Homework
Due Friday, 7 November

In addition to the following 3 problems, you will also receive a separate assignment of computer lab work, using ABAQUS.

31. A machine on a cantilever

A 100 kg machine is placed at the end of 2 m cantilever beam made of aluminum. The moment of cross section of the beam is $10^{-4}\mathrm{m}^4$. As it operates, the machine produces a harmonic force of magnitude 200 N. At what operating frequencies will the machine's steady amplitude be less than 0.5 mm. Young's modulus of aluminum is 70 GPa.

32. A beam on simple supports

In textbooks on strength of materials it is shown for a static beam

$$M = EI\frac{\partial^2 w}{\partial x^2}, \quad Q = \frac{\partial M}{\partial x}, \quad q = \frac{\partial Q}{\partial x}$$

a) Confirm these equations by using a free-body diagram of a small segment of the beam.
b) Show that a beam in vibration obeys

$$EI\frac{\partial^4 w}{\partial x^4} = -\rho A\frac{\partial^2 w}{\partial t^2}$$

c) Use the form $w(x,t) = f(x)\sin\omega t$. Determine the function $f(x)$.
d) For a beam on two simple supports show that the fundamental frequency is

$$\omega_1 = \frac{\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$$

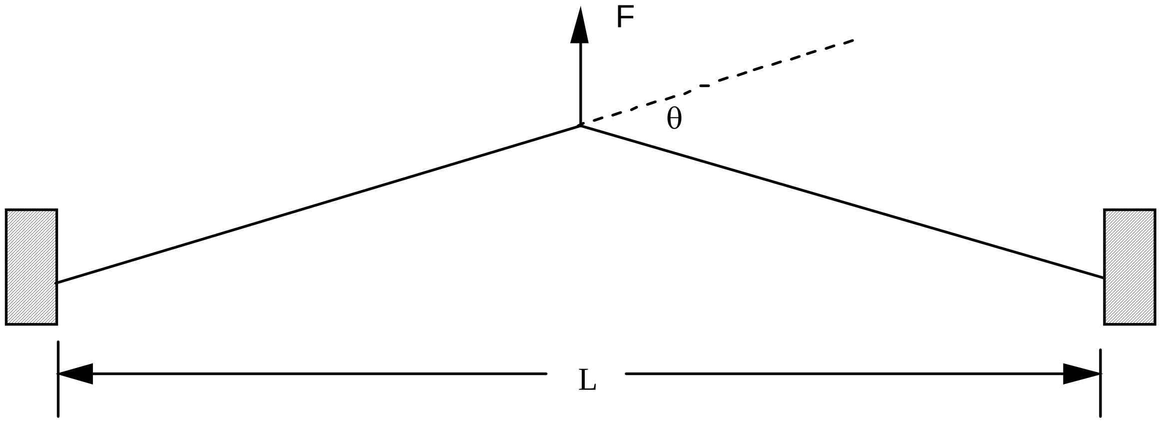
To avoid resonance, one may design a beam to have a sufficiently large natural frequency. For a beam with the same weight and length, a material with large Young's modules, and a cross-section with large second-moment (e.g., an I-beam or a tube) give the beam a high natural frequency.

33. Vibration of a piano string

Piano strings are made of high strength steel. Once hit, a string vibrates, disturbing the pressure in the air around the string. The disturbance propagates in the air as a pressure wave. When the pressure wave reaches you, it vibrates the little drums in your ears. The drums or something connected to them convert the mechanical vibration into an electrical oscillation, which then goes through your nerves to your brain. Your brain confirms that you hear a terrible sound—if I'm playing.

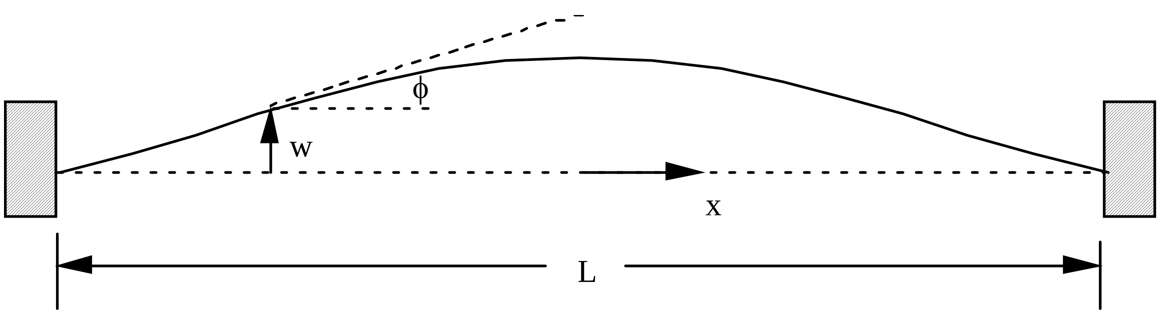
The science of sound (**acoustics**) involves many aspects, e.g., generation, transmission, reception, conversion, physiology, and psychology. Here we focus on one aspect: sound generation.

(a) The string is pre-stressed in tension. One way to measure the tension force in a string is to pull the string with a known force, and measure the angle. How do you calculate the tension in the string this way?



(b) The string typically vibrates with very small amplitude: there is negligible additional elongation of the string due to the vibration. Consequently, the tensile stress σ is constant during vibration. Furthermore, when the deflection w is small, the slope is given by $\phi = \partial w / \partial x$. Use a free body diagram of a spring segment dx to show the equation of motion:

$$\sigma \frac{\partial^2 w}{\partial x^2} = \rho \frac{\partial^2 w}{\partial t^2}$$



(c) Determine the fundamental frequency of the string.
(d) Estimate the largest fundamental frequency that can be generated by a one meter long steel string.
(e) Why does the piano have strings of variable lengths?
(f) Can we design a new piano where different frequencies are realized by different pre-stresses in a set of strings of the same length?