Unofficial ERRATA and Commentary for Continuum Mechanics for Engineers–3rd ed.
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1. The unnumbered equation at the top of page 12 is called Lagrange's formula, which is widely used, specially when substituting ∇ for $\mathbf{u}$ and $\mathbf{v}$, resulting in

$$\nabla \times (\nabla \times \mathbf{w}) = \nabla(\nabla \cdot \mathbf{w}) - (\nabla \cdot \nabla)\mathbf{w} = grad(div\mathbf{w}) - lapacian\mathbf{w}$$

2. Equation (2.20) written in the textbook as $(\mathbf{a} \otimes \mathbf{b})\mathbf{u} = \mathbf{a}\ (\mathbf{b} \cdot \mathbf{u})$ should read

$$(\mathbf{a}\ \mathbf{b}) \cdot \mathbf{u} = \mathbf{a}\ (\mathbf{b} \cdot \mathbf{u}) \tag{2.20}$$

   Otherwise the expression in the book, also found in [1, (1.53)], is difficult to read because there is a contraction implied between $(\mathbf{a} \otimes \mathbf{b})$ and $\mathbf{u}$, but not between $\mathbf{a}$ and $(\mathbf{b} \cdot \mathbf{u})$; that is, the lack of a symbol between two tensor variables may or may not imply a contraction depending on context. Such notation is very hard to read, and the textbook abandons it for the rest of the book. Better put the dot for every dot product, and as a further advantage, the symbol $\otimes$ is no longer necessary for the outer product; i.e., if a dot is used for the dot product, and a $\times$ is used for the cross product, then, lack of symbol means outer product.

3. Line immediately below (2.28) should read...

   (Note that, in other works in the literature such as [1, (1.53)], these products may be written without the dot as $\mathbf{vT}$ and $\mathbf{Tv}$ to emphasize the linear operator nature of $\mathbf{T}$ on $\mathbf{v}$.)

   See comment #2 above.

4. Line immediately below (2.96) should read...
   Called Gauss's divergence theorem, Eq. 2.95

5. Problem 2.37. Notation problem. Most of the book uses a dot to indicate a contraction. So,

- (a) should be
$$[\mathbf{A} \cdot a, \mathbf{A} \cdot b, \mathbf{A} \cdot c] = det\mathbf{A}[a, b, c]$$
- (b) should be
$$\mathbf{A}^T \cdot ((\mathbf{A} \cdot a) \times (\mathbf{A} \cdot b)) = det\mathbf{A}(a \times b)$$

6. I fount it advantageous to compare Section 5.5 on p. 176, to the part of Section 3.4 after equation (3.27) on p. 62.

7. (3.56) should read
$$\sigma_S^2 = \sigma_I^2 n_1^2 + \sigma_{II}^2 n_2^2 + \sigma_{III}^2 n_3^2 - \left[\sigma_I n_1^2 + \sigma_{II} n_2^2 + \sigma_{III} n_3^2\right]^2 \tag{3.56}$$

8. (3.57) should read
$$\begin{aligned}\sigma_S^2 = (\sigma_I^2 - \sigma_{III}^2) * n_1^2 + (\sigma_{II}^2 \\ - \sigma_{III}^2) * n_2^2 + \sigma_{III}^2 - \left[(\sigma_I - \sigma_{III}) * n_1^2 + (\sigma_{II} - \sigma_{III}) * n_2^2 + \sigma_{III}\right]^2\end{aligned} \tag{3.57}$$

9. (3.58) should read
$$\frac{\partial \sigma_S^2}{\partial n_1} = n_1 * (\sigma_I - \sigma_{III}) * \left\{\sigma_I - \sigma_{III} - 2 * \left[(\sigma_I - \sigma_{III}) * n_1^2 + (\sigma_{II} - \sigma_{III}) * n_2^2\right]\right\} \tag{3.58a}$$
$$\frac{\partial \sigma_S^2}{\partial n_2} = n_2 * (\sigma_{II} - \sigma_{III}) * \left\{\sigma_{II} - \sigma_{III} - 2 * \left[(\sigma_I - \sigma_{III}) * n_1^2 + (\sigma_{II} - \sigma_{III}) * n_2^2\right]\right\} \tag{3.58b}$$

