

## Fatigue

**Required reading.** P.C. Paris, M.P. Gomez and W.E. Anderson, A rational analytic theory of fatigue. The Trend in Engineering 13, 9-14 (1961). I went online and found that the Trend in Engineering is the alumni newsletter of the College of engineering, of the University of Washington (<http://www.engr.washington.edu/news/trend.html>). I could not find this paper online. John Hutchinson offered to write to Paul Paris for a copy of the paper, which Paris sent by airmail. A scanned copy of the paper is attached with this lecture.

Subject to monotonic bending, a metallic clip deforms, but may not break. Subject to cyclic bending, however, the clip will break after some cycles. Fracture caused by cyclic loads is known as fatigue. Examples of cyclic loads include vibration, landing and taking off of an airplane, pressurization and depressurization of a fuselage.

Fatigue is an extremely complex phenomenon. Its study has gone on well over a century. The subject can fill an entire course and more. In this course, we will give one lecture on the subject, focusing on the application of fracture mechanics to fatigue.

While all materials suffer fatigue, the effect is particularly pronounced for ductile materials, such as metals and glassy polymers. For such a ductile material, fracture may occur after a small number of cycles, when cyclic loads repeatedly cause large-scale plastic deformation. In most engineering structures, however, loads are kept small so that the structures deform elastically. Even so, a crack may initiate somewhere in such a structure, and extend by a small amount during each cycle of the load. The structure remains elastic, except for a small zone around the front of the crack. When the crack reaches a critical size, the structure breaks by rapid fracture.

Fatigue may be divided into two phases:

- the initiation of a crack and
- the extension of the crack.

The extension of a crack in a body under a cyclic load can be characterized by using fracture mechanics, as described by Paris et al. (1961). You should begin by reading this original paper. The paper is lucid and focused. I know of no better description of the essential idea. Of the paper Paris would (1998) recount years later:

The initial reaction for this young researcher at that time was that we were on to something big and important to the safety of aircraft and other structures as well. It was time to tell the world about it, so a paper was written clearly showing the correlation of data from three independent sources on two materials. The tools of Fracture Mechanics analysis needed to apply lab data to structural configurations were also illustrated. Well, that paper was very promptly rejected by three of the world's leading journals. All of the reviewers simply stated that "no elastic parameter, e.g.  $K$ , could possibly correlate fatigue cracking rates because plasticity was a dominant feature". They proceeded to somehow totally disregard the facts clearly demonstrated by the data!

Indeed, fatigue is inherently an inelastic phenomenon. It does sound logical that an elastic parameter will have nothing to say about fatigue. At this stage in our course I hope that you can see through the fallacy of this faulty logic.

**Approaches to the study of fatigue.** Fatigue is of great practical importance. It is also a complex phenomenon. The scientific study of fatigue has occupied people for more than a century. Although much is known, most basic issues remain unresolved. In particular, no approach exists to predict fatigue from first principles. Qualitative pictures of various fatigue mechanisms are often too vague to be of much use.

Fatigue has been studied at several scales:

- Atomic scale. Track accumulative activities such as dislocation generation and organization of patterns.
- Microstructural scale. Fatigue striation. Persistent slip bands. The effect of grains and inclusions.
- Component scale. In this lecture, we will focus on this scale. The Paris approach is now used in engineering design.

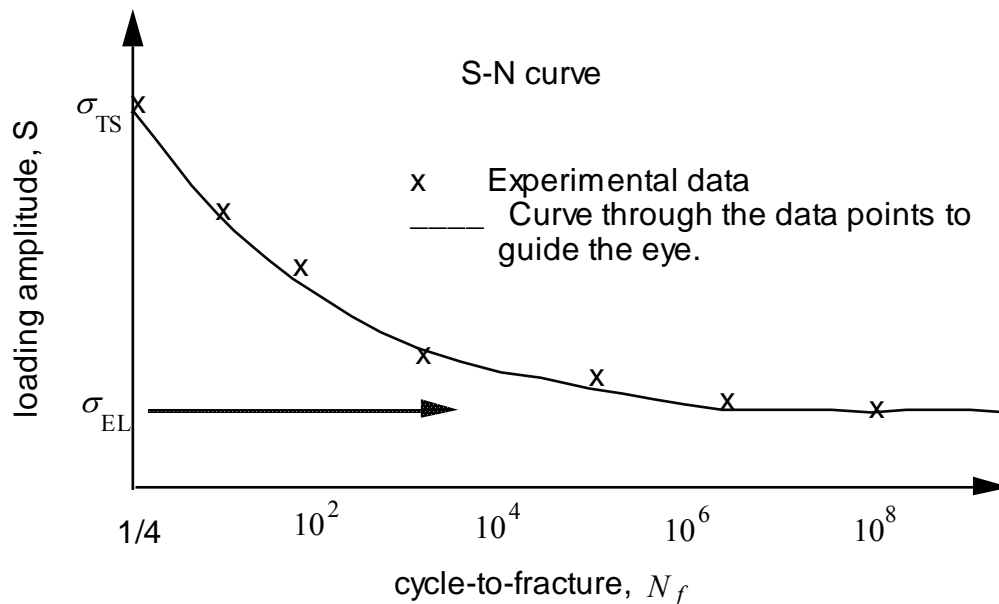
**Frequency and amplitude of the cyclic load.** In metal fatigue, the cycle-to-fracture is nearly independent of the frequency of the load, but depends on the amplitudes of the load.

