

Due in class, Thursday, 1 April 2010**21. The small-scale yielding condition**

(a) Calculate the maximum permissible crack size in a single-edge-cracked panel subject to a uniform stress of 140 MPa. The toughness is $50 \text{ MPa}\sqrt{\text{m}}$, the yield strength is 420 MPa, and the width of the sample is 0.3 m. (b) Does the situation satisfy the small-scale yielding condition?

22. Stress-corrosion cracking

A material is susceptible to stress corrosion. The threshold stress intensity factor is $K_{th} = 0.5 \text{ MPa}\sqrt{\text{m}}$. Under a load above this threshold, the crack propagates at a velocity which obeys a power law

$$v = BK^n,$$

with $n = 10$. The velocity is 10^{-7} m/s when $K = 1 \text{ MPa}\sqrt{\text{m}}$. The material is subject to a stress of 50 MPa.

- (a) Determine the flaw size a_{th} , below which the lifetime is infinite.
 (b) If the initial flaw size is $1.5 a_{th}$, estimate the lifetime.
 (c) If the initial flaw size is $2 a_{th}$, estimate the lifetime.

23. Fracture of a thick-walled pressure vessel

A thick-walled pressure vessel has inner radius $r = 10 \text{ mm}$ and outer radius $R = 40 \text{ mm}$. A flaw, depth $a = 1 \text{ mm}$, is detected on the internal surface of the vessel. The vessel is made of a steel with a yield strength $\sigma_y = 2 \text{ GPa}$ and a fracture energy 10^3 J/m^2 . Estimate the pressure that the vessel can sustain.

24. A steel has a plane strain toughness of $K_c = 100 \text{ MPa}\sqrt{\text{m}}$ and a resistance curve given by

$$K_R = K_c \left[4 - 3 \exp\left(-\frac{\Delta a}{L}\right) \right],$$

where $L = 3 \text{ mm}$. A pressure vessel has wall thickness $b = 5 \text{ cm}$. An edge crack exists in the wall of initial length 1 cm. Model the problem as an edge-cracked plate under an applied stress σ . From a handbook we find an expression for the stress intensity factor for a crack of length a :

$$K = \sigma \sqrt{\pi a} F\left(\frac{a}{b}\right),$$

where

$$F(x) = 0.265(1-x)^4 + (0.857 + 0.265x)(1-x)^{-3/2}.$$

- (a) Determine the stress σ_c at which the crack starts to advance.
 (b) Determine the stress σ^* at which the crack becomes unstable and the associated crack advance Δa^* . Plot the resistance curve and the loading curve on the same diagram.
 (c) How large would the tensile yield stress of the steel have to be for small-scale yielding condition to be valid.

25. The Rivlin-Thomas paper (1953)

Read R.S. Rivlin and A.G. Thomas, Rupture of rubber. I. Characteristic energy for tearing. Journal of Polymer Science 10, 291-318 (1953). For some of the test-pieces used by

Rivlin and Thomas, they could only determine the energy release rate by experimental measurements. Carefully describe their experimental method.

26. The trousers test

For the trousers specimen, Rivlin and Thomas (1953) could obtain an analytical expression of the energy release rate. Carefully derive their result, following the method described in their paper. Explain how this result is used to determine the fracture energy.

27. The trousers test and the J integral

Obtain the energy release rate for the trousers test by using the J integral.

28. The J integral for small deformation

In class we describe the J integral using the theory of large deformation. Develop the J integral using the theory of small deformation. A homogeneous, nonlinear elastic material is characterized by a strain energy density function

$$W = W(\varepsilon_{11}, \varepsilon_{22}, \dots),$$

such that

$$\sigma_{ij} = \frac{\partial W(\varepsilon_{11}, \varepsilon_{22}, \dots)}{\partial \varepsilon_{ij}}.$$

The J -integral is defined as

$$J = \int \left(W n_1 - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_1} \right) ds$$

- Show that the integral vanishes when evaluated over a contour that encloses no singularity.
- Consider a crack lying on the x_1 -axis in this material. The crack is loaded remotely, but the crack surfaces are traction-free. Show that the integral is path-independent when evaluated over a contour that starts on one crack surface and ends on the other.
- When the material is heterogeneous, the J integral is path-dependent. For a heterogeneous material, show the step that becomes invalid in your proof of (a).