Homework Problems 20-25

Due by submission to TFs on April 24

20. In class we derived the energy release rate G along the sides of a buckle delamination of width $2b$ and thickness h by computing the edge force and moment at the edges and substituting them into the Suo-Hutchinson relations for G and ψ along the edge. For $\sigma_0 > \sigma_C$, we found

$$\frac{G}{G_0} = \left(1 - \frac{\sigma_C}{\sigma_0}\right)\left(1 + 3\frac{\sigma_C}{\sigma_0}\right) \text{ where } G_0 = \frac{\sigma_0^{\,2} h}{2\bar{E}},\ \sigma_C = \frac{\pi^2 \bar{E}}{12}\left(\frac{h}{b}\right)^2,\ \bar{E} = \frac{E}{1-\nu^2}$$

and σ_0 is the in-plane stress component of the unbuckled layer acting in the direction perpendicular to the delamination sides. Derive this expression in a different way by (1) computing the difference between the strain energy/length in the unbuckled and buckled state, ΔU, and then (2) using $2G = d\Delta U / db$. (You can be guided by the computation for the simple symmetric specimen carried out in the notes.)

21. Read the paper "The characterization of telephone cord buckling of compressed thin films on substrates" by Moon, et al (2002) which can be downloaded from as paper #60 of the pdf files at http://www.seas.harvard.edu/hutchinson/ . In your own words in a paragraph or two, at most, discuss the map presented at the end of the paper in Fig. 19 which attempts to provide an overview of conditions for when buckle delaminations are expected and whether the morphology will be straight-sided or like the telephone cord. You need not resort to listing any equations.

22. Crack bridging and resistance curve

Review Section 2.2 of the paper, G. Bao and Z. Suo, "Remarks on crack-bridging concepts," *Applied Mechanics Review*. **45**, 355-366 (1992). (http://www.seas.harvard.edu/suo/papers/030.pdf) A material is characterized by a rectilinear bridging law (strength σ_0 and maximum opening displacement δ_0), and a crack tip fracture energy Γ_0. Away from the crack plane, the material is linear elastic. Use the bridging model to derive the resistance curve of the material. Plot your result, and explain qualitatively significant points on the curve.

23. The J-integral around a bridged crack tip

A crack is bridged over a zone near the crack tip. The bridging law is

$$\sigma = \sigma(\delta).$$

A contour starts on one crack surface and ends on the other. If both the starting and the end points are on traction-free surfaces behind the bridging zone, show that

$$J = \int_0^{\delta_{tail}} \sigma(\delta) d\delta,$$

where δ_{tail} is the crack opening displacement at the tail of the bridging zone.

24. Large-scale bridging

Read Z. Suo, S. Ho and X. Gong, " Notch ductile-to-brittle transition due to localized inelastic band," ASME *J. Engng. Mater. Tech*. **115**, 319-326 (1993). (http://www.seas.harvard.edu/suo/papers/31.pdf) Explain the physical content of Figure 5. Explain the mathematical procedure to construct this figure. List all the relevant equations.

25. The energy release rate of an interface crack

Review the notes for Lecture 9 Interfacial Fracture (http://imechanica.org/node/952).
(a) Describe the basic features in the stress and displacement fields of the Williams singularity.
(b) Use Irwin's method to derive the relation between the energy release rate and the complex stress intensity factor for the interfacial crack.