**The S-N curve.** This method had been in use before the work of Paris. The stress fluctuates as a function of time. For the time being, consider the case that the stress fluctuates within a constant range. Denote the maximum stress by  $\sigma_{\max}$ , and the minimum stress by  $\sigma_{\min}$ . The range of the stress is

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}.$$

The mean stress is

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}.$$



**Experiment:** Subject a bar to a cyclic stress. To be specific, cycles are taken to be of equal tension and compression. Denote the amplitude of the stress by  $S = \sigma_{\max} = -\sigma_{\min}$ . For a given amplitude  $S$ , cycle the bar until it fractures, and record the cycle-to-fracture,  $N_f$ . This experiment produces a data point  $(S, N_f)$ . Repeat the experiment for many amplitudes and bars, each producing a data point  $(S, N_f)$ . The final result is a curve, the S-N curve. Here are some observations:

- *Ultimate tensile strength.* If the amplitude of the stress equals the ultimate tensile strength,  $S = \sigma_{\text{TS}}$ , the fracture occurs upon first loading,  $N_f = 1/4$ .

- **Endurance limit.** For a given material (e.g., a steel), there may exist an amplitude of the stress, called the endurance limit,  $\sigma_{EL}$ . When the amplitude is below the endurance limit,  $S < \sigma_{EL}$ , the bar can sustain infinite number of cycles without fracture,  $N_f \rightarrow \infty$ . Aluminum does not have an endurance limit: under cyclic load of even very small amplitude, aluminum will fail after a finite number of cycles. Let me know if you find an endurance limit for humans. I'd love to be immortal even if it means doing sub-critical amount of work every day.
- **Cycle-to-fracture.** When the load amplitude falls between the ultimate tensile strength and the endurance limit,  $\sigma_{EL} < S < \sigma_{TS}$ , the cycle-to-fracture  $N_f$  decreases as the load amplitude increases.

**The use of the S-N curve in design.** The S-N curve is specific to each material. Once it is measured experimentally for a given material, one can use the curve to answer design questions. Two types of questions are answered with a S-N curve.

- Know the amplitude of stress, and predict the cycle-to-fracture.
- Prescribe a cycle-to-fracture, and determine the allowable amplitude of stress.

#### Difficulties in using the S-N curve

- Large scatter in the S-N curve. For a prescribed amplitude of stress, the experimentally measured cycle-to-fracture varies greatly from sample to sample.
- To generate each data point on the S-N curve, one has to fracture a sample. Many samples are needed to generate a single S-N curve, along with statistics.
- Need to prescribe ad hoc procedures to handle variable stress range  $\sigma_{max} - \sigma_{min}$  and stress ratio  $\sigma_{max} / \sigma_{min}$ .

**The Paris approach.** The Paris approach assumes that a crack pre-exists in a body. Under a cyclic load, the crack grows a little bit each cycle. When the crack is long enough, the body fractures.

The stress intensity factor is the loading parameter on a crack. For example, for a crack of length  $2a$  in an infinite panel subject to remote stress  $\sigma$ , the stress intensity factor is

$$K = \sigma \sqrt{\pi a}.$$

When the applied stress  $\sigma$  cycles with time, so does the stress intensity factor. The crack extension each cycle is much smaller than the length of the crack, so that

$$K_{max} = \sigma_{max} \sqrt{\pi a},$$

and

$$K_{min} = \sigma_{min} \sqrt{\pi a}.$$

Recall the slogan: the same  $K$ -history, the same crack-tip process. The stress intensity factor is the **only** messenger between the external load and the crack-tip process, however complex this process is. Following Paris, we focus on a particular type of  $K$ -history. While the stress intensity factor varies within each cycle, both  $K_{max}$  and  $K_{min}$  vary slowly from cycle to cycle. Paris hypothesized that the crack extension per cycle,  $da/dN$ , is a function of the instantaneous  $K_{max}$  and  $K_{min}$ . For example, the function may be written as

