# Homework Due Friday, 5 December

### **39.** A circular transverse wave

A transverse wave propagates in an isotropic material. It is observed that, at any given material particle, the magnitude of the displacement is time-independent, but the direction of the displacement rotates at frequency  $\omega$ . Write a possible displacement field.

### 40. Creep and recovery

A polymer rod is 200 mm long and has a circular cross-section of diameter 20 mm. You'd like to investigate its viscoelastic property. You subject the rod to a tensile force of 600 N along its axial direction for a very long time, and record the elongation of the rod,  $\Delta$ , as a function of time *t*. Inspired by the trend of the date, you fit them to an exponential relation, and find that they fit well with

$$\Delta(t) = 0.20 - 0.15 \exp(-0.1t)$$

where  $\Delta$  is in mm and *t* in hours.

(a) How do you verify experimentally that the polymer is linearly viscoelastic?

(b) Suppose that the polymer is indeed linearly viscoelastic. Deduce the time-dependent compliance D(t) and plot the function. Find the unrelaxed compliance, the relaxed compliance, and the relaxation time.

(c) Subject the rod to a load of 400 N for 5 hours. Calculate the elongation at the end of the 5 hours.

(d) Subject the rod to a load of 400 N for a long enough time so that the rod elongates no more. What is the elongation immediately after the load is applied? In practice, how much time is "immediately" enough? What is the final elongation? In practice, how much time is long enough, a minute, an hour, a day, or a week?

(e) Subject the rod to a load of 400 N for 5 hours and then remove the load. Plot the elongation as a function of time between 0 to 24 hours.

# 41. Temperature dependence and Mr. Arrhenius

Now you'd like to investigate the effect of temperature on viscoelasticity. The previous experiment was carried out at the room temperature,  $20^{\circ}$ C. Now you carry out an experiment at 40°C. That is, subject the rod to a load of 600 N and record the elongation of the rod as a function of time. The data obtained at 40°C fit well with

$$\Delta(t) = 0.20 - 0.15 \exp(-8.6t)$$

(a) How do you verify experimentally that the relaxation time fits the Arrhenius equation?

(b) Suppose that the relaxation time does fit the Arrhenius equation. Deduce the activation energy.

(c) Use the above data and predictions to plot the compliance as a function of time at temperature  $20^{\circ}$ C,  $30^{\circ}$ C, and  $40^{\circ}$ C.

# 42. A loose nylon bolt

Z. Suo

A nylon bolt of diameter 8 mm is used to join two ceramic plates. The nylon is taken to be linearly viscoelastic with tensile stress relaxation modulus approximated by

$$E = 5\exp\left(-t^{1/3}\right)$$

where E is in GPa, and t in hours. The bolt is tightened quickly so that the initial force in the bolt is 1000 N.

- a) Find the stain in the bolt.
- b) Find the force in the bolt after 24 hours.
- c) At the end of 24 hours, tighten the bolt again to the force 1000 N. How much is the total strain in the bolt?
- d) Find the clamping force remaining after a further 24 hours.