

where

$$\rho_{fr:in}^{v:mi} = -u_{fr:y}^{v:mi} + 2\left(f_4^{mi} + f_8^{mi} + f_{10}^{mi} + f_{17}^{mi} + f_{18}^{mi}\right) + f_0^{mi} + f_1^{mi} + f_{11}^{mi} + f_5^{mi} + f_{12}^{mi} + f_2^{mi} + f_{14}^{mi} + f_6^{mi} + f_{13}^{mi}, \tag{15}$$

$$\rho_{rf:in}^{v:mi} = -u_{fr:y}^{v:mi} + 2\left(f_3^{mi} + f_7^{mi} + f_9^{mi} + f_{15}^{mi} + f_{16}^{mi}\right) + f_0^{mi} + f_1^{mi} + f_{11}^{mi} + f_5^{mi} + f_{12}^{mi} + f_2^{mi} + f_{14}^{mi} + f_6^{mi} + f_{13}^{mi}. \tag{16}$$

2.2. South-north flow

As shown in Fig. 1b, when the electronic and ionic flow direction is from south to north, after streaming, the unknown distribution functions are $f_{sn:i}^{mi}$ (i = 5, 11, 12, 15, 17), on the contrary, the unknown distribution functions are $f_{ns:i}^{mi}$ (i = 6, 13, 14, 16, 18).

2.2.1. PC

For the south inlet and north outlet case, $f_{sn:i}^{p:mi}$ (i = 5, 11, 12, 15, 17) can be defined as

$$f_{sn:5}^{p:mi} = f_6^{mi} + \rho_{sn:in}^{p:mi} u_{sn:z}^{p:mi}/3 \quad f_{sn:11}^{p:mi} = f_{14}^{mi} + \rho_{sn:in}^{p:mi} u_{sn:z}^{p:mi}/6 - c_{sn:1}^{mi}/4 \quad f_{sn:12}^{p:mi} = f_{13}^{mi} + \rho_{sn:in}^{p:mi} u_{sn:z}^{p:mi}/6 - c_{sn:1}^{p:mi}/4, \tag{17}$$

$$f_{sn:15}^{p:mi} = f_{18}^{mi} + \rho_{sn:in}^{p:mi} u_{sn:z}^{p:mi}/6 + c_{sn:2}^{p:mi}/4 \quad f_{sn:17}^{p:mi} = f_{16}^{mi} + \rho_{sn:in}^{p:mi} u_{sn:z}^{p:mi}/6 - c_{sn:2}^{p:mi}/4. \tag{18}$$

For the north inlet and south outlet case, $f_{ns:i}^{p:mi}$ (i = 6, 13, 14, 16, 18) can be defined as

$$f_{ns:6}^{p:mi} = f_5^{mi} - \rho_{ns:in}^{p:mi} u_{ns:z}^{p:mi}/3 \quad f_{ns:14}^{p:mi} = f_{11}^{mi} - \rho_{ns:in}^{p:mi} u_{ns:z}^{p:mi}/6 - c_{sn:1}^{p:mi}/4 \quad f_{ns:13}^{p:mi} = f_{12}^{mi} - \rho_{ns:in}^{p:mi} u_{ns:z}^{p:mi}/6 + c_{sn:1}^{p:mi}/4, \tag{19}$$

$$f_{ns:18}^{p:mi} = f_{15}^{mi} - \rho_{ns:in}^{p:mi} u_{ns:z}^{p:mi}/6 - c_{sn:2}^{mi}/4 \quad f_{ns:16}^{p:mi} = f_{17}^{mi} - \rho_{in}^{mi} u_{sn:z}^{p:mi}/6 + c_{sn:2}^{p:mi}/4, \tag{20}$$

where

$$c_{sn\_\alpha}^{p:mi} = \left(f_1^{mi} - f_2^{mi} + f_7^{mi} + f_8^{mi} - f_9^{mi} - f_{10}^{mi}\right)\delta_{1\alpha} + \left(f_3^{mi} - f_4^{mi} + f_7^{mi} - f_8^{mi} + f_9^{mi} - f_{10}^{mi}\right)\delta_{2\alpha}, \tag{21}$$

$$u_{sn:z}^{p:mi} = 1 - \left[2\left(f_6^{mi} + f_{13}^{mi} + f_{14}^{mi} + f_{16}^{mi} + f_{18}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_{10}^{mi} + f_2^{mi} + f_9^{mi} + f_8^{mi} + f_1^{mi} + f_7^{mi}\right]/\rho_{sn:in}^{p:mi}, \tag{22}$$

$$u_{ns:z}^{p:mi} = 1 - \left[2\left(f_5^{mi} + f_{11}^{mi} + f_{12}^{mi} + f_{15}^{mi} + f_{17}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_8^{mi} + f_1^{mi} + f_7^{mi} + f_{10}^{mi} + f_2^{mi} + f_9^{mi}\right]/\rho_{ns:in}^{p:mi}. \tag{23}$$

2.2.2. VC

For the south inlet and north outlet case, $f_{sn:i}^{v:mi}$ (i = 5, 11, 12, 15, 17) are defined as

$$f_{sn:5}^{v:mi} = f_6^{mi} + \frac{\rho_{sn:in}^{v:mi} u_z^{mi}}{3} \quad f_{sn:11}^{v:mi} = f_{14}^{mi} + \frac{\rho_{sn:in}^{v:mi} u_z^{mi}}{6} - \frac{\rho_{sn:in}^{v:mi} c_{sn\_1}^{mi}}{2\left(\rho_{sn:in}^{v:mi} - 3\right)} \quad f_{sn:12}^{v:mi} = f_{13}^{mi} + \frac{\rho_{sn:in}^{v:mi} u_z^{mi}}{6} - \frac{\rho_{sn:in}^{v:mi} c_{sn\_1}^{v:mi}}{2\left(\rho_{sn:in}^{v:mi} - 3\right)}, \tag{24}$$

$$f_{sn:15}^{v:mi} = f_{18}^{mi} + \frac{\rho_{sn:in}^{v:mi} u_z^{mi}}{6} + \frac{\rho_{sn:in}^{v:mi} c_{sn\_2}^{v:mi}}{2\left(\rho_{sn:in}^{v:mi} - 3\right)} \quad f_{sn:17}^{v:mi} = f_{16}^{mi} + \frac{\rho_{sn:in}^{v:mi} u_z^{mi}}{6} - \frac{\rho_{sn:in}^{v:mi} c_{sn\_2}^{v:mi}}{2\left(\rho_{sn:in}^{v:mi} - 3\right)}. \tag{25}$$

For the north inlet and south outlet case, $f_{ns:i}^{v:mi}$ (i = 6, 13, 14, 16, 18) are defined as

$$f_{ns:6}^{v:mi} = f_5^{mi} - \frac{\rho_{ns:in}^{v:mi} u_z^{mi}}{3} \quad f_{ns:14}^{v:mi} = f_{11}^{mi} - \frac{\rho_{ns:in}^{v:mi} u_z^{mi}}{6} - \frac{\rho_{ns:in}^{v:mi} c_{sn\_1}^{v:mi}}{2\left(\rho_{ns:in}^{v:mi} - 3\right)} \quad f_{ns:13}^{v:mi} = f_{12}^{mi} - \frac{\rho_{ns:in}^{v:mi} u_z^{mi}}{6} + \frac{\rho_{ns:in}^{v:mi} c_{sn\_1}^{v:mi}}{2\left(\rho_{ns:in}^{v:mi} - 3\right)}, \tag{26}$$

$$f_{ns:18}^{v:mi} = f_{15}^{mi} - \frac{\rho_{ns:in}^{v:mi} u_z^{m}}{6} - \frac{\rho_{ns:in}^{v:mi} c_{sn\_2}^{mi}}{2\left(\rho_{ns:in}^{v:mi} - 3\right)} \quad f_{ns:16}^{v:mi} = f_{17}^{mi} - \frac{\rho_{ns:in}^{v:mi} u_z^{mi}}{6} + \frac{\rho_{ns:in}^{v:mi} c_{ns\_2}^{mi}}{2\left(\rho_{ns:in}^{v:mi} - 3\right)}, \tag{27}$$

where

$$\rho_{sn:in}^{v:mi} = u_z^{mi} + 2\left(f_6^{mi} + f_{13}^{mi} + f_{14}^{mi} + f_{16}^{mi} + f_{18}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_{10}^{mi} + f_2^{mi} + f_9^{mi} + f_8^{mi} + f_1^{mi} + f_7^{mi}, \tag{28}$$

$$\rho_{ns:in}^{v:mi} = u_z^{mi} + 2\left(f_5^{mi} + f_{11}^{mi} + f_{12}^{mi} + f_{15}^{mi} + f_{17}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_8^{mi} + f_1^{mi} + f_7^{mi} + f_{10}^{mi} + f_2^{mi} + f_9^{mi}. \tag{29}$$

2.3. West-east flow

As shown in Fig. 1c, when the electronic and ionic flow direction is from west to east, after streaming, the unknown distribution function are $f_{we:i}^{mi}$ (i = 1, 7, 8, 11, 13), on the contrary, the unknown distribution functions are $f_{ew:i}^{mi}$ (i = 2, 9, 10, 12, 14).

2.3.1. PC

For the west inlet and east outlet case, $f_{we:i}^{p:mi}$ (i = 1, 7, 8, 11, 13) as defined as

$$f_{we:1}^{p:mi} = f_2^{mi} + \frac{\rho_{in}^{mi} u_{we:x}^{p:mi}}{3} \quad f_{we:7}^{p:mi} = f_{10}^{mi} + \frac{\rho_{in}^{mi} u_{we:x}^{p:mi}}{6} - \frac{c_{we\_1}^{mi}}{4} \quad f_{we:8}^{p:mi} = f_9^{mi} + \frac{\rho_{in}^{mi} u_{we:x}^{p:mi}}{6} + \frac{c_{we\_1}^{mi}}{4}, \tag{30}$$

$$f_{we:11}^{p:mi} = f_{14}^{mi} + \frac{\rho_{in}^{mi} u_{we:x}^{p:mi}}{6} + \frac{c_{we\_2}^{mi}}{4} \quad f_{we:13}^{p:mi} = f_{12}^{mi} + \frac{\rho_{in}^{mi} u_{we:x}^{p:mi}}{6} - \frac{c_{we\_2}^{mi}}{4}. \tag{31}$$

For the east inlet and west outlet case, $f_{ew:i}^{p:mi}$ (i = 2,9,10,12,14) are defined as,

$$f_{ew:2}^{p:mi} = f_1^{mi} - \frac{\rho_{in}^{mi} u_{ew:x}^{p:mi}}{3} \quad f_{ew:9}^{p:mi} = f_8^{mi} - \frac{\rho_{in}^{mi} u_{ew:x}^{p:mi}}{6} - \frac{c_{we\_1}^{mi}}{4} f_{ew:10}^{p:mi} = f_7^{mi} - \frac{\rho_{in}^{mi} u_{ew:x}^{p:mi}}{6} + \frac{c_{we\_1}^{mi}}{4}, \tag{32}$$

$$f_{ew:12}^{p:mi} = f_{13}^{mi} - \frac{\rho_{in}^{mi} u_{ew:x}^{p:mi}}{6} + \frac{c_{we\_2}^{mi}}{4} \quad f_{ew:14}^{p:mi} = f_{11}^{mi} - \frac{\rho_{in}^{mi} u_{ew:x}^{p:mi}}{6} - \frac{c_{we\_2}^{mi}}{4}, \tag{33}$$

where

$$c_{we\_\alpha}^{mi} = \left(-f_3^{mi} + f_4^{mi} - f_{15}^{mi} - f_{16}^{mi} + f_{17}^{mi} + f_{18}^{mi}\right)\delta_{1\alpha} + \left(f_5^{mi} - f_6^{mi} + f_{15}^{mi} - f_{16}^{mi} + f_{17}^{mi} - f_{18}^{mi}\right)\delta_{2\alpha}, \tag{34}$$

$$u_{we:x}^{p:mi} = 1 - \left[2\left(f_2^{mi} + f_9^{mi} + f_{10}^{mi} + f_{12}^{mi} + f_{14}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_5^{mi} + f_{17}^{mi} + f_{15}^{mi} + f_{18}^{mi} + f_6^{mi} + f_{16}^{mi}\right]/\rho_{in}^{m}, \tag{35}$$

$$u_{ew:x}^{p:mi} = 1 - \left[2\left(f_1^{mi} + f_7^{mi} + f_8^{mi} + f_{11}^{mi} + f_{13}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_5^{mi} + f_{17}^{mi} + f_{15}^{mi} + f_{18}^{mi} + f_6^{mi} + f_{16}^{mi}\right]/\rho_{in}^{mi}. \tag{36}$$

2.3.2. VC

For the west inlet and east outlet case, $f_{we:i}^{v:mi}$ (i = 1,7,8,11,13), are defined as

$$f_{ew:1}^{v:mi} = f_2^{mi} + \frac{\rho_{ew:in}^{v:mi} u_x^{mi}}{3} f_{ew:7}^{v:mi} = f_{10}^{mi} + \frac{\rho_{ew:in}^{v:mi} u_x^{mi}}{6} - \frac{\rho_{ew:in}^{v:mi} c_{we\_1}^{mi}}{2\left(\rho_{ew:in}^{v:mi} - 3\right)} f_{ew:8}^{v:mi} = f_9^{mi} + \frac{\rho_{ew:in}^{v:mi} u_x^{mi}}{6} + \frac{\rho_{ew:in}^{v:mi} c_{we\_1}^{m}}{2\left(\rho_{ew:in}^{v:mi} - 3\right)}, \tag{37}$$

$$f_{ew:11}^{v:mi} = f_{14}^{mi} + \frac{\rho_{ew:in}^{v:mi} u_x^{mi}}{6} + \frac{\rho_{ew:in}^{v:mi} c_{we\_2}^{m}}{2\left(\rho_{ew:in}^{v:mi} - 3\right)} \quad f_{ew:13}^{v:mi} = f_{12}^{mi} + \frac{\rho_{ew:in}^{v:mi} u_x^{mi}}{6} - \frac{\rho_{ew:in}^{v:mi} c_{we\_1}^{mi}}{2\left(\rho_{ew:in}^{v:mi} - 3\right)}. \tag{38}$$

For the east inlet and west outlet case, $f_{ew:i}^{v:mi}$ (i = 2,9,10,12,14) are defined as,

$$f_{we:2}^{v:mi} = f_1^{mi} - \frac{\rho_{we:in}^{v:mi} u_x^{mi}}{3} \quad f_{we:10}^{v:mi} = f_7^{mi} - \frac{\rho_{we:in}^{v:mi} u_x^{mi}}{6} + \frac{\rho_{we:in}^{v:mi} c_{we\_1}^{mi}}{2\left(\rho_{we:in}^{v:mi} - 3\right)} \quad f_{we:9}^{v:mi} = f_8^{mi} - \frac{\rho_{we:in}^{v:mi} u_x^{mi}}{6} - \frac{\rho_{we:in}^{v:mi} c_{we\_1}^{mi}}{2\left(\rho_{we:in}^{v:mi} - 3\right)}, \tag{39}$$

$$f_{we:14}^{v:mi} = f_{11}^{mi} - \frac{\rho_{we:in}^{v:mi} u_x^{mi}}{6} - \frac{\rho_{we:in}^{v:mi} c_{we\_2}^{mi}}{2\left(\rho_{we:in}^{v:mi} - 3\right)} \quad f_{we:12}^{v:mi} = f_{13}^{mi} - \frac{\rho_{we:in}^{v:mi} u_x^{mi}}{6} + \frac{\rho_{we:in}^{v:mi} c_{we\_2}^{mi}}{2\left(\rho_{we:in}^{v:mi} - 3\right)}, \tag{40}$$

where

$$\rho_{ew:in}^{v:mi} = u_x^{mi} + \left[2\left(f_2^{mi} + f_9^{mi} + f_{10}^{mi} + f_{12}^{mi} + f_{14}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_5^{mi} + f_{17}^{mi} + f_{15}^{mi} + f_{18}^{mi} + f_6^{mi} + f_{16}^{mi}\right], \tag{41}$$

$$\rho_{we:in}^{v:mi} = u_x^{mi} + \left[2\left(f_1^{mi} + f_7^{mi} + f_8^{mi} + f_{11}^{mi} + f_{13}^{mi}\right) + f_0^{mi} + f_4^{mi} + f_3^{mi} + f_5^{mi} + f_{17}^{mi} + f_{15}^{mi} + f_{18}^{mi} + f_6^{mi} + f_{16}^{mi}\right]. \tag{42}$$

After we define similar the lattice velocity e_i^f and the distribution function f_i^f (i = 0,18) at position $X(x,y,z)$ and time t for the fluid flow, we can obtained the similar pressure and velocity boundary conditions for fluid flow problem (force field).

3. Boundary conditions for the thermal field

It is shown in Fig. 2. A D3Q15 lattice cubic is employed for the thermal field. The lattice velocity e_i^t (i = 0,14) at position $X(x,y,z)$ and time t for the D3Q15 model are listed in the Table 2.

The extended thermal flow density and the macroscopic flow velocity are defined in terms of the particle distribution function by

$$\rho_{in\_ini}^t u_\alpha^t = \sum_{i=0}^{14} f_i^t e_{i\alpha}^t \quad \rho_{in}^t = \sum_{i=0}^{14} f_i^t. \tag{43}$$

The extended thermal flow equilibrium distribution functions for incompressible model and compressible model can be written as follows [19–21],

$$f_i^{eq-t}(x,t) = \alpha_i^t \rho_{in}^t + \alpha_i^t \rho_{in\_ini}^t \left[\frac{e_i^t u_\alpha^t}{c_s^2} + \frac{9\left(e_i^t u_\alpha^t\right)^2}{2c_s^4} - \frac{3\left(u_\alpha^t\right)^2}{2c_s^2}\right] \quad f_i^{eq-t}(x,t) = \alpha_i^t \rho_{in}^t + \alpha_i^t \rho_{in}^t \left[\frac{e_i^t u_\alpha^t}{c_s^2} + \frac{9\left(e_i^t u_\alpha^t\right)^2}{2c_s^4} - \frac{3\left(u_\alpha^t\right)^2}{2c_s^2}\right], \tag{44}$$

where $\alpha_i^t = \frac{2}{9}\delta_{i0} + \frac{1}{9}\delta_{im} + \frac{1}{72}\delta_{in}$ $m = 1 \sim 6$ $n = 7 \sim 14$

3.1. Front-rear flow

As shown in Fig. 2a, when the thermal fluid flow direction is from front to rear, after streaming, the unknown distribution functions are $f_{fr:i}^t$ (i = 3,7,9,12,14), on the contrary, after streaming, the unknown distribution functions are $f_{rf:i}^t$ (i = 10,11,4,8,13).

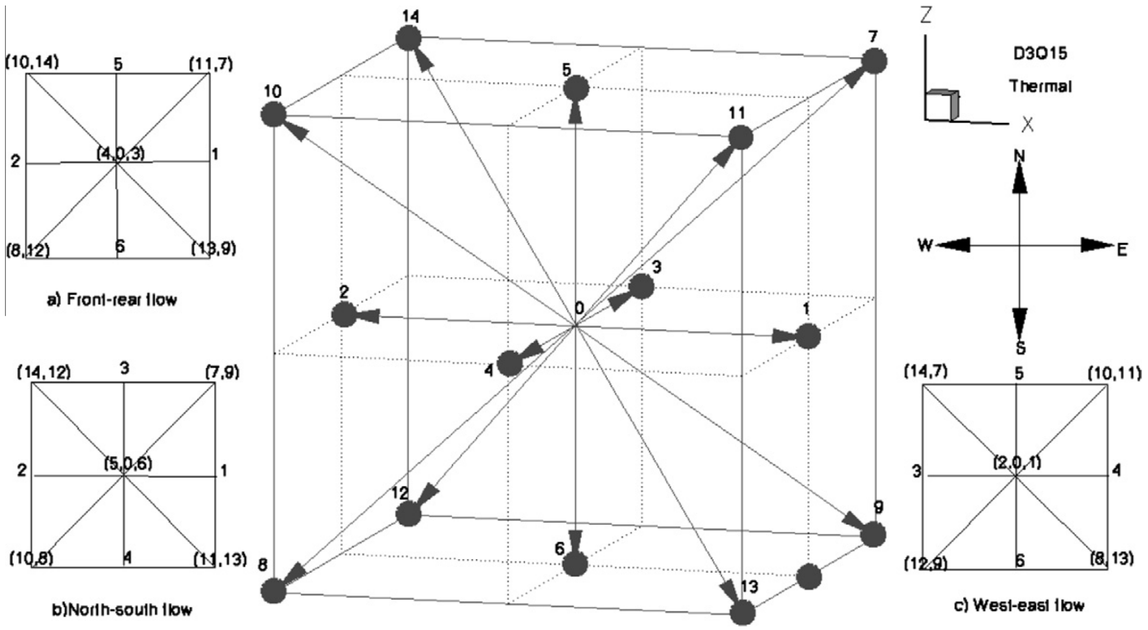

Fig. 2. Cubic lattice D3Q15 for simulating the thermal field.