$$\frac{da}{dN} = f(\Delta K, R),$$

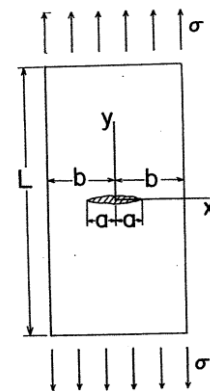


FIG. 2. A TYPICAL FATIGUE-PANEL CONFIGURATION FOR WHICH TEST DATA WERE AVAILABLE

where  $\Delta K = K_{\max} - K_{\min}$  and  $R = K_{\min} / K_{\max}$ . This function is a material property. For a given material, the function can be measured experimentally using a single sample.

To prove this hypothesis, Paris et al. (1961) plotted the experimental data of  $da/dN$  vs.  $\Delta K$  for two materials from three independent studies. The data were collected using specimens show in the figure. Included here is one of their plots. They wrote:

It is worthy of special note that the data on these curves are from three independent investigators, using many specimen sizes, i.e., widths from 1.8 to 12 in., thickness from 0.032 to 0.102 in., and lengths from 5 to 35 in. The testing frequencies varied from 50 to 2000 cpm, and the maximum stresses on the gross area varied from 6 to 30 ksi. On each graph, the materials are both clad metals and bare metals. Therefore the correlation shown is surly more than coincidental.

