

Homework Problems 14-19

Due by submission to TFs on April 3

14. Penny-shaped crack

A circular crack lies on region $x^2 + z^2 \leq a^2$, in the plane $y = 0$, of an unbounded body, subject to the remote stress field

$$\sigma_{yy} = \sigma, \quad \sigma_{xy} = \tau.$$

All other remote stress components vanish. The elasticity field of this problem has been solved analytically. The solution gives the displacements of the crack faces:

$$u_y^+ - u_y^- = \frac{4(1-\nu)\sigma}{\pi\mu} \sqrt{a^2 - x^2 - z^2}$$

$$u_x^+ - u_x^- = \frac{8(1-\nu)\tau}{(2-\nu)\pi\mu} \sqrt{a^2 - x^2 - z^2}$$

$$u_z^+ - u_z^- = 0$$

Determine the stress intensity factors of all three modes along the crack front.

15. Mixed mode loading. Crack kinking

A straight crack is cut in a material with a diamond saw. The material is loaded with a mixture of K_I and K_{II} , and the crack kinks—that is, the crack grows in a direction at an angle from the plane of the cut. Assume that the kink angle is determined by the maximum hoop stress condition. Plot the initial kink angle θ_k as a function of the loading angle, ψ , as defined by $\tan \psi = K_{II} / K_I$. Use the near-tip stress field listed in Lawn's book.

16. Four point bend specimen.

P.G. Charalambides, J. Lund, A.G. Evans, and R.M. McMeeking (1989). A test specimen for determining the fracture resistance of bimaterial interfaces. *Journal of Applied Mechanics*. **56**, 77-82.

We have discussed the four-point bend specimen in class. The above paper gave the detailed analysis. Start from basic principles, show that the energy release rate for the 4-point bend specimen is

$$G = 6 \frac{M^2}{EH^3} \left[1 - \frac{1}{(1+h/H)^3} \right]$$

where M is the applied moment per unit thickness, h and H are the thickness of the two layers. Assume that both layers have the same Young's modulus E . In particular, when $h = H$, the above result specializes to

$$G = \frac{21M^2}{4EH^3}.$$

This special configuration is often used today.

17. Mode mixity of the four-point bend specimen

In the paper cited above, the authors used the finite element method to determine the mode mixity. When the two material layers have identical Young's modulus and identical thickness, one can determine the mode mixity analytically. Show that in this special case the mode mixity is

$$\tan \psi = \frac{K_{II}}{K_I} = \frac{\sqrt{3}}{2}$$

That is, the specimen has slightly more mode I than mode II, or $\psi \approx 41^\circ$. Hint: Look p. 89 of J.W. Hutchinson and Z. Suo, Mixed-mode cracking in layered materials, *Advances in Applied Mechanics* 29, 63-191 (1992). (<http://www.deas.harvard.edu/suo/papers/17.pdf>)

18. The mode mix of an interfacial crack

Two half spaces of dissimilar materials bond at a flat interface. The materials are isotropic and linearly elastic. One material (a polymer) has the elastic constants $E_1 = 5GPa$, $\nu_1 = 0.35$, and the other material (a ceramic) has the elastic constants $E_2 = 200GPa$ and $\nu_2 = 0.25$. The bimaterial is subject to a remote tensile stress $\sigma = 10MPa$ normal to the interface crack. A crack, length $2a = 1cm$, lies on the interface. Plane strain conditions apply. We have defined the mode angle ψ by the equation

$$K = |K|l^{-i\epsilon} \exp(i\psi).$$

Calculate the mode angle when (a) $l = 1mm$ and (b) $l = 1\mu m$. Calculate the energy release rate.

19. Sandwich specimens

Z. Suo and J. W. Hutchinson, "Sandwich specimens for measuring interface crack toughness," *Materials Science and Engineering*, **A107**, 135-143 (1989). (<http://www.deas.harvard.edu/suo/papers/001.pdf>)

The content of the above paper was outlined in class.

(a) Show that the stress intensity factor of the interface crack K relates to the remote stress intensity factor K_I^∞ and K_{II}^∞ as

$$K = ch^{-i\epsilon} (K_I^\infty + iK_{II}^\infty) \exp(i\omega).$$

Calculate the constant c .

(b) Apply the above result to a thin layer of thickness h sandwiched between two substrates of thickness H . The sample is loaded under four point bend, with a crack lying on the interface between one of the substrate and the thin layer. Calculate the mode angle of the interfacial crack.