Table 2
The lattice velocity $e_i^t(X,t)$ ($i = 0, 14$) for the thermal field.

	e_0^t	e_1^t	e_2^t	e_3^t	e_4^t	e_5^t	e_6^t	e_7^t	e_8^t	e_9^t	e_{10}^t	e_{11}^t	e_{12}^t	e_{13}^t	e_{14}^t
e_{i1}^t	0	1	−1	0	0	0	0	1	−1	1	−1	1	−1	1	−1
e_{i2}^t	0	0	0	1	−1	0	0	1	−1	1	1	−1	1	−1	1
e_{i3}^t	0	0	0	0	0	1	−1	1	−1	−1	1	1	−1	−1	1
	C	E	W	R	F	S	N	NRE	SFW	SRW	NFW	NFE	SRW	SFE	NRW

3.1.1. PC

For the front inlet and rear outlet case, using Zou and He method [9], the $f_{fr:i}^{p:t}$ ($i = 3, 7, 9, 12, 14$) can be derived from the Eqs. (43) and (44), and the explicit expressions can be defined as

$$f_{fr:3}^{p:t} = f_4^t + \frac{\rho_{fr:in}^{p:t} u_{fr:y}^{p:t}}{3} f_{fr:7}^{p:t} = f_8^t + \frac{\rho_{fr:in}^{p:t}\left(u_{fr:x}^{p:t} + u_{fr:y}^{p:t} + u_{fr:z}^{p:t}]\right)}{12} \quad f_{fr:9}^{p:t} = f_{10}^t + \frac{\rho_{fr:in}^{p:t}\left(u_{fr:x}^{p:t} + u_{fr:y}^{p:t} - u_{fr:z}^{p:t}\right)}{12}, \tag{45}$$

$$f_{fr:12}^{p:t} = f_{11}^t + \frac{\rho_{fr:in}^{p:t}\left(-u_{fr:x}^{p:t} + u_{fr:y}^{p:t} - u_{fr:z}^{p:t}\right)}{12} \quad f_{fr:14}^{p:t} = f_{13}^t + \frac{\rho_{fr:in}^{p:t}\left(-u_{fr:x}^{p:t} + u_{fr:y}^{p:t} + u_{fr:z}^{p:t}\right)}{12}, \tag{46}$$

For the rear inlet and front outlet case, $f_{rf:i}^{p:t}$ ($i = 4, 8, 10, 11, 13$) can be defined as

$$f_{rf:4}^{p:t} = f_3^t - \frac{\rho_{rf:in}^{p:t} u_{rf:y}^{p:t}}{3} \quad f_{rf:8}^{p:t} = f_7^t - \frac{\rho_{rf:in}^{p:t}\left(-u_{rf:x}^{p:t} + u_{rf:y}^{p:t} + u_{rf:z}^{p:t}\right)}{12} \quad f_{rf:10}^{p:t} = f_9^t - \frac{\rho_{rf:in}^{p:t}\left(u_{rf:x}^{p:t} + u_{rf:y}^{p:t} - u_{rf:z}^{p:t}\right)}{12}, \tag{47}$$

$$f_{rf:13}^{p:t} = f_{14}^t - \frac{\rho_{rf:in}^{p:t}\left(-u_{rf:x}^{p:t} + u_{rf:y}^{p:t} - u_{rf:z}^{p:t}\right)}{12} \quad f_{rf:11}^{p:t} = f_{12}^t - \frac{\rho_{rf:in}^{p:t}\left(-u_{rf:x}^{p:t} + u_{rf:y}^{p:t} - u_{rf:z}^{p:t}\right)}{12}, \tag{48}$$

where

$$u_{fr:x}^{p:t} = \frac{3\rho_{fr:in}^{p:t}}{2}\left(f_1^t - f_2^t\right) \quad u_{fr:z}^{p:t} = -1 + \frac{6}{5}\left(f_5^t - f_6^t + \rho_{fr:in}^{p:t} u_{fr:x}^{p:t} - {}_{fr:in}^{p:t} u_{fr:y}^{p:t}\right), \tag{49}$$

$$u_{fr:y}^{p:t} = 1 - \left[2\left(f_4^t + f_8^t + f_{10}^t + f_{11}^t + f_{13}^t\right) + f_2^t + f_6^t + f_1^t + f_0^t + f_5^t\right]/\rho_{fr:in}^{p:t}, \tag{50}$$

$$u_{rf:x}^{p:t} = \frac{3\rho_{rf:in}^{p:t}}{2}\left(f_2^t - f_1^t\right) \quad u_{rf:z}^{p:t} = \frac{6}{5}\left(f_6^t - f_5^t - \rho_{rf:in}^{p:t} u_{rf:x}^{p:t} + \rho_{rf:in}^{p:t} u_{rf:y}^{p:t}\right), \tag{51}$$

$$u_{rf:y}^{p:t} = 1 - \left[2\left(f_7^t + f_9^t + f_{12}^t + f_{14}^t + f_3^t\right) + f_2^t + f_6^t + f_1^t + f_0^t + f_5^t\right]/{}_{rf:in}^{p:t}. \tag{52}$$

3.1.2. VC

At the situation, the macroscopic flow velocity should be defined as

$$u_\alpha^t = \sum_{i=0}^{14} f_i^t e_{i\alpha}^t. \tag{53}$$

After instead the Eq. (43) with above equation, through the similar derivation of pressure boundary condition, we can obtain the unknown distribution functions for velocity boundary condition. For the front inlet and rear outlet case, $f_{fr:i}^{v:t}$ (i = 3,7,9,12,14), can be defined as

$$f_{fr:3}^{v:t} = f_4^t + \frac{\rho_{in}^{v:t} u_{fr:y}^t}{3} f_{fr:7}^{v:t} = f_8^t + \frac{\rho_{in}^{v:t}\left(u_{fr:x}^t + u_{fr:y}^t + u_{fr:z}^t\right)}{12} \quad f_{fr:9}^{v:t} = f_{10}^t + \frac{\rho_{in}^{v:t}\left(u_{fr:x}^t + u_{fr:y}^t - u_{fr:z}^t\right)}{12}, \tag{54}$$

$$f_{fr:12}^{v:t} = f_{11}^t + \frac{\rho_{in}^{v:t}\left(-u_{fr:x}^t + u_{fr:y}^t - u_{fr:z}^t\right)}{12} \quad f_{fr:14}^{v:t} = f_{13}^t + \frac{\rho_{in}^{v:t}\left(-u_{fr:x}^t + u_{fr:y}^t + u_{fr:z}^t\right)}{12}. \tag{55}$$

For the rear inlet and front outlet case, $f_{fr:i}^{v:t}$ (i = 17,10,18,4,8) can be defined as

$$f_{rf:4}^{v:t} = f_3^t - \frac{\rho_{in}^{v:t} u_{rf:y}^t}{3} \quad f_{rf:8}^{v:t} = f_7^t - \frac{\rho_{in}^{v:t}\left(-u_{rf:x}^t + u_{rf:y}^t + u_{rf:z}^t\right)}{12} \quad f_{rf:10}^{v:t} = f_9^t - \frac{\rho_{in}^{v:t}\left(u_{rf:x}^t + u_{rf:y}^t - u_{rf:z}^t\right)}{12}, \tag{56}$$

$$f_{rf:13}^{v:t} = f_{14}^t - \frac{\rho_{in}^{v:t}\left(-u_{rf:x}^t + u_{rf:y}^t - u_{rf:z}^t\right)}{12} \quad f_{rf:11}^{v:t} = f_{12}^t - \frac{\rho_{in}^{v:t}\left(-u_{rf:x}^t + u_{rf:y}^t - u_{rf:z}^t\right)}{12}, \tag{57}$$

where

$$\rho_{fr:in}^{v:t} = u_y^t + \left[2\left(f_4^t + f_8^t + f_{10}^t + f_{11}^t + f_{13}^t\right) + f_2^t + f_6^t + f_1^t + f_0^t + f_5^t\right], \tag{58}$$

$$\rho_{rf:in}^{v:t} = u_y^t + \left[2\left(f_7^t + f_9^t + f_{12}^t + f_{14}^t + f_3^t\right) + f_2^t + f_6^t + f_1^t + f_0^t + f_5^t\right], \tag{59}$$

$$u_{rf:z}^t = \frac{6\left(f_5^t - f_6^t\right) - \rho_{rf:in}^{v:t} u_{rf:y}^t + \rho_{rf:in}^{v:t} u_{rf:x}^t}{\rho_{rf:in}^{v:t} - 6} \quad u_{fr:x}^t = \frac{3}{\rho_{fr:in}^{v:t} - 3}\left(f_2^t - f_1^t\right) \quad u_{rf:x}^t = \frac{3}{\rho_{fr:in}^{v:t} - 3}\left(f_1^t - f_2^t\right) \quad u_{fr:z}^t = \frac{6\left(f_6^t - f_5^t\right) - \rho_{fr:in}^{v:t} u_{fr:x}^t + \rho_{fr:in}^{v:t} u_{fr:y}^t}{\rho_{fr:in}^{v:t} - 6}. \tag{60}$$

3.2. South-north flow

As shown in Fig. 2b, when the thermal fluid flow direction is from south to north, after streaming, the unknown distribution functions are $f_{sn:i}^t$ (i = 5,7,10,11,14), on the contrary, the unknown distribution functions are $f_{ns:i}^t$ (i = 6,8,9,12,13).

3.2.1. PC

For the south inlet and north outlet case, $f_{sn:i}^{p:t}$ (i = 5,7,10,11,14) can be defined as

$$f_{sn:5}^{p:t} = f_6^t - \frac{\rho_{sn:in}^{p:t} u_{sn:z}^{p:t}}{3} f_{sn:10}^{p:t} = f_9^t - \frac{\rho_{sn:in}^{p:t}\left(u_{sn:x}^{p:t} - u_{sn:y}^{p:t} + u_{sn:z}^{p:t}\right)}{12} \quad f_{sn:14}^{p:t} = f_{13}^t - \frac{\rho_{sn:in}^{p:t}\left(-u_{sn:x}^{p:t} + u_{sn:y}^{p:t} + u_{sn:z}^{p:t}\right)}{12}, \tag{61}$$

$$f_{sn:7}^{p:t} = f_8^t - \frac{\rho_{sn:in}^{p:t}\left(-u_{sn:x}^{p:t} - u_{sn:y}^{p:t} + u_{sn:z}^{p:t}\right)}{12} \quad f_{sn:11}^{p:t} = f_{12}^t - \frac{\rho_{sn:in}^{p:t}\left(u_{sn:x}^{p:t} - u_{sn:y}^{p:t} + u_{sn:z}^{p:t}\right)}{12}. \tag{62}$$

For the north inlet and south outlet case, $f_{ns:i}^{p:t}$ (i = 6,8,9,12,13) can be defined as

$$f_{ns:6}^{p:t} = f_5^t + \frac{\rho_{ns:in}^{p:t} u_{ns:z}^{p:t}}{3} f_{ns:9}^{p:t} = f_{10}^t + \frac{\rho_{ns:in}^{p:t}\left(u_{ns:x}^{p:t} - u_{ns:y}^{p:t} + u_{ns:z}^{p:t}\right)}{12} \quad f_{ns:13}^{p:t} = f_{14}^t + \frac{\rho_{ns:in}^{p:t}\left(-u_{ns:x}^{p:t} + u_{ns:y}^{p:t} + u_{ns:z}^{p:t}\right)}{12}, \tag{63}$$

$$f_{ns:8}^{p:t} = f_7^t + \frac{\rho_{ns:in}^{p:t}\left(-u_{ns:x}^{p:t} - u_{ns:y}^{p:t} + u_{ns:z}^{p:t}\right)}{12} \quad f_{ns:12}^{p:t} = f_{11}^t + \frac{\rho_{ns:in}^{p:t}\left(u_{ns:x}^{p:t} - u_{ns:y}^{p:t} + u_{ns:z}^{p:t}\right)}{12}, \tag{64}$$

where

$$u_{sn:y}^{p:t} = \frac{3\rho_{sn:in}^{p:t}}{2}\left(f_2^t - f_1^t\right) \quad u_{sn:x}^{p:t} = \frac{6}{5}\left(f_6^t - f_5^t - \rho_{sn:in}^{p:t} u_{s:y}^{p:t} + \rho_{sn:in}^{p:t} u_{sn:z}^{p:t}\right), \tag{65}$$

$$u_{sn:z}^{p:t} = 1 - \left[2\left(f_{12}^t + f_9^t + f_6^t + f_8^t + f_3^t\right) + f_1^t + f_6^t + f_2^t + f_3^t + f_4^t\right] / \rho_{sn:in}^{p:t}, \tag{66}$$

$$u_{ns:y}^{p:t} = \frac{3\rho_{ns:in}^{p:t}}{2}\left(f_1^t - f_2^t\right) \quad u_{ns:x}^{p:t} = \frac{6}{5}\left(f_3^t - f_4^t + \rho_{ns:in}^{p:t} u_{sn:y}^{p:t} - \rho_{ns:in}^{p:t} u_{sn:z}^{p:t}\right), \tag{67}$$

$$u_{ns:z}^{p:t} = 1 - \left[2\left(f_{14}^t + f_7^t + f_5^t + f_{10}^t + f_{11}^t\right) + f_1^t + f_0^t + f_2^t + f_3^t + f_4^t\right] / \rho_{ns:in}^{p:t}. \tag{68}$$

3.2.2. VC

For the south inlet and north outlet case, $f^{v:t}_{sn:i}$ (i = 5,7,10,11,14) are defined as

$$f^{v:t}_{sn:5} = f^t_6 - \frac{\rho^{v:t}_{sn:in} u^t_{sn:z}}{3} f^{v:t}_{sn:10} = f^t_9 - \frac{\rho^{v:t}_{sn:in}\left(u^t_{sn:x} - u^t_{sn:y} + u^t_{sn:z}\right)}{12} \quad f^{v:t}_{sn:14} = f^t_{13} - \frac{\rho^{v:t}_{sn:in}\left(-u^t_{sn:x} + u^t_{sn:y} + u^t_{sn:z}\right)}{12}, \tag{69}$$

$$f^{v:t}_{sn:7} = f^t_8 - \frac{\rho^{v:t}_{sn:in}\left(-u^t_{sn:x} - u^t_{sn:y} + u^t_{sn:z}\right)}{12} \quad f^{v:t}_{sn:11} = f^t_{12} - \frac{\rho^{v:t}_{sn:in}\left(u^t_{sn:x} - u^t_{sn:y} + u^t_{sn:z}\right)}{12}. \tag{70}$$

For the north inlet and south outlet case, $f^{v:t}_{ns:i}$ (i = 6,8,9,12,13) are defined as

$$f^{v:t}_{ns:6} = f^t_5 + \frac{\rho^{v:t}_{ns:in} u^t_{ns:z}}{3} f^{v:t}_{ns:9} = f^t_{10} + \frac{\rho^{v:t}_{ns:in}\left(u^t_{ns:x} - u^t_{ns:y} + u^t_{ns:z}\right)}{12} \quad f^{v:t}_{ns:13} = f^t_{14} + \frac{\rho^{v:t}_{ns:in}\left(-u^t_{ns:x} + u^t_{ns:y} + u^t_{ns:z}\right)}{12}, \tag{71}$$

$$f^{v:t}_{ns:8} = f^t_7 + \frac{\rho^{v:t}_{ns:in}\left(-u^t_{ns:x} - u^t_{ns:y} + u^t_{ns:z}\right)}{12} \quad f^{v:t}_{ns:12} = f^t_{11} + \frac{\rho^{v:t}_{ns:in}\left(u^t_{ns:x} - u^t_{ns:y} + u^t_{ns:z}\right)}{12}, \tag{72}$$

where

$$u^t_{ns:x} = \frac{6\left(f^t_3 - f^t_4\right) - \rho^{v:t}_{ns:in} u^t_{ns:y} + \rho^{v:t}_{ns:in} u^t_{ns:z}}{\rho^{v:t}_{ns:in} - 6} \quad u^t_{sn:y} = \frac{3}{\rho^{v:t}_{sn:in} - 3}\left(f^t_1 - f^t_2\right)$$

$$u^t_{sn:x} = \frac{6\left(f^t_3 - f^t_4\right) - \rho^{v:t}_{sn:in} u^t_{sn:z} + \rho^{v:t}_{sn:in} u^t_{sn:y}}{\rho^{v:t}_{sn:in} - 6} \quad u^t_{ns:y} = \frac{3}{\rho^{v:t}_{ns:in} - 3}\left(f^t_2 - f^t_1\right), \tag{73}$$

$$\rho^{v:t}_{ns:in} = u^t_z + \left[2\left(f^t_{14} + f^t_7 + f^t_5 + f^t_{10} + f^t_{11}\right) + f^t_1 + f^t_0 + f^t_2 + f^t_3 + f^t_4\right], \tag{74}$$

$$\rho^{v:t}_{sn:in} = u^t_z + \left[2\left(f^t_{12} + f^t_9 + f^t_6 + f^t_8 + f^t_3\right) + f^t_1 + f^t_6 + f^t_2 + f^t_3 + f^t_4\right]. \tag{75}$$

3.3. West-east flow

As shown in Fig. 2c, when the thermal fluid flow direction is from west to east, after streaming, the unknown distribution function are $f^t_{we:i}$ (i = 1,7,9,11,13), on the contrary, the unknown distribution functions are $f^t_{ew:i}$ (i = 2,8,10,12,14).