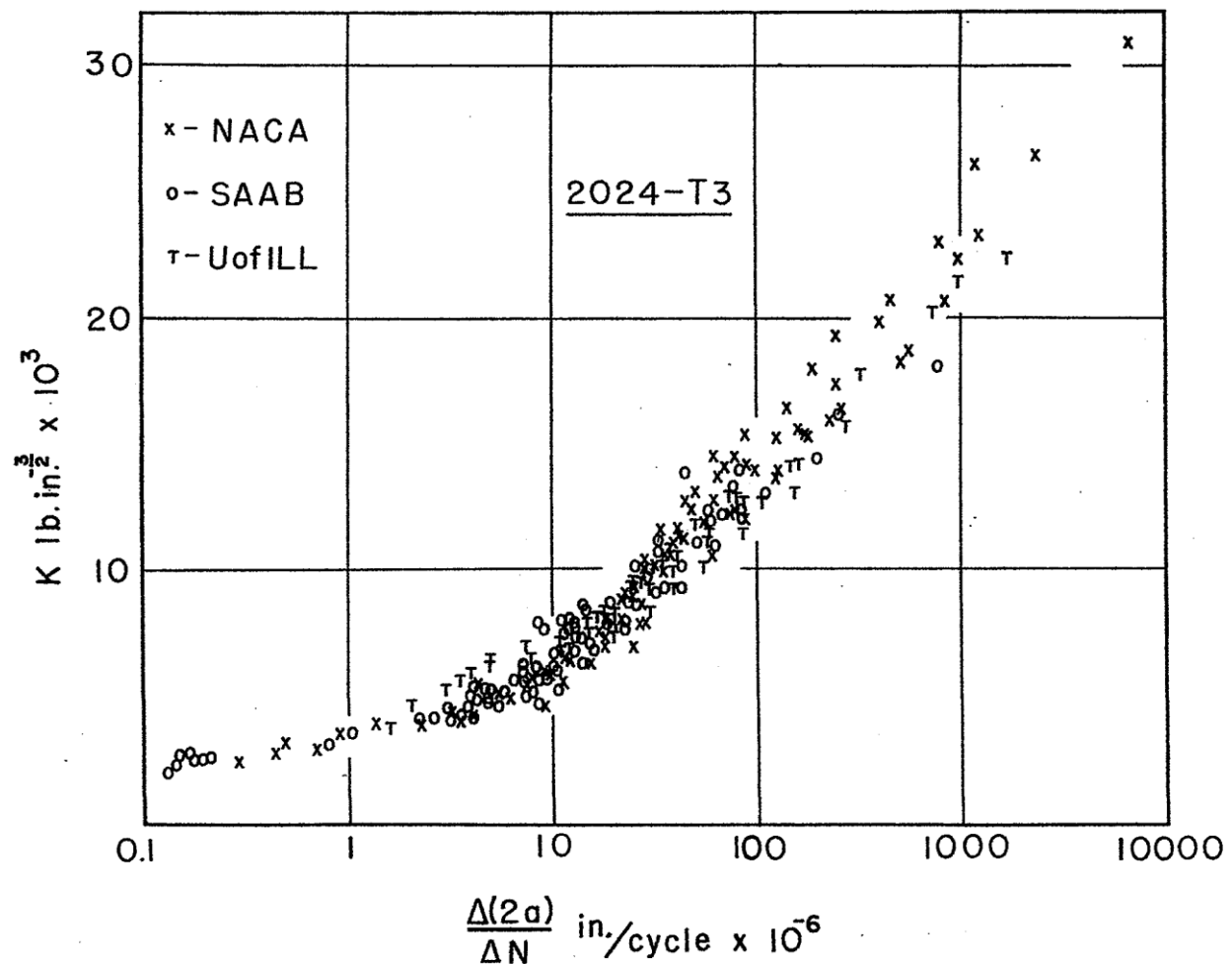


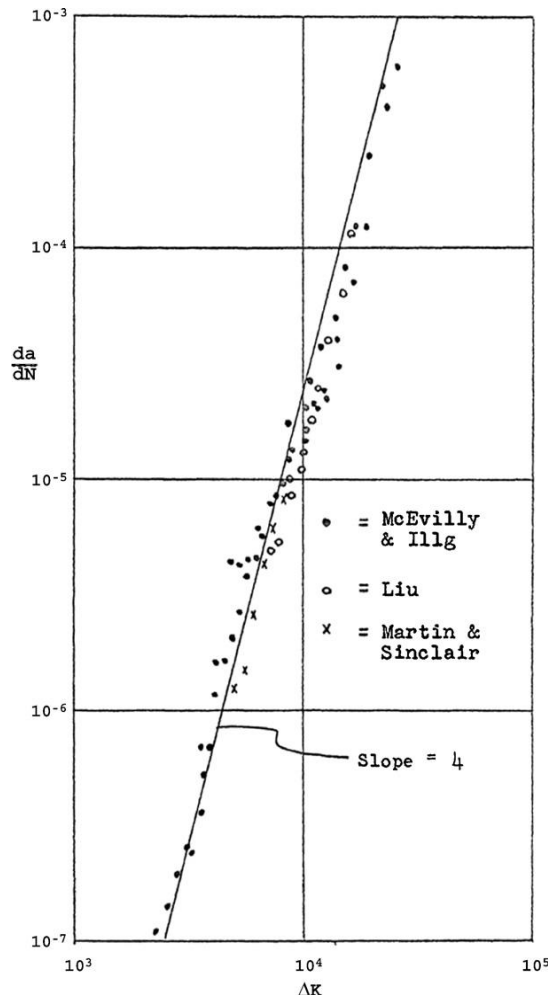
FIG. 3. CRACK EXTENSION-RATE DATA ON 2024-T3 ALUMINUM ALLOY CORRELATED FROM TESTS BY THREE INDEPENDENT INVESTIGATORS

Paris also plotted the experimental data of  $da/dN$  vs.  $\Delta K$  on a double logarithmic basis. The figure shown here is taken from Paris's PhD dissertation (1962 Lehigh University). The straight line of such a plot suggests that the experimental data fit a power law.

$$\frac{da}{dN} = A(\Delta K)^m.$$

A representative value for the exponent is  $m = 4$ . The growth also depends on  $R$ . Fatigue crack growth rate vanishes if  $R = 1$ , and increases as  $R$  reduces from 1. The curve sketched in the figure is for a specific value of  $R$ , say,  $R = 0.1$ .

The above equation is now known as the Paris law. The equation is a rather restrictive expression of Paris's original hypothesis.



**Using the Paris law in design.** In a large component, a small crack of initial size  $a_i$  has been detected. The component is subject to a cyclic stress between 0 and  $S$ . The component undergoes fast fracture when the crack size reaches  $a_f$ . How many cycles will the component last? Note that

$$\Delta K = S\sqrt{\pi a}.$$

The Paris law becomes

$$\frac{da}{dN} = A(S\sqrt{\pi a})^m.$$

This is an ordinary differential equation that governs the function  $a(N)$ . For this material, the exponent  $m = 4$ . Rearranging the above, we have

