**Due in class, Thursday, 4 March 2010**

13. A layer between two rigid grips.

Consider a semi-infinite crack in an infinite layer of thickness H. The layer is clamped in two rigid grips in the stress-free state. The grips are then subject to a separation Δ. As crack extends, the stress field around the crack does not change. Consequently, we expect the energy release rate to be independent of the length of the crack.

The stress field is complicated near the crack tip, as always. However, far away from the crack tip, the stress field is simple. The material far behind the crack tip is unstressed. The material far ahead of the crack tip is subject to a homogeneous state of uniaxial strain. Show that the energy release rate is

$$G = \frac{(1-\nu)E\Delta^2}{2(1+\nu)(1-2\nu)H}.$$
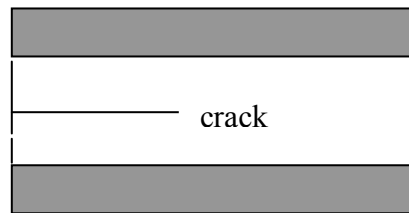
14. Penny-shaped crack

A circular crack lies on region $x^2 + z^2 < a^2$, in the plane $y = 0$, of an unbounded body, subject to the remote stress field

$$\sigma_{yy} = \sigma, \quad \sigma_{xy} = \tau.$$

All other remote stress components vanish. The elasticity field of this problem has been solved analytically. The solution gives the displacements of the crack faces:

$$u_y^+ - u_y^- = \frac{4(1-\nu)\sigma}{\pi\mu}\sqrt{a^2 - x^2 - z^2}$$

$$u_x^+ - u_x^- = \frac{8(1-\nu)\tau}{(2-\nu)\pi\mu}\sqrt{a^2 - x^2 - z^2}$$

$$u_z^+ - u_z^- = 0$$

Determine the stress intensity factors of all three modes along the crack front.

15. Stress field in the matrix around a fiber

Read Lu *et al*, *Acta metall. Mater*, **39**, 1883-1890 (1991). (http://www.seas.harvard.edu/suo/papers/013.pdf)   Assume that the matrix and the fiber have the identical elastic constants. Derive the stress distribution around the single fiber, equation (6).

16. Stress intensity factor at the tip of a crack emanating from the fiber

One method to calculate stress intensity factor is linear superposition. Consider a crack of length 2*c* in an infinite body. The crack faces are subject to a pair of forces *P*.   The stress intensity factor is given by

$$K = \frac{P}{\sqrt{\pi c}} \sqrt{\frac{c+\xi}{c-\xi}}$$

This is a standard solution in handbooks. Use this result to prove Equation (8) in the paper of Lu et al. The qualitative content of Equation (8) was discussed in class. Discuss Equation (8) again quantitatively.