3.3.1. PC

For the west inlet and east outlet case, $f^{p:t}_{we:i}$ (i = 1,7,9,11,13) as defined as

$$f^{p:t}_{we:1} = f^t_2 + \frac{\rho^{p:t}_{we:in} u^{p:t}_{we:x}}{3} f^{p:t}_{we:9} = f^t_{10} + \frac{\rho^{p:t}_{we:in}\left(u^{p:t}_{we:x} - u^{p:t}_{we:y} - u^{p:t}_{we:z}\right)}{12} \quad f^{p:t}_{we:13} = f^t_{14} + \frac{\rho^{p:t}_{we:in}\left(u^{p:t}_{we:x} - u^{p:t}_{we:y} + u^{p:t}_{we:z}\right)}{12}, \tag{76}$$

$$f^{p:t}_{we:7} = f^t_8 + \frac{\rho^{p:t}_{we:in}\left(u^{p:t}_{we:x} - u^{p:t}_{we:y} - u^{p:t}_{we:z}\right)}{12} \quad f^{p:t}_{we:11} = f^t_{12} + \frac{\rho^{p:t}_{we:in}\left(u^{p:t}_{we:x} + u^{p:t}_{we:y} - u^{p:t}_{we:z}\right)}{12}. \tag{77}$$

For the east inlet and west outlet case, $f^{p:t}_{ew:i}$ (i = 2,8,10,12,14) are defined as,

$$f^{p:t}_{ew:2} = f^t_1 - \frac{\rho^{p:t}_{ew:in} u^{p:t}_{ew:x}}{3} f^{p:t}_{ew:10} = f^t_9 - \frac{\rho^{p:t}_{ew:in}\left(u^{p:t}_{ew:x} - u^{p:t}_{ew:y} - u^{p:t}_{ew:z}\right)}{12} \quad f^{p:t}_{ew:14} = f^t_{13} - \frac{\rho^{p:t}_{ew:in}\left(u^{p:t}_{ew:x} - u^{p:t}_{ew:y} + u^{p:t}_{ew:z}\right)}{12}, \tag{78}$$

$$f^{p:t}_{ew:8} = f^t_7 + \frac{\rho^{p:t}_{ew:in}\left(u^{p:t}_{ew:x} + u^{p:t}_{ew:y} - u^{p:t}_{ew:z}\right)}{12} \quad f^{p:t}_{ew:12} = f^t_{11} + \frac{\rho^{p:t}_{ew:in}\left(u^{p:t}_{ew:x} + u^{p:t}_{ew:y} - u^{p:t}_{ew:z}\right)}{12}, \tag{79}$$

where

$$u^{p:t}_{we:z} = \frac{3\rho^{p:t}_{we:in}}{2}\left(f^t_3 - f^t_4\right) \quad u^{p:t}_{we:y} = \frac{6}{5}\left(f^t_6 - f^t_5 + \rho^{p:t}_{we:in} u^{p:t}_{we:x} - \rho^{p:t}_{we:in} u^{p:t}_{we:z}\right), \tag{80}$$

$$u^{p:t}_{we:x} = 1 - \left[2\left(f^t_7 + f^t_{11} + f^t_1 + f^t_9 + f^t_{13}\right) + f^t_3 + f^t_0 + f^t_4 + f^t_5 + f^t_6\right]/\rho^{p:t}_{we:in}, \tag{81}$$

$$u^{p:t}_{ew:z} = \frac{3\rho^{p:t}_{ew:in}}{2}\left(f^t_4 - f^t_3\right) \quad u^{p:t}_{ew:y} = \frac{6}{5}\left(f^t_5 - f^t_6 - \rho^{p:t}_{ew:in} u^{p:t}_{ew:z} - \rho^{p:t}_{ew:in} u^{p:t}_{ew:x}\right), \tag{82}$$

$$u^{p:t}_{ew:x} = 1 - \left[2\left(f^t_{14} + f^t_{10} + f^t_2 + f^t_{12} + f^t_8\right) + f^t_3 + f^t_0 + f^t_4 + f^t_5 + f^t_6\right]/\rho^{p:t}_{ew:in}. \tag{83}$$

3.3.2. VC

For the west inlet and east outlet case, $f_{we:i}^{v:t}$ (i = 1,7,9,11,13), are defined as

$$f_{we:1}^{v:t} = f_2^t + \frac{\rho_{we:in}^{v:t} u_{we:x}^t}{3} f_{we:9}^{v:t} = f_{10}^t + \frac{\rho_{we:in}^{v:t}\left(u_{we:x}^t - u_{we:y}^t - u_{we:z}^t\right)}{12} \quad f_{we:13}^{v:t} = f_{14}^t + \frac{\rho_{we:in}^{v:t}\left(u_{we:x}^t - u_{we:y}^t + u_{we:z}^t\right)}{12}, \tag{84}$$

$$f_{we:7}^{v:t} = f_8^t + \frac{\rho_{we:in}^{v:t}\left(u_{we:x}^t - u_{we:y}^t - u_{we:z}^t\right)}{12} \quad f_{we:11}^{v:t} = f_{12}^t + \frac{\rho_{we:in}^{v:t}\left(u_{we:x}^t + u_{we:y}^t - u_{we:z}^t\right)}{12}. \tag{85}$$

For the east inlet and west outlet case, $f_{ew:i}^{v:t}$ (i = 2,8,10,12,14) are defined as,

$$f_{ew:2}^{v:t} = f_1^t - \frac{\rho_{ew:in}^{v:t} u_{ew:x}^t}{3} f_{ew:10}^{v:t} = f_9^t - \frac{\rho_{ew:in}^{v:t}\left(u_{ew:x}^t - u_{ew:y}^t - u_{ew:z}^t\right)}{12} \quad f_{ew:14}^{v:t} = f_{13}^t - \frac{\rho_{ew:in}^{v:t}\left(u_{ew:x}^t - u_{ew:y}^t + u_{ew:z}^t\right)}{12}, \tag{86}$$

$$f_{ew:8}^{v:t} = f_7^t + \frac{\rho_{ew:in}^{v:t}\left(u_{ew:x}^t + u_{ew:y}^t - u_{ew:z}^t\right)}{12} \quad f_{ew:12}^{v:t} = f_{11}^t + \frac{\rho_{ew:in}^{v:t}\left(u_{ew:x}^t + u_{ew:y}^t - u_{ew:z}^t\right)}{12}, \tag{87}$$

where

$$u_{we:z}^{v:t} = \frac{3\rho_{we:in}^{v:t}}{2}\left(f_3^t - f_4^t\right) \quad u_{ew:z}^{v:t} = \frac{3\rho_{ew:in}^{v:t}}{2}\left(f_4^t - f_3^t\right) \quad u_{ew:y}^{v:t} = \frac{6}{5}\left(f_5^t - f_6^t - \rho_{ewv\backslash:in}^{v:t} u_{ew:z}^t - \rho_{ew:in}^{v:t} u_{ew:x}^t\right) u_{we:y}^{v:t}$$
$$= \frac{6}{5}\left(f_6^t - f_5^t + \rho_{we:in}^{v:t} u_{we:x}^t - \rho_{we:in}^{v:t} u_{we:z}^t\right), \tag{88}$$

$$\rho_{we:in}^{v:t} = u_{we:x}^t + \left[2\left(f_7^t + f_{11}^t + f_1^t + f_9^t + f_{13}^t\right) + f_3^t + f_0^t + f_4^t + f_5^t + f_6^t\right], \tag{89}$$

$$\rho_{ew:in}^{v:t} = u_{ew:x}^t + \left[2\left(f_{14}^t + f_{10}^t + f_2^t + f_{12}^t + f_8^t\right) + f_3^t + f_0^t + f_4^t + f_5^t + f_6^t\right]. \tag{90}$$

4. Boundary conditions for the strong coupled electromagnetic field

It is shown in Fig. 3a. D3Q13 lattice cubic is employed for the strong coupled electromagnetic field. The lattice velocity e_i^{mw} (i = 0, 12) at position $X(x,y,z)$ and time t for the D3Q13 model are listed in the Table 3.

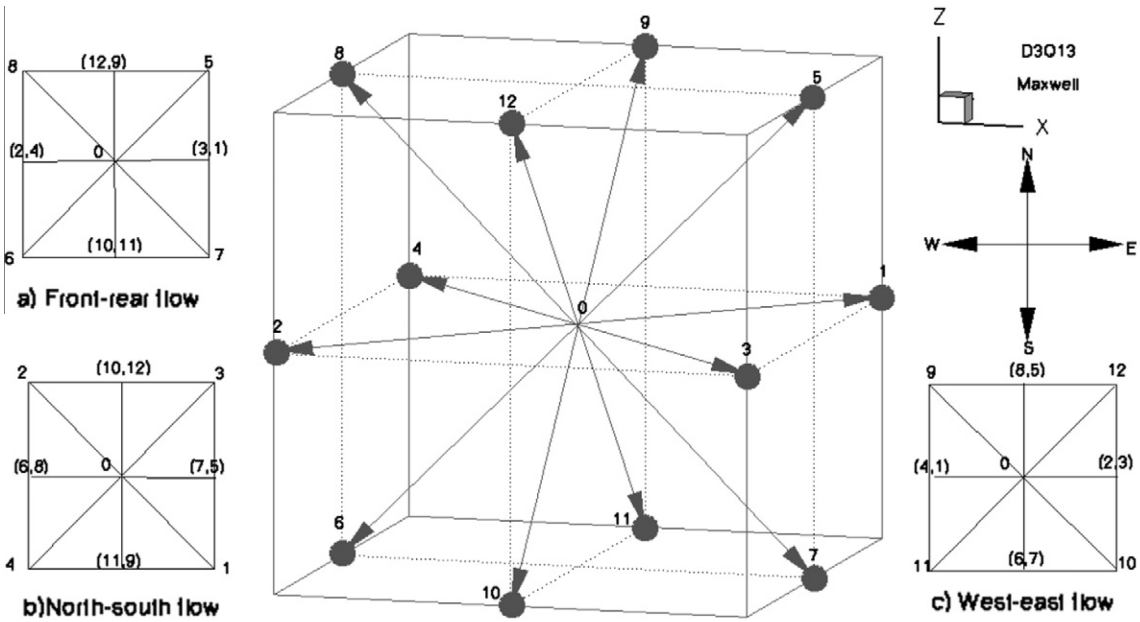

Fig. 3. Cubic lattice D3Q13 for simulating the strong coupled electromagnetic field.

Table 3
The lattice velocity $e_i^{mw}(X,t)$ $(i = 0,12)$ for the strong-coupled electromagnetic field.

	e_0^{mw}	e_1^{mw}	e_2^{mw}	e_3^{mw}	e_4^{mw}	e_5^{mw}	e_6^{mw}	e_7^{mw}	e_8^{mw}	e_9^{mw}	e_{10}^{mw}	e_{11}^{mw}	e_{12}^{mw}
e_{i1}^{mw}	0	1	−1	1	−1	1	−1	1	−1	0	0	0	0
e_{i2}^{mw}	0	1	−1	−1	1	0	0	0	0	1	−1	1	−1
e_{i3}^{mw}	0	0	0	0	0	1	−1	−1	1	1	−1	−1	−1
	C	RE	FW	FE	RW	NE	SW	SE	NW	NR	SF	SR	NF

The extended strong couple electromagnetic density and the macroscopic flow velocity are defined in terms of the particle distribution function by

$$\rho_{in\_ini}^{mw}u_\alpha^{mw} = \sum_{i=0}^{12} f_i^{mw}e_{i\alpha}^{mw} \quad \rho_{in}^{mw} = \sum_{i=0}^{12} f_i^{mw}. \tag{91}$$

The strong couple electromagnetic equilibrium distribution functions for incompressible model and compressible model can be written as follows [19–21],

$$\begin{aligned} f_i^{eq\_mw}(x,t) &= \alpha_i^{mw}\rho_{in}^{mw} + \alpha_i^{mw}\rho_{in\_ini}^{mw}\left[\frac{3e_i^{mw}u_\alpha^{mw}}{c_s^2} + \frac{9\left(e_i^{mw}u_\alpha^{mw}\right)^2}{4c_s^4} - \frac{3\left(u_\alpha^{mw}\right)^2}{2c_s^2}\right] \\ f_i^{eq\_mw}(x,t) &= \alpha_i^{mw}\rho_{in}^{mw} + \alpha_i^{mw}\rho_{in}^{mw}\left[\frac{e_i^{mw}u_\alpha^{mw}}{c_s^2} + \frac{9\left(e_i^{mw}u_\alpha^{mw}\right)^2}{4c_s^4} - \frac{3\left(u_\alpha^{mw}\right)^2}{2c_s^2}\right], \end{aligned} \tag{92}$$

where $\alpha_i^{mw} = 0\delta_{i0} + \frac{1}{8}\delta_{im}$ $m = 1 \sim 12$

4.1. Front-rear flow

As shown in Fig. 3a, when the electromagnetic fluid flow direction is from front to rear, after streaming, the unknown distribution functions are $f_{fr:i}^{mw}$ $(i = 1,4,9,11)$, on the contrary, after streaming, the unknown distribution functions are $f_{rf:i}^{mw}$ $(i = 2,3,10,12)$.

4.1.1. PC

For the front inlet and rear outlet case, using Zou and He method [9], the $f_{fr:i}^{p:mw}$ $(i = 1,4,9,11)$ can be derived from the Eqs. (91) and (92), and the explicit expression can be defined as

$$\begin{aligned} &f_{fr:1}^{p:mw} = f_2^{mw} + \frac{3\rho_{fr:in}^{p:mw}\left(u_{fr:x}^{p:mw} + u_{fr:y}^{p:mw}\right)}{4} \quad f_{fr:4}^{p:mw} = f_3^{mw} + \frac{3\rho_{fr:in}^{p:mw}\left(u_{fr:y}^{p:mw} - u_{fr:x}^{p:mw}\right)}{4} \\ &f_{fr:9}^{p:mw} = f_{10}^{mw} + \frac{3\rho_{fr:in}^{p:mw}\left(u_{fr:y}^{p:mw} + u_{fr:z}^{p:mw}\right)}{4} \quad f_{fr:11}^{p:mw} = f_{12}^{mw} + \frac{3\rho_{fr:in}^{p:mw}\left(u_{fr:y}^{p:mw} - u_{fr:z}^{p:mw}\right)}{4}. \end{aligned} \tag{93}$$

For the rear inlet and front outlet case, $f_{rf:i}^{p:mw}$ $(i = 2,3,10,12)$ can be defined as

$$\begin{aligned} &f_{rf:12}^{p:mw} = f_{11}^{mw} - \frac{3\rho_{rf:in}^{p:mw}\left(u_{rf:y}^{p:mw} - u_{rf:z}^{p:mw}\right)}{4} f_{rf2}^{p:mw} = f_1^{mw} - \frac{3\rho_{rf:in}^{p:mw}\left(u_{rf:x}^{p:mw} + u_{rf:y}^{p:mw}\right)}{4} \\ &f_{rf:10}^{p:mw} = f_9^{mw} - \frac{3\rho_{rf:in}^{p:mw}\left(u_{rf:y}^{p:mw} + u_{rf:z}^{p:mw}\right)}{4} f_{rf:3}^{p:mw} = f_4^{mw} - \frac{3\rho_{rf:in}^{p:mw}\left(u_{rf:y}^{p:mw} - u_{rf:x}^{p:mw}\right)}{4}, \end{aligned} \tag{94}$$

where

$$u_{fr:x}^{p:mw} = 2\rho_{fr:in}^{p:mw}\left(-f_5^{mw} + f_6^{mw} - f_7^{mw} + f_8^{mw}\right) \quad u_{fr:z}^{p:mw} = 2\rho_{fr:in}^{p:mw}\left(-f_5^{mw} + f_6^{mw} + f_7^{mw} - f_8^{mw}\right), \tag{95}$$

$$u_{fr:y}^{p:mw} = 1 - \left[2\left(f_2^{mw} + f_3^{mw} + f_{10}^{mw} + f_{12}^{mw}\right) + f_8^{mw} + f_5^{mw} + f_0^{mw} + f_6^{mw} + f_7^{mw}\right]/\rho_{fr:in}^{p:mw}, \tag{96}$$

$$u_{rf:x}^{p:mw} = 2\rho_{rf:in}^{p:mw}\left(f_5^{mw} - f_6^{mw} + f_7^{mw} - f_8^{mw}\right) \quad u_{rf:z}^{p:mw} = 2\rho_{rf:in}^{p:mw}\left(f_5^{mw} - f_6^{mw} - f_7^{mw} + f_8^{mw}\right), \tag{97}$$

$$u_{rf:y}^{p:mw} = 1 - \left[2\left(f_1^{mw} + f_4^{mw} + f_9^{mw} + f_{11}^{mw}\right) + f_8^{mw} + f_5^{mw} + f_0^{mw} + f_6^{mw} + f_7^{mw}\right]/\rho_{rf:in}^{p:mw}. \tag{98}$$

4.1.2. VC

At the situation, the macroscopic flow velocity should be defined as

$$u_\alpha^{mw} = \sum_{i=0}^{12} f_i^{mw}e_{i\alpha}^{mw}. \tag{99}$$

After instead the Eq. (91) with above equation, through the similar derivation of pressure boundary condition, we can obtain the unknown distribution functions for velocity boundary condition. For the front inlet and rear outlet case, $f_{fr:i}^{v:mw}$ (i = 1,4,9,11), can be defined as

$$f_{fr:1}^{v:mw} = f_2^{mw} + \frac{3\rho_{fr:in}^{v:mw}\left(u_{fr:x}^{v:mw} + u_{fr:y}^{v:mw}\right)}{4} \quad f_{fr:9}^{v:mw} = f_{10}^{mw} + \frac{3\rho_{fr:in}^{v:mw}\left(u_{fr:y}^{v:mw} + u_{fr:z}^{v:mw}\right)}{4}$$
$$f_{fr:4}^{v:mw} = f_3^{mw} + \frac{3\rho_{fr:in}^{v:mw}\left(-u_{fr:x}^{v:mw} + u_{fr:y}^{v:mw}\right)}{4} \quad f_{fr:11}^{v:mw} = f_{12}^{mw} + \frac{3\rho_{fr:in}^{v:mw}\left(u_{fr:y}^{v:mw} - u_{fr:z}^{v:mw}\right)}{4}. \tag{100}$$

For the rear inlet and front outlet case, $f_{rf:i}^{v:mw}$ (i = 2,3,10,12) can be defined as

$$f_{rf2}^{v:mw} = f_1^{mw} - \frac{3\rho_{rf:in}^{v:mw}\left(u_{rf:x}^{v:mw} + u_{rf:y}^{v:mw}\right)}{4} \quad f_{rf:3}^{v:mw} = f_4^{mw} - \frac{3\rho_{rf:in}^{v:mw}\left(-u_{rf:x}^{v:mw} + u_{rf:y}^{v:mw}\right)}{4}$$
$$f_{rf:10}^{v:mw} = f_9^{mw} - \frac{3\rho_{rf:in}^{v:mw}\left(u_{rf:y}^{v:mw} + u_{rf:z}^{v:mw}\right)}{4} \quad f_{rf:12}^{v:mw} = f_{11}^{mw} - \frac{3\rho_{rf:in}^{v:mw}\left(u_{rf:y}^{v:mw} - u_{rf:z}^{v:mw}\right)}{4}, \tag{101}$$

where

$$u_{fr:x}^{v:mw} = \frac{2}{3\rho_{fr:in}^{v:mw} - 2}\left(-f_5^{mw} + f_6^{mw} - f_7^{mw} + f_8^{mw}\right) \quad u_{fr:z}^{v:mw} = \frac{2}{3\rho_{fr:in}^{v:mw} - 2}\left(-f_5^{mw} + f_6^{mw} + f_7^{mw} - f_8^{mw}\right), \tag{102}$$
$$\rho_{fr:in}^{v:mw} = u_{fr:y}^{v:mw} + \left[2\left(f_2^{mw} + f_3^{mw} + f_{10}^{mw} + f_{12}^{mw}\right) + f_8^{mw} + f_5^{mw} + f_0^{mw} + f_6^{mw} + f_7^{mw}\right], \tag{103}$$
$$u_{rf:x}^{v:mw} = \frac{2}{3\rho_{rf:in}^{v:mw} - 2}\left(f_5^{mw} - f_6^{mw} + f_7^{mw} - f_8^{mw}\right) \quad u_{rf:z}^{v:mw} = \frac{2}{3\rho_{rf:in}^{v:mw} - 2}\left(f_5^{mw} - f_6^{mw} - f_7^{mw} + f_8^{mw}\right), \tag{104}$$
$$\rho_{rf:in}^{v:mw} = u_{rf:y}^{v:mw} + \left[2\left(f_1^{mw} + f_4^{mw} + f_9^{mw} + f_{11}^{mw}\right) + f_8^{mw} + f_5^{mw} + f_0^{mw} + f_6^{mw} + f_7^{mw}\right]. \tag{105}$$

4.2. South-north flow

As shown in Fig. 3b, when electromagnetic fluid flow direction is from south to north, after streaming, the unknown distribution functions are $f_{sn:i}^{mw}$ (i = 5,8,9,12), on the contrary, the unknown distribution functions are $f_{ns:i}^{mw}$ (i = 6,7,10,11).