10. The second term (convection part) of the symbolic term in (4.32) should be corrected as follows
$$\frac{d}{dt} = \frac{\partial()}{\partial t} + v_k \frac{\partial()}{\partial x_k} \quad ; \quad \frac{d()}{dt} = \frac{\partial()}{\partial t} + \nabla() \cdot \mathbf{v} \tag{4.32}$$
For example, the acceleration is
$$a_i = \frac{dv_i}{dt} = \frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} \tag{4.32'}$$
The MATLAB code below illustrates it by extending Example 4.4 to recalculate the acceleration in Eulerian coordinates without ever using the Lagrangian expressions, and reaching the same result as in Ex. 4.4.

```
% Ex 4.4 plus material derivative
% (c) Ever Barbero (2010)
clear, clc
```

```
syms X1 X2 X3 t x1 x2 x3

% Lagrangean motion equations
X = [X1; X2; X3]
xx1 = X1*exp(t)+X3*(exp(t)-1)
xx2 = X2+X3*(exp(t)-exp(-t))
xx3 = X3

% Invert to find the Eulerian motion equations
XX = solve(x1-xx1,x2-xx2,x3-xx3,X1,X2,X3)
X = [XX.X1; XX.X2; XX.X3]

% Lagrangean velocity and acceleration vectors (columns)
x = [xx1; xx2; xx3]
v = simplify(diff(x,t))
a = simplify(diff(v,t))

% Eulerian velocity and acceleration vectors
V1 = simplify(subs(v(1),{X1,X2,X3},{X(1),X(2),X(3)}));
V2 = simplify(subs(v(2),{X1,X2,X3},{X(1),X(2),X(3)}));
V3 = simplify(subs(v(3),{X1,X2,X3},{X(1),X(2),X(3)}));
V = [V1; V2; V3]

A1 = simplify(subs(a(1),{X1,X2,X3},{X(1),X(2),X(3)}));
A2 = simplify(subs(a(2),{X1,X2,X3},{X(1),X(2),X(3)}));
A3 = simplify(subs(a(3),{X1,X2,X3},{X(1),X(2),X(3)}));
A = [A1; A2; A3]

% Material deviravative
% Find the Eulerian acceleration using the material derivative operator
% on the Eulerian velocity vector never going back to Lagrangean
gradV(1,1) = diff(V(1),x1);
gradV(1,2) = diff(V(1),x2);
gradV(1,3) = diff(V(1),x3);
gradV(2,1) = diff(V(2),x1);
gradV(2,2) = diff(V(2),x2);
gradV(2,3) = diff(V(2),x3);
gradV(3,1) = diff(V(3),x1);
gradV(3,2) = diff(V(3),x2);
gradV(3,3) = diff(V(3),x3);
gradV

temp = gradV*V % convection term on (4.32 corrected)
```

```
% temp = (V'*gradV)' % convection term on (4.32 uncorrected) is wrong

AA = diff(V,t) % time derivative in (4.32)

AA = AA+temp %(4.32 corrected) yields the same Eulerian acceleration as A above
```

11. First line, p. 132, substitute '(unit elongation)' for '(longitudinal strain)' because $e_{(\widehat{N})}$ is not a strain in the sense that E_{11} or ε_{11} are. See (4.113) and discussion that follows. The word *strain* is used incorrectly several time in this section; e.g., line before (4.116).

12. In Problem 3.9, p. 92, should be $t_{31} = -C_{x_2}$. Note that the stress tensor must be symmetric.

13. First line in Section 4.10 should read 'In Chapter 2 (Example 2.7) we noted...'

14. Page 135, 7th line after (4.120b), substitute 'Eq 4.119'for 'Eq 4.120b'

15. p. 136, last line before Example 4.15, which in the book reads "using $\mathbf{C}^{-1} = \mathbf{F} \cdot \mathbf{F}^T = \mathbf{V}^2$" should read "using $\mathbf{c}^{-1} = \mathbf{F} \cdot \mathbf{F}^T = \mathbf{V}^2$", that is, $\mathbf{V}^2$ is the inverse of the Cauchy deformation tensor $\mathbf{c} = \mathbf{F}^{-T} \cdot \mathbf{F}^{-1}$ (4.48). Note that the Cauchy deformation tensor $\mathbf{c}$ is not the same as the Green deformation tensor $\mathbf{C}$, thus making this typo a nasty one to discover.

