Fracture of Orthotropic Materials under Mixed Mode Loading

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Outline

- Wood as orthotropic material
- Overview of fracture in wood
- Fracture toughness in mode I and mode II
- Mixed mode fracture criteria
- Comparison with experimental data
- Conclusions
Wood as Orthotropic Material

- Wood is generally considered a cylindrically orthotropic material, with the principal axes of orthotropy (R,T,L) given by the radial, tangential and longitudinal directions.
Elastic Moduli

\[ E_L \succ E_R \succ G_{LR} \sim G_{LT} \succ E_T \succ G_{RT} \]

<table>
<thead>
<tr>
<th>Species</th>
<th>Relative density RD</th>
<th>Moisture content m (%)</th>
<th>( E_T/E_L )</th>
<th>( E_R/E_L )</th>
<th>( G_{LR}/E_L )</th>
<th>( G_{LT}/E_L )</th>
<th>( G_{RT}/E_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balsa</td>
<td>0.13</td>
<td>9</td>
<td>0.015</td>
<td>0.046</td>
<td>0.054</td>
<td>0.037</td>
<td>0.005</td>
</tr>
<tr>
<td>Spruce</td>
<td>0.37</td>
<td>12</td>
<td>0.041</td>
<td>0.074</td>
<td>0.050</td>
<td>0.061</td>
<td>0.002</td>
</tr>
<tr>
<td>Yellow-poplar</td>
<td>0.38</td>
<td>11</td>
<td>0.043</td>
<td>0.092</td>
<td>0.075</td>
<td>0.069</td>
<td>0.011</td>
</tr>
<tr>
<td>Douglas-fir</td>
<td>0.50</td>
<td>12</td>
<td>0.050</td>
<td>0.068</td>
<td>0.064</td>
<td>0.078</td>
<td>0.007</td>
</tr>
<tr>
<td>Mahogany</td>
<td>0.50</td>
<td>12</td>
<td>0.073</td>
<td>0.107</td>
<td>0.098</td>
<td>0.066</td>
<td>0.028</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>0.53</td>
<td>11</td>
<td>0.050</td>
<td>0.115</td>
<td>0.089</td>
<td>0.061</td>
<td>0.021</td>
</tr>
<tr>
<td>Black Walnut</td>
<td>0.59</td>
<td>11</td>
<td>0.056</td>
<td>0.106</td>
<td>0.085</td>
<td>0.062</td>
<td>0.021</td>
</tr>
<tr>
<td>Alpine Maple</td>
<td>0.59</td>
<td>10</td>
<td>0.088</td>
<td>0.152</td>
<td>0.123</td>
<td>0.110</td>
<td>0.029</td>
</tr>
<tr>
<td>Yellow Birch</td>
<td>0.64</td>
<td>13</td>
<td>0.050</td>
<td>0.078</td>
<td>0.074</td>
<td>0.068</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Constitutive Relation

\[
\begin{pmatrix}
\varepsilon_{11} & \varepsilon_{22} & \varepsilon_{33} & \gamma_{12} & \gamma_{13} & \gamma_{23}
\end{pmatrix}
= \begin{pmatrix}
\frac{1}{E_L} & -\frac{v_{RL}}{E_R} & -\frac{v_{TL}}{E_T} & 0 & 0 & 0 \\
-\frac{v_{LR}}{E_L} & \frac{1}{E_R} & -\frac{v_{TR}}{E_T} & 0 & 0 & 0 \\
-\frac{v_{LT}}{E_L} & -\frac{v_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{LR}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{RT}}
\end{pmatrix}
\begin{pmatrix}
\sigma_{LL} \\
\sigma_{RR} \\
\sigma_{TT} \\
\tau_{LR} \\
\tau_{LT} \\
\tau_{RT}
\end{pmatrix}
\]

\[\frac{\nu_{ki}}{E_i} = \frac{\nu_{ik}}{E_k}\]
Strength

• In the radial and tangential directions the strength is 10-30% of that in longitudinal direction.

• There is also difference between tensile and compressive strengths.
Fracture Orientations

Across the grain

Parallel to the grain
Fracture along the Grain

• Cracks in wood generally grow along the grains *irrespective of both the original orientation of the crack and the mode mixity*

• Even when cross-grain notches are loaded in longitudinal tension, cracking usually occurs along the grain (*perpendicularly to the notch*)

• Orientations RL and TL are usually the primary focus.
Tension at Arbitrary Angles Relative to the Grain

- Due to anisotropy, it is essentially impossible to get a pure mode I. *Mixed mode condition* arises!
- Cracks will propagate along the weak axes of the material but *frequently jump between grain lines* when doing so maximizes the energy release.
# Fracture Toughness