4.2.1. PC

For the south inlet and north outlet case, $f_{sn:i}^{p:mw}$ (i = 5,8,9,12), can be defined a

$$f_{sn:8}^{p:mw} = f_7^{mw} + \frac{3\rho_{sn;in}^{p:mw}(u_{sn:z}^{p:mw} - u_{sn:y}^{p:mw})}{4} \quad f_{sn:12}^{p:mw} = f_{11}^{mw} + \frac{3\rho_{sn;in}^{p:mw}(u_{sn:x}^{p:mw} + u_{sn:z}^{p:mw})}{4}$$
$$f_{sn:9}^{p:mw} = f_{10}^{mw} + \frac{3\rho_{sn;in}^{p:mw}(u_{sn:z}^{p:mw} - u_{sn:x}^{p:mw})}{4} \quad f_{sn:5}^{p:mw} = f_6^{mw} + \frac{3\rho_{sn;in}^{p:mw}(u_{sn:y}^{p:mw} + u_{sn:z}^{p:mw})}{4}. \tag{106}$$

For the north inlet and south outlet case, $f_{ns:i}^{p:mw}$ (i = 6,7,10,11) can be defined as

$$f_{ns:7}^{p:mw} = f_8^{mw} - \frac{3\rho_{ns;in}^{p:mw}(u_{ns:z}^{p:mw} - u_{ns:y}^{p:mw})}{4} \quad f_{ns:11}^{p:mw} = f_{12}^{mw} - \frac{3\rho_{ns;in}^{p:mw}(u_{ns:x}^{p:mw} + u_{ns:z}^{p:mw})}{4}$$
$$f_{ns:10}^{p:mw} = f_{10}^{mw} + \frac{3\rho_{ns;in}^{p:mw}(-u_{ns:x}^{p:mw} + u_{ns:z}^{p:mw})}{4} \quad f_{ns:6}^{p:mw} = f_5^{mw} + \frac{3\rho_{ns;in}^{p:mw}(u_{ns:y}^{p:mw} + u_{ns:z}^{p:mw})}{4}, \tag{107}$$

where

$$u_{sn:z}^{p:mw} = 2\rho_{sn;in}^{p:mw}\left(-f_3^{mw} + f_4^{mw} - f_1^{mw} + f_2^{mw}\right) \quad u_{sn:x}^{p:mw} = 2\rho_{sn;in}^{p:mw}\left(-f_3^{mw} + f_4^{mw} + f_1^{mw} - f_2^{mw}\right), \tag{108}$$
$$u_{sn:x}^{p:mw} = 1 - \left[2\left(f_7^{mw} + f_6^{mw} + f_{10}^{mw} + f_{11}^{mw}\right) + f_2^{mw} + f_3^{mw} + f_0^{mw} + f_4^{mw} + f_1^{mw}\right]/\rho_{sn;in}^{p:mw}, \tag{109}$$
$$u_{ns:z}^{p:mw} = 2\rho_{ns;in}^{p:mw}\left(f_3^{mw} - f_4^{mw} + f_1^{mw} - f_2^{mw}\right) \quad u_{ns:x}^{p:mw} = 2\rho_{ns;in}^{p:mw}\left(f_3^{mw} - f_4^{mw} - f_1^{mw} + f_2^{mw}\right), \tag{110}$$
$$u_{ns:x}^{p:mw} = 1 - \left[2\left(f_8^{mw} + f_{12}^{mw} + f_9^{mw} + f_5^{mw}\right) + f_2^{mw} + f_3^{mw} + f_0^{mw} + f_4^{mw} + f_1^{mw}\right]/\rho_{ns;in}^{p:mw}. \tag{111}$$

4.2.2. VC

For the south inlet and north outlet case, $f_{sn:i}^{v:mw}$ (i = 5,8,9,12) are defined as

$$f_{sn:5}^{v:mw} = f_6^{mw} + \frac{3\rho_{sn:in}^{v:mw}\left(u_{sn:y}^{v:mw} + u_{sn:z}^{v:mw}\right)}{4} f_{sn:8}^{v:mw} = f_7^{mw} + \frac{3\rho_{sn:in}^{v:mw}\left(-u_{sn:y}^{v:mw} + u_{sn:z}^{v:mw}\right)}{4}$$
$$f_{sn:9}^{v:mw} = f_{10}^{mw} + \frac{3\rho_{sn:in}^{v:mw}(-u_{sn:x}^{v:mw} + u_{sn:z}^{v:mw})}{4} f_{sn:12}^{v:mw} = f_{11}^{mw} + \frac{3\rho_{sn:in}^{v:mw}(u_{sn:x}^{v:mw} + u_{sn:z}^{v:mw})}{4}. \tag{112}$$

For the north inlet and south outlet case, $f_{ns:i}^{v:mw}$ (i = 6,7,10,11) are defined as

$$f_{ns:7}^{v:mw} = f_8^{mw} - \frac{3\rho_{ns:in}^{v:mw}\left(-u_{ns:y}^{v:mw} + u_{ns:z}^{v:mw}\right)}{4} \quad f_{ns:11}^{v:mw} = f_{12}^{mw} - \frac{3\rho_{ns:in}^{v:mw}\left(u_{ns:x}^{v:mw} + u_{ns:z}^{v:mw}\right)}{4}$$
$$f_{ns:10}^{v:mw} = f_{10}^{mw} + \frac{3\rho_{ns:in}^{v:mw}\left(-u_{ns:x}^{v:mw} + u_{ns:z}^{v:mw}\right)}{4} f_{ns:6}^{v:mw} = f_5^{mw} + \frac{3\rho_{ns:in}^{v:mw}\left(u_{ns:y}^{v:mw} + u_{ns:z}^{v:mw}\right)}{4}, \tag{113}$$

where

$$u_{sn:z}^{v:mw} = \frac{2}{3\rho_{sn:in}^{v:mw} - 2}\left(-f_3^{mw} + f_4^{mw} - f_1^{mw} + f_2^{mw}\right) \quad u_{sn:x}^{p:mw} = \frac{2}{3\rho_{sn:in}^{v:mw} - 2}\left(-f_3^{mw} + f_4^{mw} + f_1^{mw} - f_2^{mw}\right), \tag{114}$$
$$\rho_{sn:in}^{v:mw} = u_{sn:x}^{v:mw} + \left[2\left(f_7^{mw} + f_6^{mw} + f_{10}^{mw} + f_{11}^{mw}\right) + f_2^{mw} + f_3^{mw} + f_0^{mw} + f_4^{mw} + f_1^{mw}\right], \tag{115}$$
$$u_{ns:z}^{v:mw} = \frac{2}{3\rho_{ns:in}^{v:mw} - 2}\left(f_3^{mw} - f_4^{mw} + f_1^{mw} - f_2^{mw}\right) \quad u_{ns:x}^{v:mw} = \frac{2}{3\rho_{ns:in}^{v:mw} - 2}\left(f_3^{mw} - f_4^{mw} - f_1^{mw} + f_2^{mw}\right), \tag{116}$$
$$\rho_{ns:in}^{v:mw} = u_{ns:x}^{v:mw} + \left[2\left(f_8^{mw} + f_{12}^{mw} + f_9^{mw} + f_5^{mw}\right) + f_2^{mw} + f_3^{mw} + f_0^{mw} + f_4^{mw} + f_1^{mw}\right]. \tag{117}$$

4.3. West-east flow

As shown in Fig. 3c, when the electromagnetic fluid flow direction is from west to east, after streaming, the unknown distribution function are $f_{we:i}^{mw}$ (i = 1,3,5,7), on the contrary, the unknown distribution functions are $f_{ew:i}^{mw}$ (i = 2,4,6,8).

4.3.1. PC

For the west inlet and east outlet case, $f_{we:i}^{p:mw}$ (i = 1,3,5,7) as defined as

$$f_{we:1}^{p:mw} = f_2^{mw} + \frac{3\rho_{we:in}^{p:mw}\left(u_{we:x}^{p:mw} - u_{we:z}^{p:mw}\right)}{4} \quad f_{we:3}^{p:mw} = f_4^{mw} + \frac{3\rho_{we:in}^{p:mw}\left(u_{we:x}^{p:mw} + u_{we:z}^{p:mw}\right)}{4}$$
$$f_{we:7}^{p:mw} = f_8^{mw} + \frac{3\rho_{we:in}^{p:mw}\left(u_{we:x}^{p:mw} - u_{we:y}^{p:mw}\right)}{4} f_{we:5}^{p:mw} = f_6^{mw} + \frac{3\rho_{we:in}^{p:mw}\left(u_{we:x}^{p:mw} + u_{we:y}^{p:mw}\right)}{4}. \tag{118}$$

For the east inlet and west outlet case, $f_{ew:i}^{p:mw}$ (i = 2,4,6,8) are defined as,

$$f_{ew:2}^{p:mw} = f_1^{mw} - \frac{3\rho_{ew:in}^{p:mw}\left(u_{ew:x}^{p:mw} - u_{ew:z}^{p:mw}\right)}{4} \quad f_{we:4}^{p:mw} = f_3^{mw} + \frac{3\rho_{ew:in}^{p:mw}\left(u_{ew:x}^{p:mw} + u_{ew:z}^{p:mw}\right)}{4}$$
$$f_{ew:6}^{p:mw} = f_5^{mw} - \frac{3\rho_{ew:in}^{p:mw}\left(u_{ew:x}^{p:mw} + u_{ew:y}^{p:mw}\right)}{4} f_{ew:8}^{p:mw} = f_7^{mw} + \frac{3\rho_{ew:in}^{p:mw}\left(u_{ew:x}^{p:mw} - u_{ew:y}^{p:mw}\right)}{4}, \tag{119}$$

where

$$u_{we:y}^{p:mw} = 2\rho_{we:in}^{p:mw}\left(-f_{12}^{mw} + f_{11}^{mw} + f_{10}^{mw} - f_9^{mw}\right) \quad u_{we:z}^{p:mw} = 2\rho_{we:in}^{p:mw}\left(-f_{12}^{mw} + f_{11}^{mw} - f_{10}^{mw} + f_9^{mw}\right), \tag{120}$$
$$u_{we:x}^{p:mw} = 1 - \left[2\left(f_2^{mw} + f_4^{mw} + f_6^{mw} + f_8^{mw}\right) + f_9^{mw} + f_{12}^{mw} + f_0^{mw} + f_{11}^{mw} + f_{10}^{mw}\right]/\rho_{we:in}^{p:mw}, \tag{121}$$
$$u_{ew:y}^{p:mw} = 2\rho_{ew:in}^{p:mw}\left(f_{12}^{mw} - f_{11}^{mw} - f_{10}^{mw} - f_9^{mw}\right) \quad u_{ew:z}^{p:mw} = 2\rho_{ew:in}^{p:mw}\left(-f_3^{mw} + f_4^{mw} + f_1^{mw} - f_2^{mw}\right), \tag{122}$$
$$u_{ew:x}^{p:mw} = 1 - \left[2\left(f_3^{mw} + f_1^{mw} + f_5^{mw} + f_7^{mw}\right) + f_2^{mw} + f_3^{mw} + f_0^{mw} + f_4^{mw} + f_1^{mw}\right]/\rho_{ew:in}^{p:mw}. \tag{123}$$

4.3.2. VC

For the west inlet and east outlet case, $f_{we:i}^{v:mw}$ (i = 1,3,5,7), are defined as

$$f_{we:1}^{v:mw} = f_2^{mw} + \frac{3\rho_{we:in}^{v:mw}\left(u_{we:x}^{v:mw} - u_{we:z}^{v:mw}\right)}{4} \quad f_{we:3}^{v:mw} = f_4^{mw} + \frac{3\rho_{se:in}^{v:mw}\left(u_{we:x}^{v:mw} + u_{we:z}^{v:mw}\right)}{4}$$
$$f_{we:5}^{v:mw} = f_6^{mw} + \frac{3\rho_{we:in}^{v:mw}\left(u_{we:x}^{v:mw} + u_{we:y}^{v:mw}\right)}{4} \quad f_{we:7}^{v:mw} = f_8^{mw} + \frac{3\rho_{we:in}^{v:mw}\left(u_{we:x}^{v:mw} - u_{we:y}^{v:mw}\right)}{4}. \tag{124}$$

For the east inlet and west outlet case, $f_{ew:i}^{v:mi}$ (i = 2,4,6,8) are defined as,

$$f_{ew:2}^{v:mw} = f_1^{mw} - \frac{3\rho_{ew:in}^{v:mw}\left(u_{ew:x}^{v:mw} - u_{ew:z}^{v:mw}\right)}{4} f_{ew:4}^{v:mw} = f_3^{mw} + \frac{3\rho_{ew:in}^{v:mw}\left(u_{ew:x}^{v:mw} + u_{ew:z}^{v:mw}\right)}{4}$$
$$f_{ew:6}^{v:mw} = f_5^{mw} - \frac{3\rho_{ew:in}^{v:mw}\left(u_{ew:x}^{v:mw} + u_{ew:y}^{v:mw}\right)}{4} \quad f_{ew:8}^{v:mw} = f_7^{mw} + \frac{3\rho_{ew:in}^{v:mw}\left(u_{ew:x}^{v:mw} - u_{ew:y}^{v:mw}\right)}{4}, \tag{125}$$

where

$$u_{we:y}^{v:mw} = \frac{2}{3\rho_{we:in}^{v:mw} - 2}\left(-f_{12}^{mw} + f_{11}^{mw} + f_{10}^{mw} - f_{9}^{mw}\right) \quad u_{we:z}^{v:mw} = \frac{2}{3\rho_{we:in}^{v:mw} - 2}\left(-f_{12}^{mw} + f_{11}^{mw} - f_{10}^{mw} + f_{9}^{mw}\right), \tag{126}$$

$$\rho_{we:in}^{v:mw} = u_{we:x}^{v:mw} + \left[2\left(f_{2}^{mw} + f_{4}^{mw} + f_{6}^{mw} + f_{8}^{mw}\right) + f_{9}^{mw} + f_{12}^{mw} + f_{0}^{mw} + f_{11}^{mw} + f_{10}^{mw}\right], \tag{127}$$

$$u_{ew:y}^{p:mw} = \frac{2}{3\rho_{ew:in}^{v:mw} - 2}\left(f_{12}^{mw} - f_{11}^{mw} - f_{10}^{mw} - f_{9}^{mw}\right) \quad u_{ew:z}^{p:mw} = \frac{2}{3\rho_{ew:in}^{v:mw} - 2}\left(-f_{3}^{mw} + f_{4}^{mw} + f_{1}^{mw} - f_{2}^{mw}\right), \tag{128}$$

$$\rho_{ew:in}^{v:mw} = u_{ew:x}^{p:mw} + \left[2\left(f_{3}^{mw} + f_{1}^{mw} + f_{5}^{mw} + f_{7}^{mw}\right) + f_{2}^{mw} + f_{3}^{mw} + f_{0}^{mw} + f_{4}^{mw} + f_{1}^{mw}\right]. \tag{129}$$

After we define similar the lattice velocity e_i^e and the distribution function f_i^e ($i = 0, 12$) at position $X(x, y, z)$ and time t for the electric fluid flow, we can obtained the similar pressure and velocity boundary conditions for electric fluid flow field.

5. Boundary conditions for magnetic field

It is shown in Fig. 4a. D3Q7 lattice cubic is employed for the magnetic field. The lattice velocity e_i^m ($i = 0, 6$) at position $X(x, y, z)$ and time t for the D3Q7 model are listed in the Table 4.

The extended magnetic density and the macroscopic flow velocity are defined in terms of the particle distribution function by

$$\rho_{in\_ini}^{m} u_{\alpha}^{m} = \sum_{i=0}^{6} f_i^m e_{i\alpha}^m \quad \rho_{in}^{m} = \sum_{i=0}^{6} f_i^m. \tag{130}$$

The extended magnetic equilibrium distribution functions for incompressible model and compressible model can be written as follows [19–21],

$$f_i^{eq\_m}(x,t) = \alpha_i^m \rho_{in}^m + \alpha_i^m \rho_{in\_ini}^m \left[\frac{3 e_i^m u_\alpha^m}{c_s^2} + \frac{9\left(e_i^m u_\alpha^{mi}\right)^2}{4 c_s^4} - \frac{3\left(u_\alpha^m\right)^2}{2 c_s^2}\right]$$

$$f_i^{eq\_m}(x,t) = \alpha_i^m \rho_{in}^m + \alpha_i^m \rho_{in}^m \left[\frac{3 e_i^m u_\alpha^m}{c_s^2} + \frac{9\left(e_i^m u_\alpha^{m}\right)^2}{4 c_s^4} - \frac{3\left(u_\alpha^m\right)^2}{2 c_s^2}\right], \tag{131}$$

where $\alpha_i^{mi} = \frac{1}{6}\delta_{i0} + \frac{1}{72}\delta_{im} + \frac{1}{36}\delta_{in}$ $m = 1 \sim 4$ $n = 5 \sim 6$

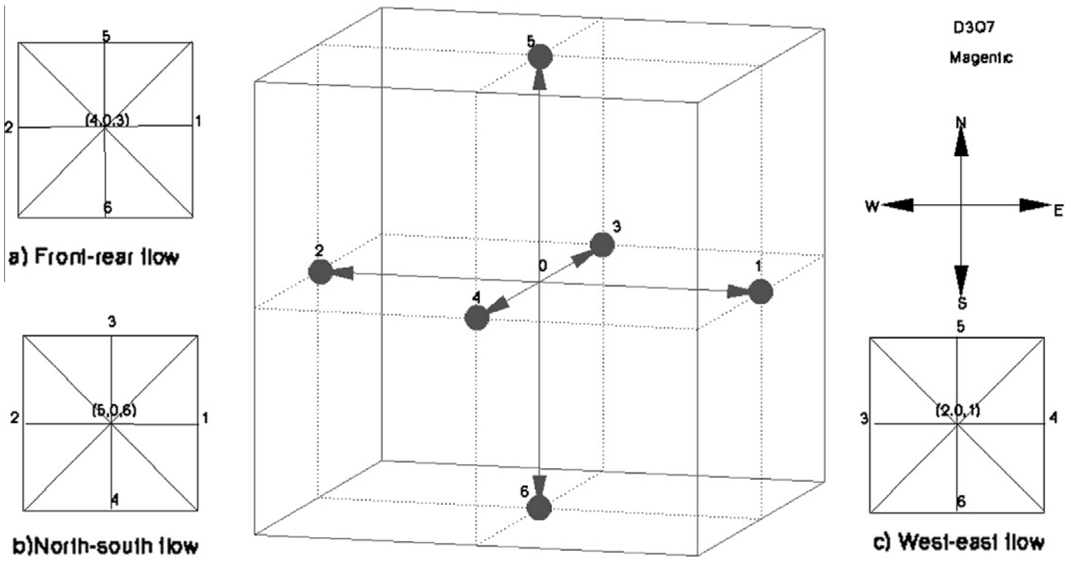

Fig. 4. Cubic Lattice D3Q7 for simulating the magnetic field.

Table 4
The lattice velocity $e_i^m(X,t)$ ($i = 0, 6$) for the magnetic field.

	e_0^m	e_1^m	e_2^m	e_3^m	e_4^m	e_5^m	e_6^m
e_{i1}^m	0	1	−1	0	0	0	0
e_{i2}^m	0	0	0	1	−1	0	0
e_{i3}^m	0	0	0	0	0	1	−1
	C	E	W	R	F	S	N

5.1. Front-rear flow

As shown in Fig. 4a, when the magnetic fluid flow direction is from front to rear, after streaming, the unknown distribution functions are $f^{m}_{fr:3}$, on the contrary, after streaming, the unknown distribution functions are $f^{mi}_{rf:4}$.

