IOSO Global Optimization software benchmarking from Japan

All examples are taken from two public sources (written in Japanese):

1) **Global Optimization by Generalized Random Tunneling Algorithm (2nd Report: Examination on the accuracy of solution and its efficiency)** Satoshi KITAYAMA and Koetsu YAMAZAKI Department of Human & Mechanical Systems Engineering, Kanazawa University 2-40-20, Kodatsuno, Kanazawa, Ishikawa, 920-8667, Japan

2) **Global Optimization by Generalized Random Tunneling Algorithm (5th Report: Approximate Optimization Using RBF Network)** Satoshi KITAYAMA, Masao ARAKAWA, Koetsu YAMAZAKI Department of Human & Mechanical Systems Engineering, Kanazawa University Kakuma-machi, Kanazawa, 920-1192, Japan

Example 1
Task formulation

\[
 f(x) = \frac{1}{2} \sum_{i=1}^{2} (x_i^4 - 16x_i^2 + 5x_i) \rightarrow \min 
\]

\[
 g_1(x) = x_1^2 + x_2^2 - 9 \leq 0 
\]

Position of global optimum is

\[
 (x_1, x_2)^T = (-2.121, -2.121)^T 
\]

![Contour of functions and the position of global minimum](image)

**Result given by IOSO**

IOSO found the global solution easily and quickly

<table>
<thead>
<tr>
<th>Call No</th>
<th>x1</th>
<th>x2</th>
<th>f</th>
<th>g1</th>
</tr>
</thead>
<tbody>
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</table>

Prepared by [http://www.wavefront.co.jp/](http://www.wavefront.co.jp/)
Example 2
Task formulation

\[ f(x) = -x_1 - x_2 \rightarrow \min \]
\[ g_1(x) = -2 - 2x_1^4 + 8x_1^3 - 8x_1^2 + x_2 \leq 0 \]
\[ g_2(x) = -36 - 4x_1^4 + 32x_1^3 - 88x_1^2 + 96x_1 + x_2 \leq 0 \]
\[ 0 \leq x_1 \leq 3, \quad 0 \leq x_2 \leq 4 \]

Position of global optimum is

\[ (x_1, x_2)^T = (2.329, 3.178)^T \]

where \( f = -5.508 \)

Result given by IOSO
IOSO found the global solution easily and quickly

Fig 2 Contour of functions and the position of global minimum

<table>
<thead>
<tr>
<th>Call No</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( f )</th>
<th>( g_1 )</th>
<th>( g_2 )</th>
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<tbody>
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Example 3 (Infeasible region)

Task formulation

\[
\begin{align*}
    f(x) &= -(x_1 - 10)^2 - (x_2 - 15)^2 \rightarrow \min \\
    g_1(x) &= (x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6)^2 \\
    + 10(1 - \frac{1}{8\pi}) \cos x_1 + 5 \leq 0 \\
    -5 \leq x_1 &\leq 10 \\
    0 \leq x_2 &\leq 15
\end{align*}
\]

Global solution

\[x_G = (3.271, 0.0496)^T\]

where

\[f(x_G) = -268.788\]

Result given by IOSO

IOSO found the global solution without any problem

<table>
<thead>
<tr>
<th>Call No</th>
<th>x1</th>
<th>x2</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
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</table>
Example 4 (to minimize weight of spring-coil)

Task formulation

\[ f(x) = (2 + x_1)x_1^2x_2 \rightarrow \min \]

\[ g_1(x) = 1 - x_2^2x_3/(71785x_4^4) \leq 0 \]

\[ g_2(x) = 4x_3^2 - x_1x_2 \left/ \left(12566(x_2x_1^3 - x_1^4) + 5108x_1^2\right) \right. - 1 \leq 0 \]

\[ g_3(x) = 1 - 140.45x_1/(x_2^2x_3) \leq 0 \]

\[ g_4(x) = (x_1 + x_2)/(1.5) - 1 \leq 0 \]

0.05 ≤ x₁ ≤ 2.00
0.25 ≤ x₂ ≤ 1.30
2.00 ≤ x₃ ≤ 15.0

Result given by IOSO

IOSO easily found the global solution that is the same as given by Hu

Table 1 Comparison of the results

<table>
<thead>
<tr>
<th>Variables</th>
<th>Arora (18)</th>
<th>Coello (19)</th>
<th>Ray (20)</th>
<th>Hu (21)</th>
<th>Kitavan (22)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁ (d)</td>
<td>0.053396</td>
<td>0.051480</td>
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<td>0.051466</td>
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<td>x₂ (D)</td>
<td>0.399180</td>
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<td>x₃ (N)</td>
<td>9.185400</td>
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<tr>
<td>g₁(x)</td>
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<td>g₃(x)</td>
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<td>g₄(x)</td>
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<td>f(x)</td>
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<td>0.013060</td>
<td>0.012667</td>
<td>0.014469</td>
</tr>
</tbody>
</table>

Various results are presented by various scientists for comparison (the result found by Hu is the best one)