$$\frac{da}{A\pi^2 S^4 a^2} = dN.$$

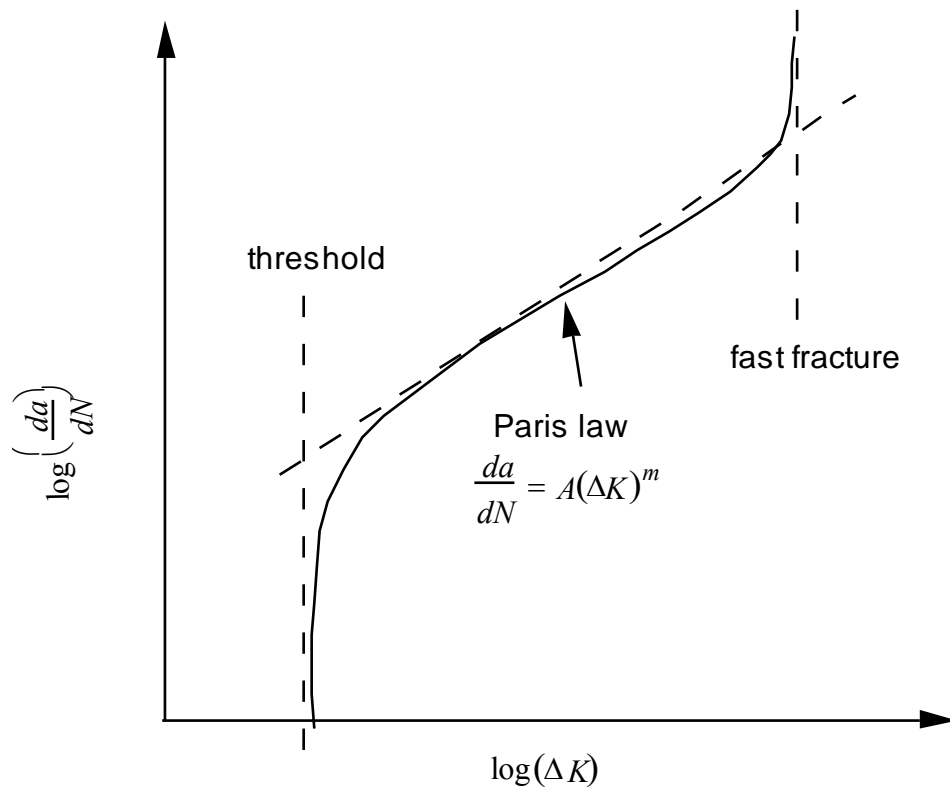
An integration from  $a_i$  to  $a_f$  gives the number of cycles to fast fracture:

$$N_f = \frac{1}{A\pi^2 S^4} \left( \frac{1}{a_i} - \frac{1}{a_f} \right).$$

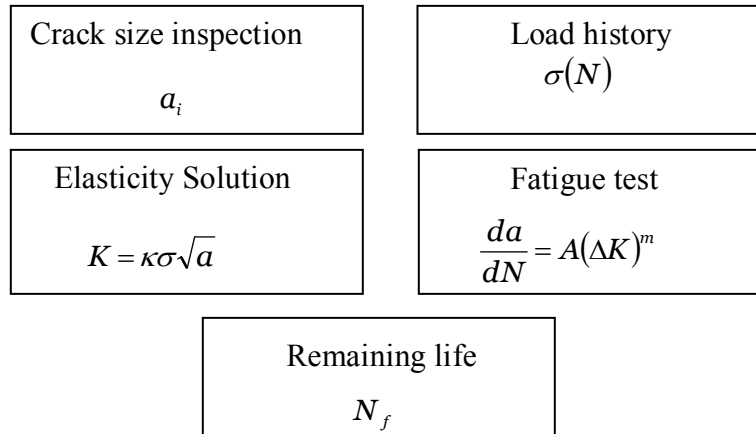
**Main observations:**

- The formula provides an explanation for the  $S$ - $N$  curve.
- Typically, the initial crack size is much smaller than the final size,  $a_i \ll a_f$ , so that the term involving  $a_f$  is negligible.
- The  $S$ - $N$  curve depends on the initial crack size. If the component has a large initial flaw, the lifetime will be much reduced.
- Also, using the  $S$ - $N$  curve without the knowledge of the initial flaw can be dangerous

**Threshold.** Let us return to the experimental data of the crack extension per cycle as a function of the stress intensity factor. The main features of the data are indicated in the figure: threshold, fast fracture, and what in between.



**Lifetime prediction: division of labor.** J.D. Achenbach, Quantitative nondestructive evaluation. *International Journal of Solids and Structures* 37, 13-37 (2000).



### References

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- M.F. Ashby and D.R.H. Jones, *Engineering Materials* 1, Pergamon Press, New York. Chapter 15 provides a background of fatigue.
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