5.1.1. PC

For the front inlet and rear outlet case, using Zou and He method [9], the $f^{p:m}_{fr:3}$ can be derived from the Eqs. (130) and (131), and the explicit expression can be defined as

$$f^{p:m}_{fr:3} = f^{m}_{4} + \left(\rho^{p:m}_{fr:in} - 2f^{m}_{4} - f^{m}_{0} - f^{m}_{1} - f^{m}_{5} - f^{m}_{2} - f^{m}_{6}\right)/3. \tag{132}$$

For the rear inlet and front outlet case, $f^{p:m}_{rf:4}$ can be defined as

$$f^{p:m}_{rf:4} = f^{m}_{3} - \left(-\rho^{p:m}_{rf:in} + 2f^{m}_{3} + f^{m}_{0} + f^{m}_{1} + f^{m}_{5} + f^{m}_{2} + f^{m}_{6}\right)/3. \tag{133}$$

5.1.2. VC

At the situation, the macroscopic flow velocity should be defined as

$$u^{m}_{\alpha} = \sum_{i=0}^{6} f^{m}_{i} e^{m}_{i\alpha}. \tag{134}$$

After instead the Eq. (130) with above equation, through the similar derivation of pressure boundary condition, we can obtain the unknown distribution functions for velocity boundary condition. For the front inlet and rear outlet case, $f^{v:m}_{fr:3}$, can be defined as

$$f^{v:m}_{fr:3} = f^{m}_{4} + u^{v:m}_{fr:y}\left(-u^{m}_{y} + 2f^{m}_{4} + f^{m}_{0} + f^{m}_{1} + f^{m}_{5} + f^{m}_{2} + f^{m}_{6}\right)/3. \tag{135}$$

For the rear inlet and front outlet case, $f^{v:m}_{rf:4}$, can be defined as

$$f^{v:m}_{rf:4} = f^{m}_{3} - u^{v:m}_{rf:y}\left(-u^{m}_{y} + 2f^{m}_{3} + f^{m}_{0} + f^{m}_{1} + f^{m}_{5} + f^{m}_{2} + f^{m}_{6}\right)/3. \tag{136}$$

5.2. South-north flow

As shown in Fig. 4b, when the magnetic fluid flow direction is from south to north, after streaming, the unknown distribution functions are $f^{m}_{sn:5}$, on the contrary, the unknown distribution functions are $f^{mi}_{ns:6}$.

5.2.1. PC

For the south inlet and north outlet case, $f^{p:m}_{sn:5}$ can be defined as

$$f^{p:m}_{sn:5} = f^{m}_{6} + \left(\rho^{p:m}_{sn:in} - 2f^{m}_{6} - f^{m}_{0} - f^{m}_{4} - f^{m}_{3} - f^{m}_{2} - f^{m}_{1}\right)/3. \tag{137}$$

For the north inlet and south outlet case, $f^{p:m}_{ns:6}$ can be defined as

$$f^{p:m}_{ns:6} = f^{m}_{5} - \left(\rho^{p:m}_{ns:in} - 2f^{m}_{5} - f^{m}_{0} - f^{m}_{4} - f^{m}_{3} - f^{m}_{1} - f^{m}_{2}\right)/3. \tag{138}$$

5.2.2. VC

For the south inlet and north outlet case, $f^{v:m}_{sn:5}$ is defined as

$$f^{v:m}_{sn:5} = f^{m}_{6} + u^{v:m}_{sn:z}\left(u^{m}_{z} + 2f^{m}_{6} + f^{m}_{0} + f^{m}_{4} + f^{m}_{3} + f^{m}_{2} + f^{m}_{1}\right)/3. \tag{139}$$

For the north inlet and south outlet case, $f^{v:m}_{ns:6}$ is defined as

$$f^{v:m}_{ns:6} = f^{m}_{5} - u^{v:m}_{ns:z}\left(u^{m}_{z} + 2f^{m}_{5} + f^{m}_{0} + f^{m}_{4} + f^{m}_{3} + f^{m}_{1} + f^{m}_{2}\right)/3. \tag{140}$$

5.3. West-east flow

As shown in Fig. 4c, when the magnetic fluid flow direction is from west to east, after streaming, the unknown distribution function is $f^{m}_{we:1}$, on the contrary, the unknown distribution functions is $f^{m}_{ew:2}$.

5.3.1. PC

For the west inlet and east outlet case, $f_{we:1}^{p:m}$ is defined as,

$$f_{we:1}^{p:m} = f_2^m + \left(\rho_{we;in}^{p:m} - 2f_2^m - f_0^m - f_4^m - f_3^m - f_5^m - f_6^m\right)/3. \tag{141}$$

For the east inlet and west outlet case, $f_{ew:2}^{p:m}$ is defined as,

$$f_{ew:2}^{p:m} = f_1^m - \left(\rho_{ew;in}^{p:m} - 2f_1^m - f_0^m - f_4^m - f_3^m - f_5^m - f_6^m\right)/3. \tag{142}$$

5.3.2. VC

For the west inlet and east outlet case, $f_{we:1}^{v:m}$ are defined as

$$f_{we:1}^{v:m} = f_2^m + u_{we:x}^{v:m}(u_x^m + 2f_2^m + f_0^m + f_4^m + f_3^m + f_5^m + f_6^m)/3. \tag{143}$$

For the east inlet and west outlet case, $f_{ew:2}^{v:m}$ are defined as,

$$f_{ew:2}^{v:m} = f_1^m - u_{ew:x}^{v:m}(u_x^m + 2f_1^m + f_0^m + f_4^m + f_3^m + f_5^m + f_6^m)/3. \tag{144}$$

6. Boundary conditions for multiple coupled fields

The extended hybrid cubic lattice D3Q27 model is defined by combined D3Q19 (electric and ionic field) model, D3Q19 (force field) model, D3Q15 (thermal fields) model, D3Q13 (Maxwell equation-strong electromagnetic coupled field) model, D3Q13 (electric field) model and D3Q7 (magnetic field) model for multiple coupled fields, the extended multiple coupled pressure and velocity condition and derive the extended distribution functions for every kind of possible case was established, see Figs. 5 and 6.

The brief presentations of the extended lattice velocity $e_i(X,t)$ (i = 0∼ 26) at position $X(x,y,z)$ and time t, and the distribution functions for hybrid D3Q27 model are given in Table 5, where c_{kn}^l $(l = 1 \sim 27, k = 1 \sim 6, n = 1 \sim 6)$ is coupled coefficient matrix $(27 \times 6 \times 6)$, further details can be found in Refs. [22,23].

The extended multiple coupled fields density and the macroscopic flow velocity are defined in terms of the particle distribution function by

$$\rho_{in\_ini} u_\alpha = \sum_{i=0}^{26} f_i e_{i\alpha} \quad \rho_{in} = \sum_{i=0}^{26} f_i. \tag{145}$$

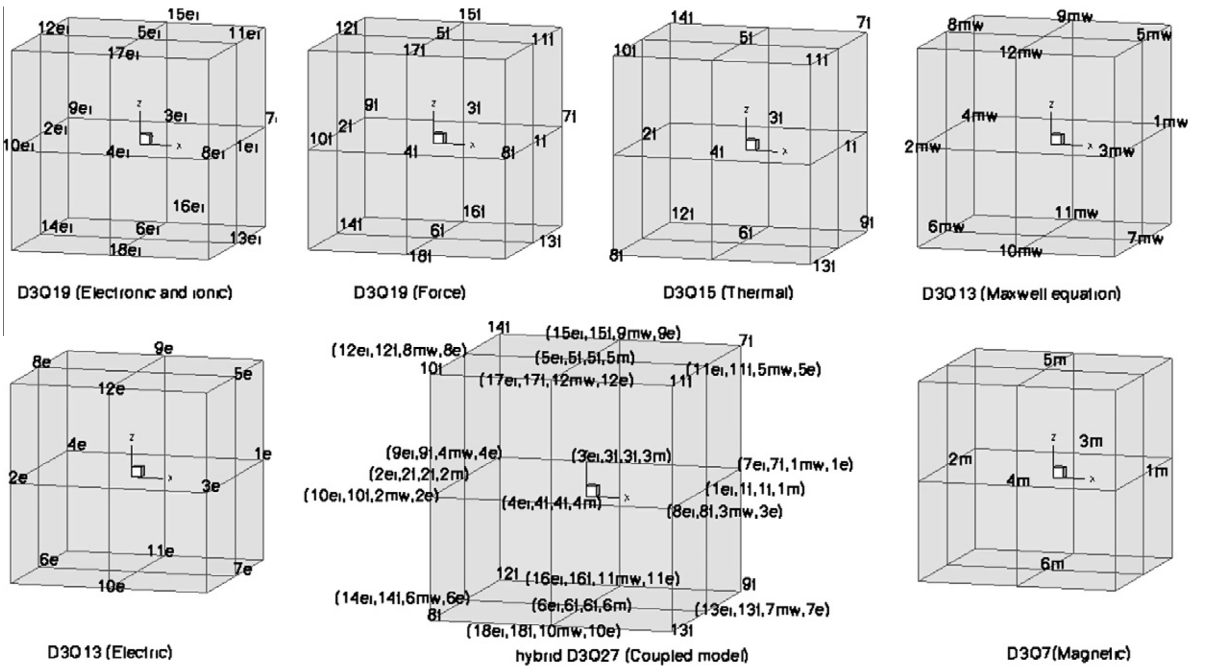

Fig. 5. Hybrid cubic Lattice D3Q27 multiple coupled fields.

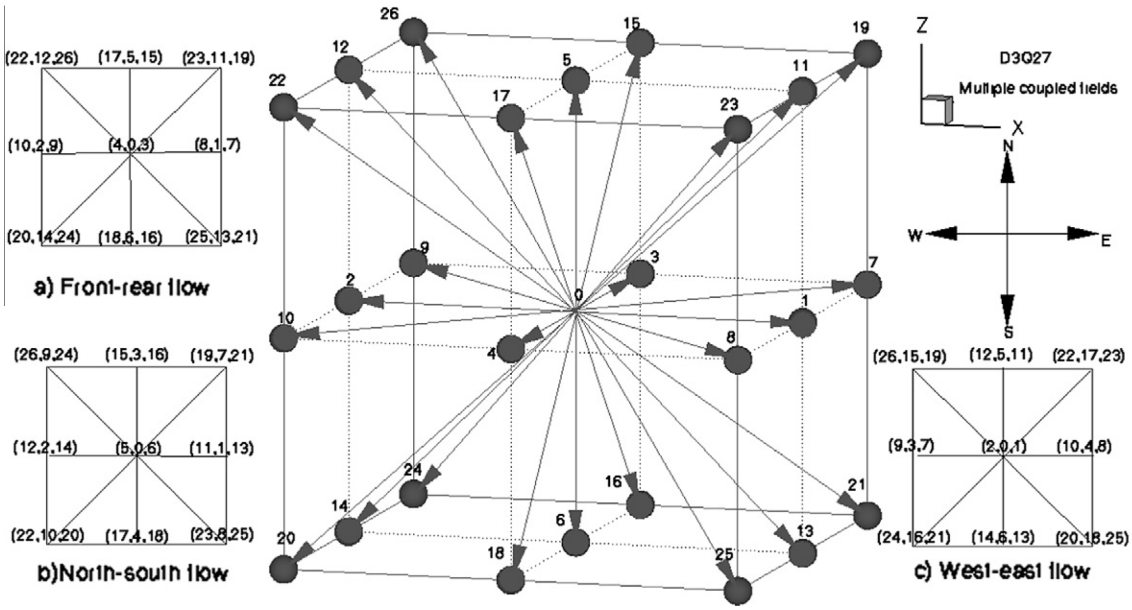

Fig. 6. Extended boundary for hybrid cubic lattice D3Q27 model.

Table 5
The lattice velocity e_i ($i = 0 \sim 26$) for the multiple coupled fields.

e_i	f_i	e_{i1}	e_{i2}	e_{i3}	
e_0	$\sum_{n=1}^{6}\left(f_0^{ei}c_{1n}^1+f_0^f c_{2n}^1+f_0^t c_{3n}^1+f_0^{mw}c_{4n}^1+f_0^e c_{5n}^1+f_0^m c_{6n}^1\right)$	0	0	0	C
e_1	$\sum_{n=1}^{6}\left(f_1^{ei}c_{1n}^2+f_1^f c_{2n}^2+f_1^t c_{3n}^2+f_1^m c_{6n}^2\right)$	1	0	0	E
e_2	$\sum_{n=1}^{6}\left(f_2^{ei}c_{1n}^3+f_2^f c_{2n}^3+f_2^t c_{3n}^3+f_2^m c_{6n}^3\right)$	−1	0	0	W
e_3	$\sum_{n=1}^{6}\left(f_3^{ei}c_{1n}^4+f_3^f c_{2n}^4+f_3^t c_{3n}^4+f_3^m c_{6n}^4\right)$	0	1	0	R
e_4	$\sum_{n=1}^{6}\left(f_4^{ei}c_{1n}^5+f_4^f c_{2n}^5+f_4^t c_{3n}^5+f_4^m c_{6n}^5\right)$	0	−1	0	F
e_5	$\sum_{n=1}^{6}\left(f_5^{ei}c_{1n}^6+f_5^f c_{2n}^6+f_5^t c_{3n}^6+f_5^m c_{6n}^6\right)$	0	0	1	N
e_6	$\sum_{n=1}^{6}\left(f_6^{ei}c_{1n}^7+f_6^f c_{2n}^7+f_6^t c_{3n}^7+f_6^m c_{6n}^7\right)$	0	0	−1	S
e_7	$\sum_{n=1}^{6}\left(f_7^{ei}c_{1n}^8+f_7^f c_{2n}^7+f_1^{mw}c_{4n}^7+f_1^e c_{5n}^7\right)$	1	1	0	RE
e_8	$\sum_{n=1}^{6}\left(f_8^{ei}c_{1n}^9+f_8^f c_{2n}^9+f_3^{mw}c_{4n}^9+f_3^e c_{5n}^9\right)$	1	−1	0	FE
e_9	$\sum_{n=1}^{6}\left(f_9^{ei}c_{1n}^{10}+f_9^f c_{2n}^{10}+f_4^{mw}c_{4n}^{10}+f_4^e c_{5n}^{10}\right)$	−1	1	0	RW
e_{10}	$\sum_{n=1}^{6}\left(f_{10}^{ei}c_{1n}^{11}+f_{10}^f c_{2n}^{11}+f_2^{mw}c_{4n}^{11}+f_2^e c_{5n}^{11}\right)$	−1	−1	0	FW
e_{11}	$\sum_{n=1}^{6}\left(f_{11}^{ei}c_{1n}^{12}+f_{11}^f c_{2n}^{12}+f_5^{mw}c_{4n}^{12}+f_5^e c_{5n}^{12}\right)$	1	0	1	NE
e_{12}	$\sum_{n=1}^{6}\left(f_{12}^{ei}c_{1n}^{13}+f_{12}^f c_{2n}^{13}+f_8^{mw}c_{4n}^{13}+f_8^e c_{5n}^{13}\right)$	−1	0	1	NW
e_{13}	$\sum_{n=1}^{6}\left(f_{13}^{ei}c_{1n}^{14}+f_{13}^f c_{2n}^{14}+f_7^{mw}c_{4n}^{14}+f_7^e c_{5n}^{14}\right)$	1	0	−1	SE
e_{14}	$\sum_{n=1}^{6}\left(f_{14}^{ei}c_{1n}^{15}+f_{14}^f c_{2n}^{15}+f_6^{mw}c_{4n}^{15}+f_6^e c_{5n}^{15}\right)$	−1	0	−1	SW
e_{15}	$\sum_{n=1}^{6}\left(f_{15}^{ei}c_{1n}^{16}+f_{15}^f c_{2n}^{16}+f_9^{mw}c_{4n}^{16}+f_9^e c_{5n}^{16}\right)$	0	1	1	NR
e_{16}	$\sum_{n=1}^{6}\left(f_{16}^{ei}c_{1n}^{17}+f_{16}^f c_{2n}^{17}+f_{11}^{mw}c_{4n}^{17}+f_{11}^e c_{5n}^{17}\right)$	0	1	−1	SR
e_{17}	$\sum_{n=1}^{6}\left(f_{17}^{ei}c_{1n}^{18}+f_{17}^f c_{2n}^{18}+f_{12}^{mw}c_{4n}^{18}+f_{12}^e c_{5n}^{18}\right)$	0	−1	1	NF
e_{18}	$\sum_{n=1}^{6}f_7^t c_{3n}^{20}$	0	−1	−1	SF
e_{19}	$\sum_{n=1}^{6}f_7^t c_{3n}^{20}$	1	1	1	RNE
e_{20}	$\sum_{n=1}^{6}f_8^t c_{3n}^{21}$	−1	−1	−1	FSW
e_{21}	$\sum_{n=1}^{6}f_9^t c_{3n}^{22}$	1	1	−1	RSE
e_{22}	$\sum_{n=1}^{6}f_{10}^t c_{3n}^{23}$	−1	−1	1	FNW
e_{23}	$\sum_{n=1}^{6}f_{11}^t c_{3n}^{24}$	1	−1	1	FNE
e_{24}	$\sum_{n=1}^{6}f_{12}^t c_{3n}^{25}$	−1	1	−1	RSW
e_{25}	$\sum_{n=1}^{6}f_{13}^t c_{3n}^{26}$	1	−1	−1	FSE
e_{26}	$\sum_{n=1}^{6}f_{14}^t c_{3n}^{27}$	−1	−1	1	RNW

6.1. Front-rear flow

As shown in Fig. 6a, when the electronic and ionic flow direction is from front to rear, after streaming, the unknown distribution functions are $f_{fr:i}$ (i = 26, 15, 19, 9, 3, 7, 24, 16, 21), on the contrary, after streaming, the unknown distribution functions are $f_{rf:i}$ (i = 22, 17, 23, 10, 4, 8, 20, 18, 25).

6.1.1. PC

For the front inlet and rear outlet case, the $f^p_{fr:i}$ (i = 26, 15, 19, 9, 3, 7, 24, 16, 21) can be defined as

$$f^p_{fr:26} = f_{13} - \frac{\rho^{p:t}_{in}\left(u^{p:t}_{fr:x} - u^{p:t}_{fr:y} - u^{p:t}_{fr:z}\right)}{12} f^p_{fr:21} = f_{10} + \frac{\rho^{t}_{in}\left(u^{p:t}_{fr:x} + u^{p:t}_{fr:y} - u^{p:t}_{fr:z}\right)}{12}$$

$$f^p_{fr:24} = f_{11} + \frac{\rho^{t}_{in}\left(-u^{t}_{fr:x} + u^{t}_{fr:y} - u^{t}_{fr:z}\right)}{12} \quad f^p_{fr:19} = f_{8} + \frac{\rho^{p:t}_{in}\left(u^{p:t}_{fr:x} + u^{p:t}_{fr:y} + u^{p:t}_{fr:z}\right)}{12}, \tag{146}$$

$$f^p_{fr:9} = \frac{\rho^{mi}_{in}u^{mi}_{fr:y} + \rho^{f}_{in}u^{f}_{fr:y}}{6} + \frac{4f_8 + 4f_3 - f_1 + f_2 - f_{11} + f_{12} - f_{13} + f_{14} + 3\rho^{mw}_{in}\left(-u^{mw}_{fr:x} + u^{mw}_{fr:y}\right) + 3\rho^{m}_{in}\left(-u^{m}_{fr:x} + u^{m}_{fr:y}\right)}{4}, \tag{147}$$

$$f^p_{fr:3} = \frac{\rho^{mi}_{in}u^{mi}_{fr:y} + \rho^{mw}_{in}u^{mw}_{fr:y} + \rho^{f}_{in}u^{f}_{fr:y} + \rho^{m}_{in}u^{m}_{fr:y} + 7f_4 - f_0 - f_1 - f_5 - f_2 - f_6}{3}, \tag{148}$$

$$f^p_{fr:7} = \frac{\rho^{mi}_{in}u^{mi}_{fr:y} + \rho^{f}_{in}u^{f}_{fr:y}}{6} + \frac{3\rho^{p:mw}_{in}\left(u^{p:mw}_{fr:x} + u^{p:mw}_{fr:y}\right) + 3\rho^{p:e}_{in}\left(u^{p:e}_{fr:x} + u^{p:e}_{fr:y}\right) - 4f_{10} - 4f_2 + f_1 - f_2 + f_{11} - f_{12} + f_{13} - f_{14}}{4}, \tag{149}$$

$$f^p_{fr:15} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{fr:y} + \rho^{p:f}_{in}u^{p:f}_{fr:y}}{6} + \frac{3\rho^{p:mw}_{in}\left(u^{p:mw}_{fr:y} + u^{p:mw}_{fr:z}\right) + 3\rho^{p:m}_{in}\left(u^{p:m}_{fr:y} + u^{p:m}_{fr:z}\right) + 4f_{18} + 4f_{10} - f_5 + f_6 - f_{11} - f_{12} + f_{13} + f_{14}}{4}, \tag{150}$$

$$f^p_{fr:16} = \frac{\rho^{mi}_{in}u^{mi}_{fr:y} + \rho^{f}_{in}u^{f}_{fr:y}}{6} + \frac{3\rho^{p:mw}_{in}\left(u^{p:mw}_{fr:y} - u^{p:mw}_{fr:z}\right) + 3\rho^{p:e}_{in}\left(u^{p:e}_{fr:y} - u^{p:e}_{fr:z}\right) + 4f_{17} + 4f_{12} - f_5 + f_6 - f_{11} - f_{12} + f_{13} + f_{14}}{4}. \tag{151}$$