16. Equation (4.159) is called Nanson's formula. It relates the infinitesimal area between the undeformed $d\mathbf{S^0}$ and deformed $d\mathbf{S}$ configurations. Obviously, $X_{A,q} = \mathbf{F}^{-T}$, so, Nanson's formula is usually written to highlight the participation of (the inverse of) the deformation gradient as follows

$$d\mathbf{S} = J\ \mathbf{F}^{-T} \cdot d\mathbf{S}^0$$

    Immediately after this, or at the end of this section 4.12, I teach section 5.4, which I feel is otherwise orphan in Chapter 5.

17. In Problem 4.36, (b) should read '(b) the stretch rate per unit stretch...', not 'per unit length...'

18. In Problem 4.42, $v_2 = e^{x_3 - kt} \sin \omega t$ should read $v_2 = e^{x_3 - ct} \sin \omega t$

19. Problem 4.42, solution for part (c) is wrong because the statement of the problem calls for a direction

$$\widehat{\mathbf{n}} = (\widehat{\mathbf{e}}_1 + \widehat{\mathbf{e}}_2)/\sqrt{2}$$

    The correct solution is

$$\frac{d \ln \Lambda}{dt} = 0$$

Otherwise, if you change the statement of the problem for a direction

$$\widehat{\mathbf{n}} = (\widehat{\mathbf{e}}_1 + \widehat{\mathbf{e}}_3)/\sqrt{2}$$

then, the solution given in the book is correct.

20. To derive the equation before (5.2), use equation (4.161), $\dot{J} = v_{k,k} J$.

21. Equation above (5.3) has the subscript k repeated three times, which violates one of the rules of indicial notation. You must delete the second k inside the parenthesis,

$$= \int_V \left[... + \left(v_k P^*_{ij...} \right)_{,k} \right] dV$$

which upon application of the divergence theorem [specifically (2.95)] becomes

22. Equation above (5.18) is missing a J

$$= \int_{V^0} \dot{A}_{ij...} \; \rho \; J \; dV^0 = ...$$

23. Line after (5.57), does not agree with terminology in other references (e.g., [1, (4.84)]) in that $\mathbf{S}$ defined in (5.57) is the stress power, not the stress work. In fact, the stress work is

$$\int_V t_{ij} \epsilon_{ij} dV$$

where $\epsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ is the linear strain.

Errata for the Solutions Manual

This section applies only if you are an instructor and thus are legally entitled to have a copy of the solutions manual. *If you are a student that somehow got a copy of the solutions manual and you are consulting it, your behavior is unethical and it only hurts you. The textbook exercises at the end of chapters are designed to force you to understand the material. If you don't, you should ask the instructor. Otherwise, just looking at the solutions makes everything look easy and you are missing a big chunk of your learning; you are not developing your skills in the subject.*

- Problem 2.19. There are several typos and mistakes on the last four lines of equations. Make sure you work them out yourself before you assign it to the students.

- Problem 2.21. The third equation of section (b) of the solution has a redundant '$= \frac{1}{\sqrt{2}}$' on its LHS.

- Problem 3.15. There is a typo repeated twice in the solution. To correct it, note that the statement on p. 95 reads (a) the principal stresses $\sigma_I, \sigma_{II}, \sigma_{III}$.

- Problem 4.12. The answer is correct but the solution is incomplete.
- Problem 4.15. The second part of the first line in the solution

$$u_i^{(1)} = A_{ij}^{(1)} X_j \quad ; \quad u_i^{(2)} = A_{ij}^{(2)} X_j$$

dealing with the second deformation (2) is confusing as stated. The solution path becomes more clear and the students are able to successfully complete the problem if said line is given as

$$u_i^{(1)} = A_{ij}^{(1)} X_j \quad ; \quad u_i^{(2)} = A_{ij}^{(2)} X_j^{(1)}$$

where

$$X_j^{(1)} = X_j + u_j^{(1)}$$

are the updated Lagrangean coordinates of the particle after deformation (1) is completed.

- Problem 4.36, part (d), it is much easier and intuitive to calculate

$$\dot{\gamma}_{max} = \frac{\lambda_{(1)} - \lambda_{(3)}}{2}$$

i.e., one half of the difference between the largest and smallest principal values of the rate of deformation tensor.

References

[1] Holzapfel, G. A. (2000) Nonlinear Solid Mechanics, Wiley.