

Due in class, Thursday, 18 February 2010**9. Obreimoff's Experiments on Mica**

- Obreimoff, The splitting strength of mica, *Proc. Roy. Soc. Lond.* **A127**, 290-297 (1930). This short paper is pleasant to read. The paper also contains a lucid description of experiments.

Mica splits readily into flakes along a certain crystal plane. Obreimoff carried out the following experiments. A glass wedge is inserted beneath a thin flake of mica attached to a parent block, and is made to drive along the cleavage plane.

a) Show that the energy release rate is

$$G = \frac{3}{8} E d^3 h^2 c^{-4}.$$

b) Take $E = 200$ GPa, $h = 480 \mu\text{m}$, $d = 75 \mu\text{m}$. The experimentally measured crack length is $c = 10$ mm. Determine the fracture energy of the mica.

c) **Sticky microbeam.** In MEMS, a cantilever beam is often fabricated on a substrate. It is sometimes found that a long cantilever beam can stick to the substrate. Say the cantilever is made of silicon, $1 \mu\text{m}$ thick and $20 \mu\text{m}$ above the substrate. Estimate the critical beam length, below which the beam cannot stick to the substrate. Take Young's modulus of Si to be 160 GPa, and the work of adhesion between the beam and the substrate to be 0.1 J/m^2 .

10. Tunneling crack

Shrwai Ho (<http://imechanica.org/user/12399>) was my first Ph.D. student when I started at the University of California at Santa Barbara. He came from Taiwan, and returned after his study. The UCSB campus was near the beach. I could walk from my office to the beach in three minutes. Here are several papers Shrwai wrote with me. You don't need to read them to do this homework problem, but they are a click away in case you need them.

- S. Ho and Z. Suo, Tunneling cracks in constrained layers. *J. Appl. Mech.* **60**, 890-894 (1993). <http://www.seas.harvard.edu/suo/papers/o24.pdf>
- S. Ho and Z. Suo, Microcracks tunneling in brittle matrix composites driven by thermal expansion mismatch. *Acta Metall. Mater.* **40**, 1685-1690 (1992). <http://www.seas.harvard.edu/suo/papers/o28.pdf>
- S. Ho, C. Hillman, F.F. Lange and Z. Suo, Surface cracking in layers under biaxial, residual compressive stress. *J. Am. Ceram. Soc.* **78**, 2353-2359 (1995). <http://www.seas.harvard.edu/suo/papers/o51.pdf>

For this homework problem, you will look at a special case that can be worked out analytically. A brittle layer, thickness h , is bonded between two thick substrates. The three materials have similar elastic constants. The layer is under residual tensile stress σ . Cracks can tunnel in the layer.

(a) Show that the steady state energy release rate at the front of a tunnel is

$$G = \frac{\pi(1-\nu^2)\sigma^2 h}{4E}.$$

(b) The brittle layer has fracture energy $\Gamma = 10 \text{ J/m}^2$, Young's modulus 100 GPa. It is $1 \mu\text{m}$ thick, and is under a tensile stress 500 MPa. Will tunnel cracks form?

Hint. The boundary-value problem of a Griffith crack has been solved analytically. The crack opening displacement of the Griffith crack is

$$\delta(x) = \frac{4\sigma(1-\nu^2)}{E} \sqrt{a^2 - x^2}.$$

11. Anti-plane shear deformation. Mode III elastic crack tip.

A solid is said to be in a state of anti-plane shear when the component of displacement w only varies with x and y , and the other two components of displacement vanish.

- (a) Derive all the field equations for the anti-plane shear deformation. Assume that the material is linearly elastic and isotropic.
- (b) Derive the square-root singular field of a mode III crack. Obtain the stress field and displacement field.

12. Irwin's relation for a mode III crack

In his 1957 paper, Irwin derived a relation between the energy release rate and the stress intensity factor for a mode I crack. Show that for a mode III crack the stress intensity factor relates to the energy release rate as

$$G = \frac{K_{III}^2}{2\mu}.$$

Recommend a book that you think will help students in this course

- Post your recommendation as a new blog entry on iMechanica by **Wednesday, 3 March 2010**.
- If you are unsure how to post on iMechanica, read this note: <http://imechanica.org/node/410>

Even though the lectures and homework problems are reasonably self-contained, you should get familiar with a few books on the subject. Please restrict your recommendation to a printed book, which you have spent some time reading. Please focus on one book, and include in your post the following items.

1. Title of the post. The title of the book and the names of authors (Example: Fracture of Brittle Solids by B. Lawn).
2. Use 4 free tags: **ES 247, fracture mechanics, Spring 2010, textbooks**.
3. If there are already helpful reviews of the book online, please make a hyperlink in your post to the web page of the review.
4. Why do you think that this book complements this course?
5. Anything particular that makes you like this book?
6. Any weakness of the book do you see?