For rear inlet and front outlet case, $f^p_{rf:i}$ (i = 22, 17, 23, 10, 4, 8, 20, 18, 25) can be defined as

$$f^p_{rf:22} = f_9 - \frac{\rho^{t}_{in}\left(u^{p:t}_{rf:x} + u^{p:t}_{rf:y} - u^{p:t}_{rf:z}\right)}{12} \quad f^p_{rf:25} = f_{14} - \frac{\rho^{p:t}_{in}\left(-u^{p:t}_{rf:x} + u^{p:t}_{rf:y} - u^{p:t}_{rf:z}\right)}{12}$$

$$f^p_{rf:23} = f_{12} - \frac{\rho^{p:t}_{in}\left(-u^{p:t}_{rf:x} + u^{p:t}_{rf:y} - u^{p:t}_{rf:z}\right)}{12} \quad f^p_{rf:20} = f_7 - \frac{\rho^{p:t}_{in}\left(-u^{p:t}_{rf:x} + u^{p:t}_{rf:y} + u^{p:t}_{rf:z}\right)}{12}, \tag{152}$$

$$f^p_{rf:17} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{rf:y} - \rho^{p:f}_{in}u^{p:f}_{rf:y}}{6} + \frac{3\rho^{p:mw}_{in}\left(u^{p:mw}_{rf:y} - u^{p:mw}_{rf:z}\right) + 3\rho^{p:e}_{in}\left(u^{p:e}_{rf:y} - u^{p:e}_{rf:z}\right) - 3f_{11} + 4f_{16} + f_5 - f_6 + f_{12} - f_{13} - f_{14}}{4}, \tag{153}$$

$$f^p_{rf:4} = \frac{7f_3 - f_0 - f_1 - f_5 - f_2 - f_6 + \rho^{mi}_{in}u^{p:mi}_{rf:y} + \rho^{p:f}_{in}u^{p:f}_{rf:y} + \rho^{p:t}_{in}u^{p:t}_{rf:y} + \rho^{p:m}_{in}u^{p:m}_{u:y}}{3}, \tag{154}$$

$$f^p_{rf:8} = \frac{4f_9 + 4f_4 + f_1 - f_2 + f_{11} - f_{12} + f_{13} - f_{14} - 3\rho^{p:mw}_{in}\left(-u^{p:mw}_{rf:x} + u^{p:mw}_{rf:y}\right) - 3\rho^{p:e}_{in}\left(-u^{p:e}_{rf:x} + u^{p:e}_{rf:y}\right)}{4} - \frac{\rho^{p:mi}_{in}u^{p:mi}_{rf:y} + \rho^{p:f}_{in}u^{p:f}_{rf:y}}{6}, \tag{155}$$

$$f^p_{rf:18} = \frac{4f_{15} + 4f_9 - f_5 + f_6 - f_{11} - f_{12} + f_{13} + f_{14} - 3\rho^{p:mw}_{in}\left(u^{p:mw}_{rf:y} + u^{p:mw}_{rf:z}\right) - 3\rho^{p:e}_{in}\left(u^{p:e}_{rf:y} + u^{p:e}_{rf:z}\right)}{4} - \frac{\rho^{p:mi}_{in}u^{p:mi}_{rf:y} - \rho^{p:f}_{in}u^{p:f}_{rf:y}}{6}. \tag{156}$$

6.1.2. VC

For the front inlet and rear outlet case, $f^v_{fr:i}$ ($i = 26, 15, 19, 9, 3, 7, 24, 16, 21$), can be defined as

$$f^v_{fr:26} = f_{13} + \frac{\rho^{v:t}_{in}\left(-u^t_{fr:x} + u^t_{fr:y} + u^t_{fr:z}\right)}{12} \quad f^v_{fr:24} = f_{11} + \frac{\rho^{v:t}_{in}\left(u^t_{fr:y} - u^t_{fr:x} - u^t_{fr:z}\right)}{12}$$
$$f^v_{fr:21} = f_{10} + \frac{\rho^{v:t}_{in}\left(u^{v:t}_{fr:x} + u^{v:t}_{fr:y} - u^{v:t}_{fr:z}\right)}{12} \quad f^v_{fr:19} = f_8 + \frac{\rho^{v:t}_{in}\left(u^{v:t}_{fr:x} + u^{v:t}_{fr:y} + u^{v:t}_{fr:z}\right)}{12}, \tag{157}$$

$$f^v_{fr:15} = \frac{\rho^{v:mi}_{fr:in}u^{v:mi}_y + \rho^{v:f}_{fr:in}u^{v:f}_y}{6} - \left(\frac{\rho^{v:mi}_{fr:in}}{\rho^{v:mi}_{fr:in} - 3} + \frac{\rho^{v:f}_{fr:in}}{\rho^{v:f}_{fr:in} - 3}\right)\left(\frac{f_6 - 4f_{18} - 4f_{10} - f_5 - f_{11} - f_{12} + f_{13} + f_{14}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{fr:in}\left(u^{v:mw}_{fr:y} + u^{v:mw}_{fr:z}\right) + 3\rho^{v:e}_{fr:in}\left(u^{v:e}_{fr:y} + u^{v:e}_{fr:z}\right)}{4}, \tag{158}$$

$$f^v_{fr:9} = \frac{\rho^{v:mi}_{fr:in}u^{v:mi}_y + \rho^{v:f}_{fr:in}u^{v:f}_y}{6} + \left(\frac{\rho^{v:mi}_{fr:in}}{\rho^{v:mi}_{fr:in} - 3} + \frac{\rho^{v:f}_{fr:in}}{\rho^{v:f}_{fr:in} - 3}\right)\left(\frac{4f_8 + 4f_3 + f_2 - f_1 - f_{11} + f_{12} - f_{13} + f_{14}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{fr:in}\left(-u^{v:mw}_{fr:x} + u^{v:mw}_{fr:y}\right) + 3\rho^{v:e}_{fr:in}\left(-u^{v:e}_{fr:x} + u^{v:e}_{fr:y}\right)}{4}, \tag{159}$$

$$f^v_{fr:3} = \frac{\rho^{v:mi}_{fr:in}u^{v:mi}_y + \rho^{v:e}_{fr:in}u^{v:e}_y + \rho^{v:t}_{in}u^{v:t}_{fr:y} + \rho^{v:m}_{in}u^{v:m}_{fr:y} + 11f_4 + f_1 + f_5 + f_2 + f_6}{3}, \tag{160}$$

$$f^v_{fr:7} = \frac{\rho^{v:mi}_{fr:in}u^{v:mi}_y + \rho^{v:f}_{fr:in}u^{v:f}_y}{6} - \left(\frac{\rho^{v:mi}_{fr:in}}{\rho^{v:mi}_{fr:in} - 3} + \frac{\rho^{v:f}_{fr:in}}{\rho^{v:f}_{fr:in} - 3}\right)\left(\frac{f_2 - 4f_{10} - 4f_2 - f_1 - f_{11} + f_{12} - f_{13} + f_{14}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{fr:in}\left(u^{v:mw}_{fr:x} + u^{v:mw}_{fr:y}\right) + 3\rho^{v:e}_{fr:in}\left(u^{v:e}_{fr:x} + u^{v:e}_{fr:y}\right)}{4}, \tag{161}$$

$$f_{fr:16} = \frac{\rho^{v:mi}_{fr:in}u^{v:mi}_y + \rho^{v:f}_{fr:in}u^{v:f}_y}{6} + \left(\frac{\rho^{v:mi}_{fr:in}}{\rho^{v:mi}_{fr:in} - 3} + \frac{\rho^{v:f}_{fr:in}}{\rho^{v:f}_{fr:in} - 3}\right)\left(\frac{4f_{17} + 4f_{12} + f_6 - f_5 - f_{11} - f_{12} + f_{13} + f_{14}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{fr:in}\left(u^{v:mw}_{fr:y} - u^{v:mw}_{fr:z}\right) + 3\rho^{v:e}_{fr:in}\left(u^{v:e}_{fr:y} - u^{v:e}_{fr:z}\right)}{4}, \tag{162}$$

For rear inlet and front outlet case, $f^v_{rf:i}$ ($i = 22, 17, 23, 10, 4, 8, 20, 18, 25$) can be defined as

$$f^v_{rf:22} = f^v_9 - \frac{\rho^{v:t}_{in}\left(u^{v:t}_{rf:x} + u^{v:t}_{rf:y} - u^{v:t}_{rf:z}\right)}{12} \quad f^v_{rf:20} = f_7 - \frac{\rho^{v:t}_{in}\left(-u^{v:t}_{rf:x} + u^{v:t}_{rf:y} + u^{v:t}_{rf:z}\right)}{12}$$
$$f^v_{rf:23} = f_{12} - \frac{\rho^{v:t}_{in}\left(-u^{v:t}_{rf:x} + u^{v:t}_{rf:y} - u^{v:t}_{rf:z}\right)}{12} \quad f^v_{rf:25} = f_{14} - \frac{\rho^{v:t}_{in}\left(-u^{v:t}_{rf:x} + u^{v:t}_{rf:y} - u^{v:t}_{rf:z}\right)}{12}, \tag{163}$$

$$f^v_{rf:17} = \frac{\rho^{v:mi}_{rf:in}u^{v:mi}_y + \rho^{v:f}_{rf:in}u^{v:f}_y}{6} - \left(\frac{\rho^{v:mi}_{rf:in}}{\rho^{v:mi}_{rf:in} - 3} + \frac{\rho^{v:f}_{rf:in}}{\rho^{v:f}_{rf:in} - 3}\right)\left(\frac{2f_{16} + f_{11} + f_6 - f_5 - f_{12} + f_{13} + f_{14}}{2}\right)$$
$$- \frac{3\rho^{v:mw}_{rf:in}\left(u^{v:mw}_{rf:y} - u^{v:mw}_{rf:z}\right) + 3\rho^{v:e}_{rf:in}\left(u^{v:e}_{rf:y} - u^{v:e}_{rf:z}\right)}{4}, \tag{164}$$

$$f^v_{rf:10} = \left(\frac{\rho^{v:mi}_{rf:in}}{2\left(\rho^{v:mi}_{rf:in} - 3\right)} + \frac{\rho^{v:e}_{rf:in}}{2\left(\rho^{v:e}_{rf:in} - 3\right)}\right)\left(\frac{2f_7 + f_1 + f_2 - f_{11} + f_{12} - f_{13} + f_{14}}{2}\right) - \frac{\rho^{v:mi}_{in}u^{v:mi}_{rf:y} + \rho^{v:e}_{in}u^{v:e}_{rf:y}}{6}$$
$$- \frac{3\rho^{v:mw}_{rf:in}\left(u^{v:mw}_{rf:x} + u^{v:mw}_{rf:y}\right) + 3\rho^{v:e}_{rf:in}\left(u^{v:e}_{rf:x} + u^{v:e}_{rf:y}\right)}{4}, \tag{165}$$

$$f^v_{rf:4} = \frac{11f_3 - f_0 - f_1 - f_5 - f_2 - f_6 - \rho^{v:mi}_{rf:in}u^{v:mi}_{rf} - \rho^{v:e}_{rf:in}u^{v:e}_{rf} - \rho^{v:t}_{in}u^{v:t}_{rf:y} - \rho^{v:m}_{in}u^{v:m}_{rf:y}}{3}, \tag{166}$$

$$f^{v}_{rf:8} = \left(\frac{\rho^{v:mi}_{rf:in}}{\rho^{v:mi}_{rf:in} - 3} + \frac{\rho^{v:f}_{rf:in}}{\rho^{v:e}_{rf:in} - 3}\right)\left(\frac{2f_9 - f_2 + 2f_4 + f_1 + f_{11} - f_{12} + f_{13} - f_{14}}{2}\right) - \frac{\rho^{v:mi}_{rf:in}u^{mi}_{y} + \rho^{v:f}_{rf:in}u^{v:f}_{y}}{6} - \frac{3\rho^{v:mw}_{rf:in}\left(-u^{v:mw}_{rf:x} + u^{v:mw}_{rf:y}\right) + 3\rho^{v:e}_{rf:in}\left(-u^{v:e}_{rf:x} + u^{v:e}_{rf:y}\right)}{4}, \tag{167}$$

$$f^{v}_{rf:18} = \left(\frac{\rho^{v:mi}_{rf:in}}{\rho^{v:mi}_{rf:in} - 3} + \frac{\rho^{v:f}_{rf:in}}{\rho^{v:f}_{rf:in} - 3}\right)\left(\frac{2f_{15} + 2f_9 + f_6 - f_5 - f_{11} - f_{12} + f_{13} + f_{14}}{2}\right) - \frac{\rho^{v:mi}_{rf:in}u^{v:mi}_{y} + \rho^{v:f}_{rf:in}u^{v:f}_{y}}{6} - \frac{3\rho^{v:mw}_{rf:in}\left(u^{v:mw}_{rf:y} + u^{v:mw}_{rf:z}\right) + 3\rho^{v:e}_{rf:in}\left(u^{v:e}_{rf:y} + u^{v:e}_{rf:z}\right)}{4}. \tag{168}$$

6.2. South-north flow

As shown in Fig. 6b, when the extended fluid flow direction is from south to north, after streaming, the unknown distribution functions are $f_{sn:i}$ ($i = 26, 12, 22, 15, 5, 17, 19, 11, 23$), on the contrary, the unknown distribution functions are $f_{ns:i}$ ($i = 20, 14, 24, 18, 6, 16, 25, 13, 21$).

6.2.1. PC

For the south inlet and north outlet case, $f^{p:mi}_{sn:i}$ ($i = 26, 12, 22, 15, 5, 17, 19, 11, 23$) can be defined as

$$f^{p}_{sn:26} = f^{t}_{13} - \frac{\rho^{t}_{in}\left(-u^{p:t}_{sn:x} + u^{p:t}_{sn:y} + u^{p:t}_{sn:z}\right)}{12} \quad f^{p}_{sn:19} = f_8 - \frac{\rho^{p:t}_{in}\left(-u^{p:t}_{sn:x} - u^{p:t}_{sn:y} + u^{p:t}_{sn:z}\right)}{12}$$
$$f^{p}_{sn:26} = f_{12} - \frac{\rho^{p:t}_{in}\left(u^{p:t}_{sn:x} - u^{p:t}_{sn:y} + u^{p:t}_{sn:z}\right)}{12} \quad f^{p}_{sn:22} = f_9 - \frac{\rho^{p:t}_{in}\left(u^{p:t}_{ns:x} - u^{p:t}_{ns:y} + u^{p:t}_{ns:z}\right)}{12}, \tag{169}$$

$$f^{p}_{sn:12} = \frac{3\rho^{p:mw}_{in}\left(-u^{p:mw}_{sn:y} + u^{p:mw}_{sn:z}\right) + 3\rho^{p:e}_{in}\left(-u^{p:e}_{sn:y} + u^{p:e}_{sn:z}\right) - f_8 + 4f_{13} + 3f_7 - f_1 + f_2 + f_9 + f_{10}}{4} + \frac{\rho^{p:mi}_{in}u^{p:mi}_{sn:z} + \rho^{p:f}_{in}u^{p:f}_{sn:z}}{6}, \tag{170}$$

$$f^{p}_{sn:15} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{sn:z} + \rho^{p:f}_{in}u^{p:f}_{sn:z}}{6} + \frac{4f_{18} + 4f_{10} + f^{mi}_3 - f^{mi}_4 + f^{mi}_7 - f^{mi}_8 + f^{mi}_9 - f^{mi}_{10} + 3\rho^{mw}_{in}\left(-u^{p:mw}_{sn:x} + u^{p:mw}_{sn:z}\right) + 3\rho^{p:e}_{in}\left(-u^{p:e}_{sn:x} + u^{p:e}_{sn:z}\right)}{4}, \tag{171}$$

$$f^{p}_{sn:5} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{sn:z} + \rho^{p:e}_{in}u^{p:e}_{sn:z} + \rho^{p:m}_{in}u^{p:m}_{sn:z} + 7f_6 - f_0 - f_4 - f_3 - f_2 - f_1}{3} + \frac{3\rho^{p:e}_{in}\left(u^{p:e}_{sn:y} + u^{p:e}_{sn:z}\right)}{4}, \tag{172}$$

$$f^{p}_{sn:17} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{sn:z} + \rho^{p:f}_{in}u^{p:f}_{sn:z}}{6} + \frac{4f_{16} + 4f_{11} - f_3 + f_4 - f_7 + f_8 - f_9 + f_{10} + 3\rho^{p:mw}_{in}\left(u^{p:mw}_{sn:x} + u^{p:mw}_{sn:z}\right) + 3\rho^{p:e}_{in}\left(u^{p:e}_{sn:x} + u^{p:e}_{sn:z}\right)}{4}, \tag{173}$$

$$f^{p}_{sn:11} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{sn:z} + \rho^{p:f}_{in}u^{p:f}_{sn:z}}{6} + \frac{4f_{14} + 4f_6 - f^{mi}_1 + f^{mi}_2 - f^{mi}_7 - f^{mi}_8 + f^{mi}_9 + f^{mi}_{10} + 3\rho^{p:mw}_{in}\left(u^{p:mw}_{sn:y} + u^{p:mw}_{sn:z}\right) + 3\rho^{p:e}_{in}\left(u^{p:e}_{sn:y} + u^{p:e}_{sn:z}\right)}{4}. \tag{174}$$

For the north inlet and south outlet case, $f^{p:mi}_{ns:i}$ ($i = 20, 14, 24, 18, 6, 16, 25, 13, 21$) can be defined as

$$f^{p}_{ns:20} = f_7 + \frac{\rho^{p:t}_{in}\left(-u^{p:t}_{ns:x} - u^{p:t}_{ns:y} + u^{p:t}_{ns:z}\right)}{12} \quad f^{p}_{ns:21} = f_{10} + \frac{\rho^{p:t}_{in}\left(u^{p:t}_{ns:x} - u^{p:t}_{ns:y} + u^{p:t}_{ns:z}\right)}{12}$$
$$f^{p}_{ns:24} = f_{11} + \frac{\rho^{p:t}_{in}\left(u^{p:t}_{ns:x} - u^{p:t}_{ns:y} + u^{p:t}_{ns:z}\right)}{12} \quad f^{p}_{ns:25} = f_{14} + \frac{\rho^{p:t}_{in}\left(-u^{p:t}_{ns:x} + u^{p:t}_{ns:y} + u^{p:t}_{ns:z}\right)}{12}, \tag{175}$$

$$f^{p}_{ns:14} = \frac{3\rho^{p:mw}_{in}\left(u^{p:mw}_{ns:y} + u^{p:mw}_{ns:z}\right) + 3\rho^{p:e}_{in}\left(u^{p:e}_{ns:y} + u^{p:e}_{ns:z}\right) - f_1 + f_2 - f_7 - f_8 + f_9 + f_{10} + 2f_{11} + 2f_5}{4} - \frac{\rho^{p:mi}_{in}u^{p:mi}_{ns:z} + \rho^{p:f}_{in}u^{p:f}_{ns:z}}{6}, \tag{176}$$

$$f^{p}_{ns:18} = \frac{2f_{15} + 2f_{10} - f_3 + f_4 - f_7 + f_8 - f_9 + f_{10} + 3\rho^{p:mw}_{in}\left(-u^{p:mw}_{ns:x} + u^{p:mw}_{ns:z}\right) + 3\rho^{e:mw}_{in}\left(-u^{e:mw}_{ns:x} + u^{e:mw}_{ns:z}\right)}{4} - \frac{\rho^{p:mi}_{in}u^{p:mi}_{ns:z} + \rho^{p:f}_{in}u^{p:f}_{ns:z}}{6}, \tag{177}$$

$$f^p_{ns:6} = \frac{\rho^{p:t}_{in} u^{p:t}_{ns:z} + \rho^{p:m}_{in} u^{p:m}_{ns:z} - \rho^{p:mi}_{in} u^{p:mi}_{ns:z} - \rho^{p:f}_{in} u^{p:f}_{ns:z} - 11f_5 - f_0 - f_4 - f_3 - f_1 - f_2}{3}, \tag{178}$$

$$f^p_{ns:16} = \frac{2f_{17} + 2f_{12} + f_3 - f_4 + f_7 - f_8 + f_9 - f_{10} - 3\rho^{p:mw}_{in}(u^{p:mw}_{ns:x} + u^{p:mw}_{ns:z}) - 3\rho^{p:e}_{in}(u^{p:e}_{ns:x} + u^{p:e}_{ns:z})}{4} - \frac{\rho^{p:mi}_{in} u^{p:mi}_{sn:z} + \rho^{p:f}_{in} u^{p:f}_{sn:z}}{6}, \tag{179}$$

$$f^p_{ns:13} = \frac{4f_{12} + 5f_8 + f_1 - f_2 + f_7 - f_9 - f_{10} - 3\rho^{p:mw}_{in}\left(-u^{p:mw}_{ns:y} + u^{p:mw}_{ns:z}\right) - 3\rho^{p:e}_{in}\left(-u^{p:e}_{ns:y} + u^{p:e}_{ns:z}\right)}{4} - \frac{\rho^{p:mi}_{in} u^{p:mi}_{ns:z} + \rho^{p:f}_{in} u^{p:f}_{ns:z}}{6}. \tag{180}$$

