Homework
Hand in this set of homework (except for Problem 16) **before 11 am on Friday, 17 October**
to Yuhang Hu in her office, Pierce Hall 404

16. Recommend a textbook that you think will help students in this course

- Post your recommendation as a new blog entry on iMechanica by **Wednesday, 22 October**.
- See recommendations from students who took this course before (http://www.imechanica.org/taxonomy/term/179,176,34).

Even though the lectures and homework problems are reasonably self-contained, you should always get familiar with a few textbooks on the subject. Please restrict your recommendation to a printed book. You have multiple resources to help you locate textbooks: the books reserved in the library for the course, Amazon.com, and word of mouth. Ultimately, you have to decide which books are good for you and others with similar tastes. Please select one textbook, and post a new blog entry on iMechanica. Please include the following items.

- Title of the post. The title of the book and the names of authors (Example: Theory of Elasticity by S.P. Timoshenko and J.N. Goodier).
- Use 4 free tags: **ES 240**, **solid mechanics**, **Fall 2008**, **textbooks**.
- If there are already helpful reviews of the book online, please make a hyperlink in your post to the web page of the review.
- Outline the content of the book.
- Why do you think that this book complements this course?
- Anything particular that makes you like this book and recommend it to others?
- Feel free to write anything else about the book and its relation to you and to the course.

17. Disclination (the cut-and-weld operation)

In class I followed the approach in Timoshenko and Goodier (p. 79, Fig. 45), and interpreted the physical significance of the term $\phi = Br^2 \log r$. The approach seemed to be too surprising and too cute. The surprise came about because the region after welding was multiply connected, and we had to invoke displacement field explicitly in solving the problem. Ultimately, the surprise came about because we formulated the problem in terms of the Airy stress function. Well, we don't have to.

Now let us study the disclination, the cut-and weld problem, directly in terms of displacement field. Upon expecting the geometry of the operation, we can readily convince ourselves that the hoop displacement must be

$$v(r,\theta) = \frac{\alpha r \theta}{2\pi}.$$

The inspection should also convince us that the radial displacement must be independent of θ, namely,

$$u(r,\theta) = f(r).$$

(i) Convince yourself and your teaching fellow the above expressions.

(ii) Insert the above expressions into the general equations of linear elasticity and derive the ODE for the function $f(r)$.

(iii) Solve the ODE, along with appropriate boundary conditions.

18. Design a rotating disk to avert plastic deformation

In class we have reduced the plane elasticity problem to a single PDE for the Airy stress function. This approach may not be always viable or convenient. Another approach is to express stresses and strains in terms of displacements, and then establish PDEs for the displacements. Here is an example.

The problem of the stresses in disks spinning at high speed is of vital importance in the design of many machines. We now model a solid disk in the polar coordinates (r,θ), and calculate the stresses in the disk caused by the centrifugal force. The nonzero stress components are σ_r and σ_θ. Recall the relations between the radial displacement u and the strains:

$$\varepsilon_\theta = \frac{u}{r}, \varepsilon_r = \frac{du}{dr}.$$

(a) Apply Newton's second law to a differential element in the polar coordinates, and show that

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = -\rho\omega^2 r,$$

where ω is the angular velocity, and ρ the density.

(b) Assume the disk deforms elastically. Formulate a boundary value problem and determine the stress field in the disk.

(c) If you want to design a steel disk to rotate at 3600 rotations per minute (rpm), what is the largest radius that the disk can have? ($\rho = 7800 \text{kg/m}^2, E = 210\text{GPa}, \nu = 0.3, \sigma_Y = 500\text{MPa}$).

19. A half space of an elastic material subject to a periodic traction on the surface.

An elastic material occupies the half space $z < 0$. The surface of the material, $z = 0$, is subject to a traction in shear, $\tau_{xz}(x, y, z) = \tau_0 \sin kx$, where τ_0 is the amplitude and k the wave number. Determine the stress field and displacement field inside the elastic material.

20. Orthotropy rescaling

When I was a graduate student, I noticed a relation, which I called orthotropy rescaling. I made an application of this idea and published a paper (http://www.deas.harvard.edu/suo/papers/006.pdf). The idea is outlined in this problem.

For orthotropic materials, Hooke's law is

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}
=
\begin{bmatrix}
s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\
s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\
s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & s_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & s_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & s_{66}
\end{bmatrix}
$$

where s_{ij} are the components of the compliance matrix.

   i) Consider plane stress problems in the (x,y)-plane. Prove that the Airy stress function $\phi(x,y)$ is governed by

$$\frac{\partial^4 \phi}{\partial x^4} + 2\rho\sqrt{\lambda}\,\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \lambda\frac{\partial^4 \phi}{\partial y^4} = 0\,,$$
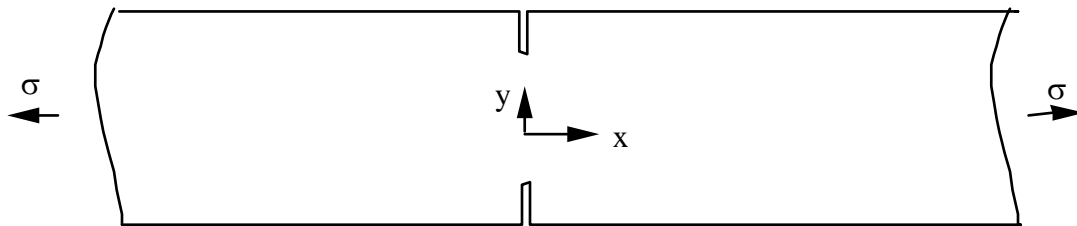
where

$$\lambda = \frac{s_{11}}{s_{22}}\,, \qquad \rho = \frac{2s_{12} + s_{66}}{2\sqrt{s_{11}s_{22}}}\,.$$

   ii) Argue that the governing equation is of the same form for plane strain problems but s_{ij} in i) should be replaced by

$$s'_{ij} = s_{ij} - \frac{s_{i3}s_{j3}}{s_{33}}\,.$$

   iii) For traction-prescribed plane problems on simply-connected domains, argue that the in-plane stresses depend on elastic constants only through λ and ρ.

   iv) Consider edge cracks (slits with no thickness) in an infinite strip subjected to a tensile load. Show that stress component σ_x is independent of λ. Consequently, σ_x depends on elastic constants only through a single parameter, ρ.

Strip with edge cracks.