<table>
<thead>
<tr>
<th>Species</th>
<th>$K_{IC}$ (kNm$^{-3/2}$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TL</td>
<td>RL</td>
</tr>
<tr>
<td>Douglas-fir</td>
<td>320</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>309$^b$</td>
<td>410$^b$</td>
</tr>
<tr>
<td></td>
<td>260$^c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>847$^c$</td>
<td></td>
</tr>
<tr>
<td>Western hemlock</td>
<td>375</td>
<td></td>
</tr>
<tr>
<td>Western white pine</td>
<td>250</td>
<td>260</td>
</tr>
<tr>
<td>Scots pine</td>
<td>440</td>
<td>500</td>
</tr>
<tr>
<td>Southern pine</td>
<td>375</td>
<td></td>
</tr>
<tr>
<td>Ponderosa pine</td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>Red spruce</td>
<td>420</td>
<td></td>
</tr>
<tr>
<td>Northern red oak</td>
<td>410</td>
<td></td>
</tr>
<tr>
<td>Sugar maple</td>
<td>480</td>
<td></td>
</tr>
<tr>
<td>Yellow-poplar</td>
<td>517</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Species</th>
<th>$K_{IIC}$ (kNm$^{-3/2}$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TL</td>
<td>RL</td>
</tr>
<tr>
<td>Douglas-fir</td>
<td>1370$^d$</td>
<td>1562/1746$^b$</td>
</tr>
<tr>
<td>Western hemlock</td>
<td>2240$^d$</td>
<td></td>
</tr>
<tr>
<td>Western white pine</td>
<td>2420/2250$^c$</td>
<td></td>
</tr>
<tr>
<td>Scots pine</td>
<td>2050</td>
<td></td>
</tr>
<tr>
<td>Southern pine</td>
<td>2070</td>
<td></td>
</tr>
<tr>
<td>Ponderosa pine</td>
<td>1930$^d$</td>
<td></td>
</tr>
<tr>
<td>Red spruce</td>
<td>2190</td>
<td>1665</td>
</tr>
<tr>
<td>Poplar</td>
<td>2232$^e$</td>
<td></td>
</tr>
</tbody>
</table>
Mixed Mode Fracture

• Crack growth depends on not only $K_{IC}$ and $K_{IIC}$ but also on the interaction between the two

• Theories for predicting mixed mode fracture in anisotropic homogeneous materials usually predict that a crack subjected to mixed mode loading will grow out of its original plane. *They cannot be directly applied to wood*
Mixed mode fracture

• In isotropic materials, the crack will typically turn so that its plane is perpendicular to the load axis, becoming mode I.
• In case of wood, it will continue to propagate under mixed mode.
Mixed mode fracture criteria

- **Empirical criterion**
  - based on experiments with balsa wood and fiber-glass-reinforced plastic plates

\[
\frac{K_I}{K_{IC}} + \left( \frac{K_{II}}{K_{IIC}} \right)^2 = 1
\]
Mixed mode fracture criteria

- Empirical criterion
  - Eastern Red Spruce

\[
\left( \frac{K_I}{K_{IC}} \right)^a + \left( \frac{K_{II}}{K_{IIC}} \right)^b = 1
\]

\[2 < b < 3.5\]
Mixed mode fracture criteria

• **Critical Energy Release Rate**
  
  – fracture takes place when the strain energy release rate during crack propagation equals the energy rate needed to tear the material apart.

  \[
  \left( \frac{K_I}{K_{IC}} \right)^2 + \left( \frac{K_{II}}{K_{IC}} \right)^2 = 1
  \]

  \[
  \frac{K_{III}}{K_{IC}} = \left( \frac{C_{12}}{C_{11}} \right)^{1/4}
  \]
Mixed Mode Fracture Criteria

• **Critical Strain Energy Density**
  
  – Crack growth occurs when the strain energy density at some distance from the crack tip reaches a critical value.

\[
\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1
\]

\[
\frac{K_{IIC}}{K_{IC}} = \left(\frac{C_{11} f_{11}^2(0) + C_{22} f_{22}^2(0) + 2C_{12} f_{11}(0) f_{22}(0)}{C_{ee} g_{12}^2(0)}\right)^{1/2}
\]

\[
a_{ij} = \frac{K_I f_{ij}(\theta)}{\sqrt{2\pi r}} + \frac{K_{II} g_{ij}(\theta)}{\sqrt{2\pi r}}
\]
Mixed Mode Fracture Criteria

- **Critical In-Plane Maximum Principal Stress**
  - Fracture takes place when the maximum principal stress at some distance in front of the crack tip reaches a critical value.

\[
\frac{f_{11}(0) + f_{22}(0)}{2f_{11}(0)} \frac{K_I}{K_{IC}} + \sqrt{\frac{2[f_{11}(0) - f_{22}(0)]^2}{[f_{11}(0) + f_{22}(0)]^2 + 2[f_{11}(0) - f_{22}(0)]^2} \left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IC}}\right)^2} = 1
\]

\[
\frac{K_{IIc}}{K_{IC}} = f_{11}(0)
\]
Mixed Mode Fracture Criteria

- Non-local Stress Fracture Criterion (2008)
  - Fracture takes place when the mean value of the function of decohesive normal and shear stress over a segment $d$, the length of the damage zone, reaches its critical value

\[
\max_{\varphi} R(\sigma_n, \tau_n) = \max_{\varphi} \left[ \frac{1}{d} \int_{0}^{d} R(\sigma_n, \tau_n) \right] = 1
\]

\[
\left( \frac{K_I}{K_{IC}} \right)^2 + \left( \frac{K_{II}}{K_{IIc}} \right)^2 = 1
\]

Crack parallel to the grain

\[
\lambda_{11} K_I^2 + \lambda_{12} K_I K_{II} + \lambda_{22} K_{II}^2 = K_{IC}^2
\]

Crack arbitrarily oriented
Discussion

Scots Pine

Eastern Red Spruce

![Graph of Scots Pine and Eastern Red Spruce data comparing Energy Release Rate, Strain Energy Density, and Max Principal Stress]
Discussion

Norway Spruce
Discussion

Pine Wood – crack arbitrarily oriented
Conclusions

• Empirical criteria require a large number of material constants determined for each crack configuration.
• The energy based criteria don’t predict well the mixed mode fracture in wood.
• Cracks oriented along the grain
  – Critical In-Plane Maximum Principal Stress criterion
  – Non-Local Stress Fracture Criterion
• Cracks oriented arbitrarily
  • Non-Local Stress Fracture Criterion
References