6.2.2. VC

For the south inlet and north outlet case, $f^{v:mi}_{sn:i}$ (i = 26, 12, 22, 15, 5, 17, 19, 11, 23) are defined as

$$f^v_{sn:26} = f_{13} - \frac{\rho^{v:t}_{in}\left(-u^{v:t}_{sn:x} + u^{v:t}_{sn:y} + u^{v:t}_{sn:z}\right)}{12} \quad f^v_{sn:19} = f_8 - \frac{\rho^{v}_{in}\left(-u^{v:t}_{sn:x} - u^{v:t}_{sn:y} + u^{v:t}_{sn:z}\right)}{12}$$
$$f^v_{sn:22} = f_9 - \frac{\rho^{v:t}_{in}\left(u^{v:t}_{sn:x} - u^{v:t}_{sn:y} + u^{v:t}_{sn:z}\right)}{12} \quad f^v_{sn:23} = f_{12} - \frac{\rho^{v:t}_{in}\left(u^{v:t}_{sn:x} - u^{v:t}_{sn:y} + u^{v:t}_{sn:z}\right)}{12}, \tag{181}$$

$$f^v_{sn:12} = \frac{\rho^{v:mi}_{sn:in} u^{mi}_z + \rho^{v:f}_{sn:in} u^{v:f}_z}{6} - \left(\frac{\rho^{v:mi}_{sn:in}}{\rho^{v:mi}_{sn:in} - 3} + \frac{\rho^{v:f}_{sn:in}}{\rho^{v:f}_{sn:in} - 3}\right)\left(\frac{f_1 - f_2 - f_7 + f_8 - f_9 - f_{10} - 2f_{13}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{sn:in}\left(-u^{v:mw}_{sn:y} + u^{v:mw}_{sn:z}\right) + 3\rho^{v:e}_{sn:in}\left(-u^{v:e}_{sn:y} + u^{v:e}_{sn:z}\right)}{4}, \tag{182}$$

$$f^v_{sn:15} = \frac{\rho^{v:mi}_{sn:in} u^{mi}_z + \rho^{v:f}_{sn:in} u^{f}_z}{6} + \left(\frac{\rho^{v:mi}_{sn:in}}{2\left(\rho^{v:mi}_{sn:in} - 3\right)} + \frac{\rho^{v:f}_{sn:in}}{2\left(\rho^{v:f}_{sn:in} - 3\right)}\right)\left(\frac{f_3 - f_4 + f_7 - f_8 + f_9 + f_{10} + 2f_{18}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{sn:in}\left(-u^{v:mw}_{sn:x} + u^{v:mw}_{sn:z}\right) + 3\rho^{v:e}_{sn:in}\left(-u^{v:e}_{sn:x} + u^{v:e}_{sn:z}\right)}{4}, \tag{183}$$

$$f^v_{sn:5} = \frac{\rho^{v:mi}_{sn:in} u^{v:mi}_z + \rho^{v:f}_{sn:in} u^{v:f}_z - \rho^{v:t}_{in} u^{v:t}_{sn:z} + \rho^{v:m}_{in} u^{v:m}_{sn:z} + 11f_6 + f_0 + f_4 + f_3 + f_2 + f_1}{3}, \tag{184}$$

$$f^v_{sn:17} = \frac{\rho^{v:mi}_{sn:in} u^{v:mi}_z + \rho^{v:f}_{sn:in} u^{v:f}_z}{6} - \left(\frac{\rho^{v:mi}_{sn:in}}{\rho^{v:mi}_{sn:in} - 3} + \frac{\rho^{v:f}_{sn:in}}{\rho^{v:f}_{sn:in} - 3}\right)\left(\frac{f_3 - f_4 + f_7 - f_8 + f_9 - f_{10} - 2f_{16} - 2f_{11}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{sn:in}\left(u^{v:mw}_{sn:x} + u^{v:mw}_{sn:z}\right) + 3\rho^{v:e}_{sn:in}\left(u^{v:e}_{sn:x} + u^{v:e}_{sn:z}\right)}{4}, \tag{185}$$

$$f^v_{sn:11} = \frac{\rho^{v:mi}_{sn:in} u^{v:mi}_z + \rho^{v:f}_{sn:in} u^{v:f}_z}{6} - \left(\frac{\rho^{v:mi}_{sn:in}}{\rho^{v:mi}_{sn:in} - 3} - \frac{\rho^{v:f}_{sn:in}}{\rho^{v:f}_{sn:in} - 3}\right)\left(\frac{f_1 - f_2 + f_7 + f_8 - f_9 - f_{10} - 2f_{14} - 2f_6}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{sn;in}\left(u^{v:mw}_{sn:y} + u^{v:mw}_{sn:z}\right) + 3\rho^{v:e}_{sn;in}\left(u^{v:e}_{sn:y} + u^{v:e}_{sn:z}\right)}{4}. \tag{186}$$

For north inlet and south outlet case, $f^{v:mi}_{ns:i}$ (i = 20, 14, 24, 18, 6, 16, 25, 13, 21) are defined as

$$f^v_{ns:20} = f_7 + \frac{\rho^{v:t}_{in}\left(-u^{v:t}_{ns:x} - u^{v:t}_{ns:y} + u^{v:t}_{ns:z}\right)}{12} \quad f^v_{ns:21} = f_{10} + \frac{\rho^{v:t}_{in}\left(u^{v:t}_{ns:x} - u^{v:t}_{ns:y} + u^{v:t}_{ns:z}\right)}{12}$$
$$f^v_{ns:24} = f_{11} + \frac{\rho^{v:t}_{in}\left(u^{t}_{ns:x} - u^{t}_{ns:y} + u^{t}_{ns:z}\right)}{12} \quad f^v_{ns:25} = f_{14} + \frac{\rho^{v:t}_{in}\left(-u^{t}_{ns:x} + u^{t}_{ns:y} + u^{t}_{ns:z}\right)}{12}, \tag{187}$$

$$f^v_{ns:14} = -\frac{\rho^{v:mi}_{ns:in} u^{v:mi}_z + \rho^{v:f}_{ns:in} u^{v:f}_z}{6} - \left(\frac{\rho^{v:mi}_{ns:in}}{\rho^{v:mi}_{ns:in} - 3} + \frac{\rho^{v:f}_{ns:in}}{\rho^{v:f}_{ns:in} - 3}\right)\left(\frac{f_7 - 2f_{11} - 2f_5 + f_1 - f_2 + f_8 - f_9 - f_{10}}{2}\right)$$
$$+ \frac{3\rho^{v:mw}_{ns:in}\left(u^{v:mw}_{ns:y} + u^{v:mw}_{ns:z}\right) + 3\rho^{v:e}_{ns:in}\left(u^{v:e}_{ns:y} + u^{v:e}_{ns:z}\right)}{4}, \tag{188}$$

$$f^{v}_{ns:18} = -\frac{\rho^{v:mi}_{ns:in}u^{v:mi}_{z} + \rho^{v:f}_{ns:in}u^{v:f}_{z}}{6} - \left(\frac{\rho^{v:mi}_{ns:in}}{\rho^{v:mi}_{ns:in}-3} + \frac{\rho^{v:f}_{ns:in}}{\rho^{v:f}_{ns:in}-3}\right)\left(\frac{f_7 - 2f_{15} - 2f_{10} - f_3 - f_4 + -f_8 + f_9 - f_{10}}{2}\right) + \frac{3\rho^{v:mw}_{ns:in}(u^{v:mw}_{ns:z} + u^{v:mw}_{ns:x}) + 3\rho^{v:e}_{ns:in}(u^{v:e}_{ns:z} + u^{v:e}_{ns:x})}{4}, \quad (189)$$

$$f^{v}_{ns:6} = \frac{f_0 + f_4 + f_3 + f_1 + f_2 - 7f_5 - \rho^{v:mi}_{ns:in}u^{v:mi}_{z} - \rho^{v:f}_{ns:in}u^{v:f}_{z} - \rho^{v:t}_{in}u^{v:t}_{ns:z} - \rho^{m:t}_{in}u^{m:t}_{ns:z}}{3}, \quad (190)$$

$$f^{v}_{ns:16} = \left(\frac{\rho^{v:mi}_{ns:in}}{\rho^{v:mi}_{ns:in}-3} + \frac{\rho^{v:f}_{ns:in}}{\rho^{v:f}_{ns:in}-3}\right)\left(\frac{2f_{17} + 2f_{12} + f_3 - f_4 + f_7 - f_8 + f_9 - f_{10}}{2}\right) - \frac{\rho^{v:mi}_{ns:in}u^{v:mi}_{z} + \rho^{v:f}_{ns:in}u^{v:f}_{z}}{6} + -\frac{3\rho^{v:mw}_{ns:in}(u^{v:mw}_{ns:x} + u^{v:mw}_{ns:z}) + 3\rho^{v:e}_{ns:in}(u^{v:e}_{ns:x} + u^{v:e}_{ns:z})}{4}, \quad (191)$$

$$f^{v}_{ns:13} = \left(\frac{\rho^{v:mi}_{ns:in}}{\rho^{v:mi}_{ns:in}-3} + \frac{\rho^{v:f}_{ns:in}}{\rho^{v:f}_{ns:in}-3}\right)\left(\frac{f_1 - f_2 + f_7 + f_8 - f_9 - f_{10} + 2f_{12} + 2f_8}{2}\right) - \frac{\rho^{v:mi}_{ns:in}u^{v:mi}_{z} + \rho^{v:f}_{ns:in}u^{v:f}_{z}}{6} + -\frac{3\rho^{v:mw}_{ns:in}\left(u^{v:mw}_{ns:z} - u^{v:mw}_{ns:y}\right) + 3\rho^{v:e}_{ns:in}\left(u^{v:e}_{ns:z} - u^{v:e}_{ns:y}\right)}{4}. \quad (192)$$

6.3. West-east flow

As shown in Fig. 6c, when the extended fluid flow direction is from west to east, after streaming, the unknown distribution functions are $f_{we:i}$ ($i = 23,11,19,8,1,7,25,13,21$), on the contrary, the unknown distribution functions are $f_{ew:i}$ ($i = 22,12,26,10,2,9,20,14,24$).

6.3.1. PC

For the west inlet and east outlet case, $f^{p}_{we:i}$ ($i = 23,11,19,8,1,7,25,13,21$) as defined as

$$f^{p}_{we:23} = f_{12} + \frac{\rho^{p:t}_{in}(u^{p:t}_{we:x} + u^{p:t}_{we:y} - u^{p:t}_{we:z})}{12} \quad f^{p}_{we:21} = f_{10} + \frac{\rho^{p:t}_{in}(u^{p:t}_{we:x} - u^{p:t}_{we:y} - u^{p:t}_{we:z})}{12}$$
$$f^{p}_{we:25} = f_{14} + \frac{\rho^{p:t}_{in}(u^{p:t}_{we:x} - u^{p:t}_{we:y} + u^{p:t}_{we:z})}{12} f^{p}_{we:19} = f_8 + \frac{\rho^{p:t}_{in}(u^{p:t}_{we:x} - u^{p:t}_{we:y} - u^{p:t}_{we:z})}{12}, \quad (193)$$

$$f^{=}_{we:11}\frac{\rho^{p:mi}_{in}u^{p:mi}_{we:x} + \rho^{p:f}_{in}u^{p:f}_{we:x}}{6} + \frac{4f_{14} + 3f_6 + f_5 + f_{15} - f_{16} + f_{17} - f_{18} + 3\rho^{p:mw}_{in}(u^{p:mw}_{we:x} + u^{p:mw}_{we:y}) + 3\rho^{p:e}_{in}(u^{p:e}_{we:x} + u^{p:e}_{we:y})}{4}, \quad (194)$$

$$f^{p}_{we:8} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{we:x} + \rho^{p:f}_{in}u^{p:f}_{we:x}}{6} + \frac{4f_9 + 5f_4 - f_3 - f_{15} - f_{16} + f_{17} + f_{18} + 3\rho^{p:mw}_{in}(u^{p:mw}_{we:x} + u^{p:mw}_{we:z}) + 3\rho^{p:e}_{in}(u^{p:e}_{we:x} + u^{p:e}_{we:z})}{4}, \quad (195)$$

$$f^{p}_{we:1} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{we:x} + \rho^{p:f}_{in}u^{p:f}_{we:x} + \rho^{p:t}_{in}u^{p:t}_{we:x} + \rho^{p:m}_{in}u^{p:m}_{we:x} - f_0 - f_4 - f_3 - f_5 - f_6 + 7f_2}{3}, \quad (196)$$

$$f^{p}_{we:7} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{we:x} + \rho^{p:f}_{in}u^{p:f}_{we:x}}{6} + \frac{4f_{10} + 4f_8 + f_3 - f_4 + f_{15} + f_{16} - f_{17} - f_{18} + 3\rho^{p:mw}_{in}(u^{p:mw}_{we:x} - u^{p:mw}_{we:y}) + 3\rho^{p:e}_{in}(u^{p:e}_{we:x} - u^{p:e}_{we:y})}{4}, \quad (197)$$

$$f^{p}_{we:13} = \frac{\rho^{p:mi}_{in}u^{p:mi}_{we:x} + \rho^{p:f}_{in}u^{p:f}_{we:x}}{6} - \frac{4f_{12} + 4f_8 + f_5 - f_6 + f_{15} - f_{16} + f_{17} - f_{18} + 3\rho^{mw}_{in}(u^{p:mw}_{we:x} - u^{p:mw}_{we:y}) + 3\rho^{e}_{in}(u^{p:e}_{we:x} - u^{p:e}_{we:y})}{4}. \quad (198)$$

For the east inlet and west outlet case, $f^{p}_{ew:i}$ ($i = 22,12,26,10,2,9,20,14,24$) are defined as,

$$f^{p}_{ew:22} = f^{p:t}_{9} - \frac{\rho^{p:t}_{in}(u^{p:t}_{ew:x} - u^{p:t}_{ew:y} - u^{p:t}_{ew:z})}{12} f^{p}_{ew:24} = f_{11} + \frac{\rho^{p:t}_{in}(u^{p:t}_{ew:x} + u^{p:t}_{ew:y} - u^{p:t}_{ew:z})}{12}$$
$$f^{p}_{ew:20} = f_7 + \frac{\rho^{p:t}_{in}(u^{p:t}_{ew:x} + u^{p:t}_{ew:y} - u^{p:t}_{ew:z})}{12} f^{p}_{ew:26} = f_{13} - \frac{\rho^{p:t}_{in}(u^{p:t}_{ew:x} - u^{p:t}_{ew:y} + u^{p:t}_{ew:z})}{12}, \quad (199)$$

$$f^{p}_{ew:12} = \frac{\rho^{p:mi}_{in} u^{p:mi}_{ew:x} + \rho^{p:f}_{in} u^{p:f}_{ew:x}}{6} + \frac{4f_{13} + 4f_7 + f_5 - f_6 + f_{15} - f_{16} + f_{17} - f_{18} + 3\rho^{mw}_{in}\left(u^{p:mw}_{ew:x} - u^{p:mw}_{ew:y}\right) + 3\rho^{e}_{in}\left(u^{p:e}_{ew:x} - u^{p:e}_{ew:y}\right)}{4}, \tag{200}$$

$$f^{p:}_{ew:10} = -\frac{\rho^{p:mi}_{in} u^{p:mi}_{ew:x} + \rho^{p:f}_{in} u^{p:f}_{ew:x}}{6} + \frac{4f_7 + 4f_1 - f_3 + f_4 - f_{15} - f_{16} + f_{17} + f_{18} - 3\rho^{p:mw}_{in}(u^{p:mw}_{ew:x} - u^{p:mw}_{ew:z}) - 3\rho^{p:e}_{in}(u^{p:e}_{ew:x} - u^{p:e}_{ew:z})}{4}, \tag{201}$$

$$f^{p}_{ew:2} = \frac{9f_1 + f_0 + f_4 + f_3 + f_5 + f_6 - \rho^{p:mi}_{in} u^{p:mi}_{ew:x} - \rho^{p:f}_{in} u^{p:f}_{ew:x} - \rho^{p:t}_{in} u^{p:t}_{ew:x} - \rho^{p:m}_{in}}{3}, \tag{202}$$

$$f^{p}_{ew:9} = f_8 + f_3 - \frac{\rho^{p:mi}_{in} u^{p:mi}_{ew:x} + \rho^{p:f}_{in} u^{p:f}_{ew:x}}{6} - \frac{-f_3 + f_4 - f_{15} - f_{16} + f_{17} + f_{18} + 3\rho^{p:mw}_{in}(u^{p:mw}_{ew:x} + u^{p:mw}_{ew:z}) + 3\rho^{p:e}_{in}(u^{p:e}_{ew:x} + u^{p:e}_{ew:z})}{4}, \tag{203}$$

$$f^{p}_{ew:14} = f_{11} + f_5 - \frac{\rho^{p:mi}_{in} u^{p:mi}_{ew:x} + \rho^{p:f}_{in} u^{p:f}_{ew:x}}{6} - \frac{-f_3 + f_4 - f_{15} - f_{16} + f_{17} + f_{18} - 3\rho^{p:mw}_{in}\left(u^{p:mw}_{ew:x} + u^{p:mw}_{ew:y}\right) - 3\rho^{p:e}_{in}\left(u^{p:e}_{ew:x} + u^{p:e}_{ew:y}\right)}{4}. \tag{204}$$

6.3.2. VC

For the west inlet and east outlet case, $f^{v}_{we:i}$ ($i = 23, 11, 19, 8, 1, 7, 25, 13, 21$), are defined as

$$f^{v}_{we:23} = f_{12} + \frac{\rho^{v:t}_{we:in}\left(u^{v:t}_{we:x} + u^{v:t}_{we:y} - u^{v:t}_{we:z}\right)}{12} \quad f^{v}_{we:25} = f_{14} + \frac{\rho^{v:t}_{we:in}\left(u^{v:t}_{we:x} - u^{v:t}_{we:y} + u^{v:t}_{we:z}\right)}{12}$$

$$f^{v}_{we:21} = f_{10} + \frac{\rho^{v:t}_{we:in}\left(u^{v:t}_{we:x} - u^{v:t}_{we:y} - u^{v:t}_{we:z}\right)}{12} \quad f^{v}_{we:19} = f_8 + \frac{\rho^{v:t}_{we:in}\left(u^{v:t}_{we:x} - u^{v:t}_{we:y} - u^{v:t}_{we:z}\right)}{12}, \tag{205}$$

$$f^{v}_{ew:11} = \left(\frac{\rho^{v:mi}_{ew:in}}{\rho^{v:mi}_{ew:in} - 3} + \frac{\rho^{v:f}_{ew:in}}{\rho^{v:f}_{ew:in} - 3}\right)\left(\frac{f_{14} + f_5 + f_6 + f_{15} - f_{16} + f_{17} - f_{18}}{2}\right) + \frac{\rho^{v:mi}_{ew:in} u^{v:mi}_{x} + \rho^{v:f}_{ew:in} u^{v:f}_{x}}{6} + \frac{3\rho^{v:mw}_{we:in}\left(u^{v:mw}_{we:x} + u^{v:mw}_{we:y}\right) + 3\rho^{v:e}_{we:in}\left(u^{v:e}_{we:x} + u^{v:e}_{we:y}\right)}{4}, \tag{206}$$

$$f^{v:mi}_{ew:8} = \left(\frac{\rho^{v:mi}_{ew:in}}{\rho^{v:mi}_{ew:in} - 3} + \frac{\rho^{v:f}_{ew:in}}{\rho^{v:f}_{ew:in} - 3}\right)\left(\frac{2f_9 - f_3 - f_4 - f_{15} - f_{16} + f_{17} + f_{18}}{2}\right) + \frac{\rho^{v:mi}_{ew:in} u^{v:mi}_{x} + \rho^{v:f}_{ew:in} u^{v:f}_{x}}{6} + \frac{3\rho^{v:mw}_{se:in}\left(u^{v:mw}_{we:x} + u^{v:mw}_{we:z}\right) + 3\rho^{v:e}_{se:in}\left(u^{v:e}_{we:x} + u^{v:e}_{we:z}\right)}{4}, \tag{207}$$

$$f^{v}_{ew:1} = \frac{\rho^{v:mi}_{ew:in} u^{v:mi}_{x} + \rho^{v:f}_{ew:in} u^{v:f}_{x} + \rho^{v:t}_{we:in} u^{v:m}_{we:x} + \rho^{v:m}_{we:in} u^{v:m}_{we:x} + 11f_2 + f_0 + f_4 + f_3 + f_5 + f_6}{3}, \tag{208}$$

$$f^{v}_{ew:7} = \left(\frac{\rho^{v:mi}_{ew:in}}{\rho^{v:mi}_{ew:in} - 3} + \frac{\rho^{v:f}_{ew:in}}{\rho^{v:f}_{ew:in} - 3}\right)\left(\frac{f_{15} - f_{10} - f_2 + f_3 - f_4 + f_{16} - f_{17} - f_{18}}{2}\right) + \frac{\rho^{v:mi}_{ew:in} u^{v:mi}_{ew:x} + \rho^{v:f}_{ew:in} u^{v:f}_{ew:x}}{6} + \frac{3\rho^{v:mw}_{we:in}\left(u^{v:mw}_{we:x} - u^{v:mw}_{we:z}\right) + 3\rho^{v:e}_{we:in}\left(u^{v:e}_{we:x} - u^{v:e}_{we:z}\right)}{4}, \tag{209}$$

$$f^{v}_{ew:13} = \left(\frac{\rho^{v:mi}_{ew:in}}{\rho^{v:mi}_{ew:in} - 3} + \frac{\rho^{v:f}_{ew:in}}{\rho^{v:f}_{ew:in} - 3}\right)\left(\frac{f_3 + 2f_{12} + 2f_8 - f_4 + f_{15} + f_{16} - f_{17} - f_{18}}{2}\right) + \frac{\rho^{v:mi}_{ew:in} u^{v:mi}_{x} + \rho^{v:f}_{ew:in} u^{v:f}_{x}}{6} + \frac{\rho^{v:mw}_{we:in}\left(u^{v:mw}_{we:x} - u^{v:mw}_{we:z}\right) + \rho^{v:e}_{we:in}\left(u^{v:e}_{we:x} - u^{v:e}_{we:z}\right)}{4}. \tag{210}$$

For the east inlet and west outlet case, $f^{v}_{ew:i}$ $(i = 22, 12, 26, 10, 2, 9, 20, 14, 24)$ are defined as,

$$f^{v}_{ew:26} = f_{13} - \frac{\rho^{v:t}_{ew:in}\left(u^{v:t}_{ew:x} - u^{v:t}_{ew:y} + u^{v:t}_{ew:z}\right)}{12} \quad f^{v}_{ew:22} = f_{9} - \frac{\rho^{v:t}_{ew:in}\left(u^{v:t}_{ew:x} - u^{v:t}_{ew:y} - u^{v:t}_{ew:z}\right)}{12}$$
$$f^{v}_{ew:24} = f_{11} + \frac{\rho^{v:t}_{ew:in}\left(u^{v:t}_{ew:x} + u^{v:t}_{ew:y} - u^{v:t}_{ew:z}\right)}{12} \quad f^{v}_{ew:20} = f_{7} + \frac{\rho^{v:t}_{ew:in}\left(u^{v:t}_{ew:x} + u^{v:t}_{ew:y} - u^{v:t}_{ew:z}\right)}{12}, \tag{211}$$

$$f^{v}_{ew:12} = \left(\frac{\rho^{v:mi}_{we:in}}{\rho^{v:mi}_{we:in} - 3} + \frac{\rho^{v:f}_{we:in}}{\rho^{v:f}_{we:in} - 3}\right)\frac{(2f_{13} + 2f_{7} + f_{5} - f_{6} + f_{15} - f_{16} + f_{17} - f_{18})}{2} - \frac{\rho^{v:mi}_{we:in}u^{mi}_{x} + \rho^{v:f}_{we:in}u^{v:f}_{x}}{6}$$
$$+ \frac{3\rho^{v:mw}_{ew:in}\left(u^{v:mw}_{ew:x} - u^{v:mw}_{ew:y}\right) + 3\rho^{v:e}_{ew:in}\left(u^{v:e}_{ew:x} - u^{v:e}_{ew:y}\right)}{4}, \tag{212}$$

$$f^{v}_{ew:10} = \left(\frac{\rho^{v:mi}_{we:in}}{\rho^{v:mi}_{we:in} - 3} + \frac{\rho^{v:f}_{we:in}}{\rho^{v:f}_{we:in} - 3}\right)\frac{(2f_{7} + 2f_{1} - f_{3} + f_{4} - f_{15} - f_{16} + f_{17} + f_{18})}{2} - \frac{\rho^{v:mi}_{we:in}u^{v:mi}_{x} + \rho^{v:f}_{we:in}u^{v:f}_{x}}{6}$$
$$- \frac{3\rho^{v:mw}_{in}\left(u^{v:mw}_{ew:x} - u^{v:mw}_{ew:z}\right) + 3\rho^{v:e}_{in}\left(u^{v:e}_{ew:x} - u^{v:e}_{ew:z}\right)}{4}, \tag{213}$$

$$f^{v}_{ew:2} = \frac{7f_{1} + f_{0} + f_{4} + f_{3} + f_{5} + f_{6} - \rho^{v:mi}_{we:in}u^{v:mi}_{x} - \rho^{v:f}_{we:in}u^{v:f}_{x} - \rho^{v:t}_{ew:in}u^{t}_{ew:x} - \rho^{v:m}_{we:in}u^{v:m}_{x}}{3}, \tag{214}$$

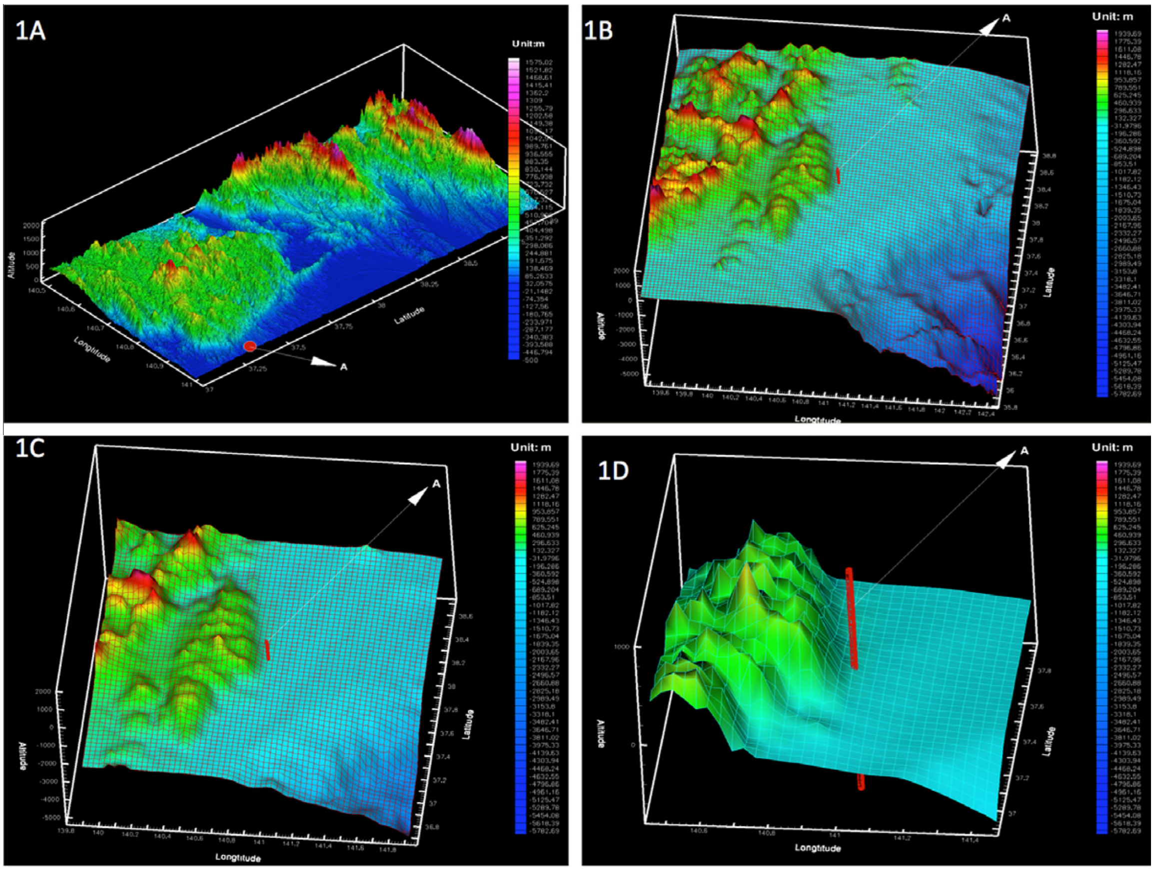

Fig. 7. Fukushima Cesium-137 penetration and diffusion D3Q27 model. (1A)-The land penetration and diffusion model [E140.25° ~ 141.021°; N37° ~ 39°]; (1B)-150 Km range penetration and diffusion model [E139.4° ~ 142.4°; N35.8° ~ 38.8°]; (1C)-100 Km range penetration and diffusion model [E140.15° ~ 141.65°; N36.55° ~ 38°]; (1D)-50 Km range penetration and diffusion model [E140.4° ~ 141.4°; N36.8° ~ 37.8°].

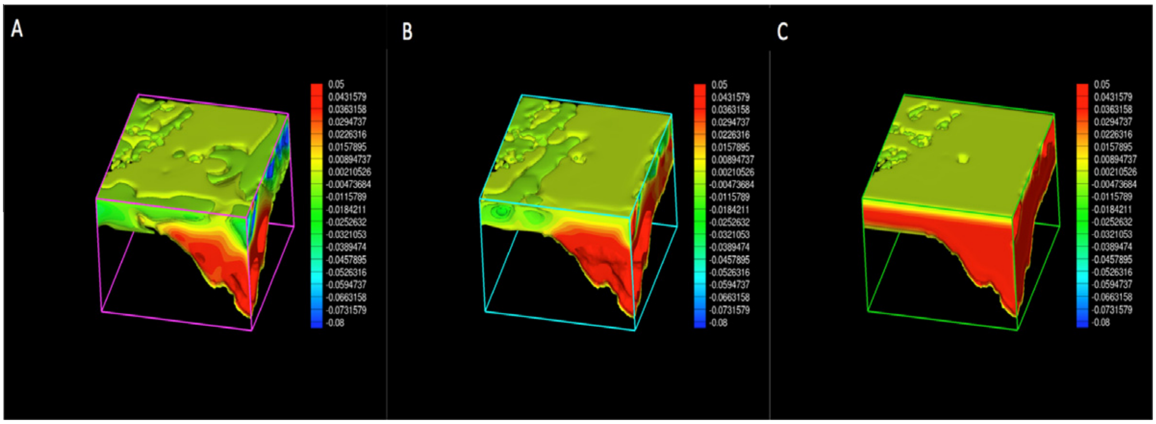

Fig. 8. Penetration and diffusion process as function of time and spatial variables. (A.) 6 months after 2009 Japan-Honshu 9.0 Earthquake; (B.) 12 months after 2009 Japan-Honshu 9.0 Earthquake; (C.) 18 months after 2009 Japan-Honshu 9.0 Earthquake.

$$f^{v}_{ew:9} = \left(\frac{\rho^{v:mi}_{we:in}}{\rho^{v:mi}_{we:in}-3}+\frac{\rho^{v:f}_{we:in}}{\rho^{v:f}_{we:in}-3}\right)\left(\frac{2f_8+f_3+f_4-f_{15}-f_{16}+f_{17}+f_{18}}{2}\right)-\frac{\rho^{v:mi}_{we:in}u^{v:mi}_x+\rho^{v:f}_{we:in}u^{v:f}_x}{6} + \frac{3\rho^{v:mw}_{ew:in}(u^{v:mw}_{ew:x}+u^{v:mw}_{ew:z})+3\rho^{v:e}_{ew:in}(u^{v:e}_{ew:x}+u^{v:e}_{ew:z})}{4}, \tag{215}$$

$$f^{v}_{ew:14} = \left(\frac{\rho^{v:mi}_{we:in}}{\rho^{v:mi}_{we:in}-3}+\frac{\rho^{v:f}_{we:in}}{\rho^{v:f}_{we:in}-3}\right)\left(\frac{f_6-f_{15}+f_{16}-f_{17}+f_{18}+2f_{11}+f_5}{2}\right)-\frac{\rho^{v:mi}_{we:in}u^{v:mi}_x+\rho^{v:f}_{we:in}u^{v:f}_x}{6} - \frac{3\rho^{v:mw}_{in}\left(u^{v:mw}_{ew:x}+u^{v:mw}_{ew:y}\right)+3\rho^{v:e}_{in}\left(u^{v:e}_{ew:x}+u^{v:e}_{ew:y}\right)}{4}. \tag{216}$$

7. Application

To verify the precision, stability and convergent valid of the extended hybrid boundary conditions, the typical extended 3D flow driven pore-crack network Fukushima nuclear plant leak accident model is considered and the penetration process of Cesium-137 at different temporal spatial scales have been calculated.

As shown in Fig. 7, the multi temporal spatial scale Fukushima Cesium-137 penetration and diffusion D3Q27 LBM model is established.

The penetration and diffusion process as function of time and spatial variables are shown in the Fig. 8. After the disaster, Cesium-137 penetration and diffusion into seawater/stratum fastly and the preferred direction of diffusion is Pacific Ocean zone. After six months, the nuclear leak will reach to the bottom of Pacific Ocean, and the preferred direction of diffusion is change to crust zone. Over another twelve months the nuclear leak will diffuse to all zones (100 Km $\times$ 100 Km) and at this time the content of Cesium-137 close to the surface of the Earth begin reduce. In engineer practice, this can help understand the extended fluid flow mechanism in various porosity composites and analyze the extended fluid flow varying mechanism on nuclear leak problem.

8. Conclusions

In this paper, the extended hybrid electronic-ionic, thermal, electromagnetic (weak and strong coupled conditions) and force couple fields pressure and velocity boundary conditions for the lattice Boltzmann model is established.

The numerical model of an extended fluid flow driven pore-crack network is proposed to examine the accurate of the hybrid boundary condition. The simulation verify that the precision, stability and convergent valid is satisfied.

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References

[1] J.M. Zhan, Y.Y. Luo, Y.S. Li, A high accuracy hybrid method for two-dimensional Navier–Stokes equations, Appl. Math. Model. 32 (2008) 873.
[2] Y.H. Qian, D. D'Humières, P. Lallemand, Lattice BGK models for Navier–Stokes equation, Europhys. Lett. 17 (1992) 479.
[3] Q. Zou, S. Hou, S. Chen, G.D. Doolen, A improved incompressible lattice Boltzmann model for time-independent flows, J. Stat. Phys. 81 (1995) 35.
[4] Z. Guo, B. Shi, N. Wang, Lattice BGK model for incompressible Navier–Stokes equation, J. Comput. Phys. 165 (2000) 288.
[5] D.R. Noble, S. Chen, J.G. Georgiadis, A consistent hydrodynamic boundary condition for the lattice Boltzmann method, Phys. Fluids 7 (1995) 203.
[6] S. Chen, D. Martínez, R. Mei, On boundary conditions in lattice boltzmann methods, Phys. Fluids 8 (1996) 2527.
[7] S. Chen, G.D. Doolen, lattice boltzmann method for fluid flows, Annu. Rev. Fluid Mech. 30 (1998) 329.
[8] J. Bear, The transition zone between fresh and salt waters in coastal aquifers, Ph.D thesis 1960; Berkeley, University of California.
[9] Q.S. Zou, X.Y. He, On pressure and velocity boundary conditions for the lattice Boltzmann BGK model, Phys. Fluids 9 (1997) 1591.
[10] R.S. Maier, R.S. Bernard, D.W. Grunau, Boundary conditions for the lattice Boltzmann method, Phys. Fluids 8 (1996) 1788.
[11] D.R. Noble, S.Y. Chen, J.G. Georgiadis, R.O. Buckius, A consistent hydrodynamic boundary-condition for the lattice Boltzmann method, Phys. Fluids 7 (1995) 203.
[12] D. Gary, Doolen, Lattice gas methods: theory, applications, and hardware, MIT Press, Cambridge, Mass., 1991.
[13] X.Y. He, G.D. Doolen, Thermodynamic foundations of kinetic theory and Lattice Boltzmann models for multiphase flows, J. Stat. Phys. 107 (2002) 309.
[14] D. Gary, Doolen, M. Frisch, B. Hasslacher, S. Orszag, S. Wolfram, Lattice gas methods for partial differential equations, Addison-Wesley, Redwood City, California, Wokingham, 1990. 4.
[15] H.I. Zhang, S.D. Hu, G.L. Wang, J.Y. Zhu, Modeling and simulation of plasma jet by lattice Boltzmann method, Appl. Math. Model. 31 (2007) 1124.
[16] Z.H. Chai, B.C. Shi, A novel lattice Boltzmann model for the poisson equation, Appl. Math. Model. 32 (2008) 2050.
[17] U. Frisch, B. Hasslacher, Y. Pomeau, Lattice-gas automata for the Navier–Stokes equation, Phys. Rev. Lett. 56 (1986) 1505.
[18] R. Jeremy, Henderson, G. Ian, Main, M. Calum, G. Michael, Norman, A fracture-mechanical cellular automaton model of seismicity, Pure Appl. Geophys. 142 (1994) 545.
[19] X.Y. He, L.S. Luo, Theory of the lattice Boltzmann method: from the Boltzmann equation to the lattice Boltzmann equation, Phys. Rev. E 56 (1997) 6811.
[20] X.Y. He, L.S. Luo, Lattice Boltzmann model for the incompressible Navier–Stokes equation, J. Stat. Phys. 88 (1997) 927.
[21] Z.L. Guo, B.C. Shi, N.C. Wang, Lattice BGK model for incompressible Navier–Stokes equation, J. Comput. Phys. 165 (2000) 288.
[22] B. Zhu, T. Qin, Hypersingular integral equation method for a three-dimensional crack in anisotropic electro-magneto-elastic bimaterials, Theor. Appl. Fract. Mech. 47 (2007) 219.
[23] B. Zhu, T. Qin, 3D modeling of crack growth in electro-magneto-thermo-elastic coupled viscoplastic multiphase composites, Appl. Math. Model. 33 (2009) 1014.