Numerical Results for shear-lock free finite elements based on Mindlin-Reissner plate and Timoshenko beam theories

Plate Elements

N x N	Mesh Size (Full Plate)
Exact Solution	Srinivasa Rao and AK Rao and Theory of Plates and Shells
CF	Convergence Factor
ASR	Aspect Ratio
MR_FE_1 MR_FE_2 MR_FE_3 MR_FE_4	Finite Elements based on Mindlin-Reissner theory using new shape functions
Material:	E=1.092E06; $\upsilon = 0.3$;
Geometry:	Square plate a=10.0; h=2.0, 1.4, 1.0, 0.5;
Type of plate	Simply supported (SS)
Load:	UDL=10.0

Timoshenko beam elements

N	Number of Elements (Full beam)
Exact Solution	Theory of Elasticity
CF	Convergence Factor
ASR	Aspect Ratio
Timo_FE_1 Timo_FE_2 Timo_FE_3 Timo_FE_4	Finite Elements based on Timoshenko beam theory using new shape functions
Material:	29,000; $\upsilon = 0.3$;
Geometry:	L=5,10, 25,100, 200, 400; h=1.0; b=1.0;
Load:	Concentrated Load q=100.0 and UDL=10.0

Higher order beam elements (capable of accurately predicting three dimensional stresses) based on the higher order shear deformation theories developed by me

HFE_1 - based on Lagrangian polynomials
HFE_3 - based on Lagrangian polynomials
FE_NSF_1 - based on new shape functions
FE_NSF_3 - based on new shape functions

Advantages of the present new finite elements

1. The primary aim of this research work is to replace the finite elements based on the first order shear deformation theory available in the general purpose finite element packages by these new finite elements.

2. Presently all general purpose finite element packages like MSC-NASTRAN, NX- NASTRAN, ABAQUS, LS-DYNA, and ANSYS use the finite elements based on Timoshenko beam theory, Reissner-Mindlin plate theory and Kirchhoff-Love shell theory (all are called first order shear deformation theories). The drawback of these finite elements is that they can not be used for the analysis of thin structures due to shear lock problem. To eliminate this problem, special integration scheme must be used.

   The new finite elements based on the Timoshenko beam theory, Reissner-Mindlin Plate theory developed by me using special shape functions and standard finite element procedure are applied to the analysis of beams and plates. The numerical results show that accurate solution is obtained for less number of elements. The specialties of these finite elements are that (i) thick and thin structures can be analyzed, (ii) No special integration scheme is required, (iii) a new concept, Convergence Factor (CF), is introduced in the formulation of these elements to accelerate convergence, and (iv) Accurate solution is obtained by keeping the number of elements as constant and increasing the value of the CF. Hence, this procedure reduces modelling effort, computational time and increasing accuracy.

3. A comparison study was carried out among the finite elements based on the new shape functions and Lagrangian shape functions using two higher order shear deformation theories developed by me for the analysis of simply supported beam under transverse load. The convergence is achieved faster than that of the finite elements based on the Lagrangian shape functions.

4. Considering the above points, study related to the development of triangular finite elements based on the new shape functions is in progress.

5. Compare to the development of other thin plate finite elements like Discrete Kirchhoff Theory (DKT) finite element and Mixed Interpolated Tensorial Component (MITC4) element, the development of these finite elements is very simple.

6. Since these new shape functions are very effective in accurately predicting displacements, strains and stresses, finite elements based on these shape functions can be developed for various applications, for example, multiscale modelling, crack propagation and gradient-enhanced damage models.

7. It satisfies Partition of unity condition.

8. It satisfies Kronecker Delta condition and hence imposition of essential boundary condition is not a problem.

9. The last two conditions can not be achieved in Isogeometric Analysis.

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 5

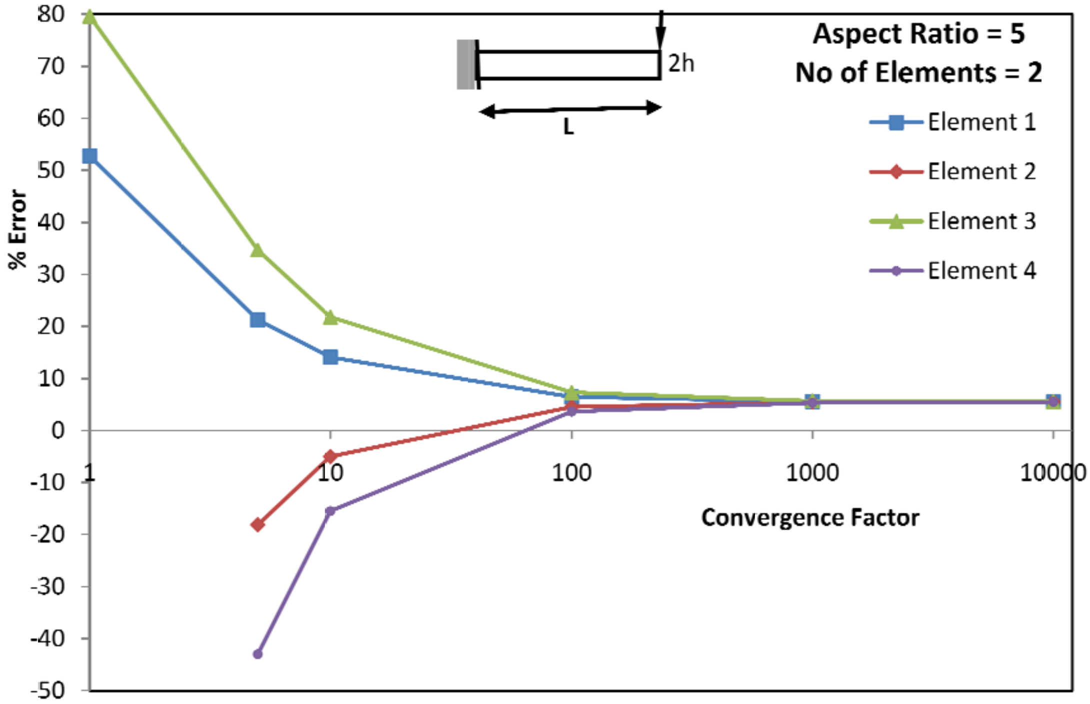


Fig-1_CB_AR_5_NEL_2

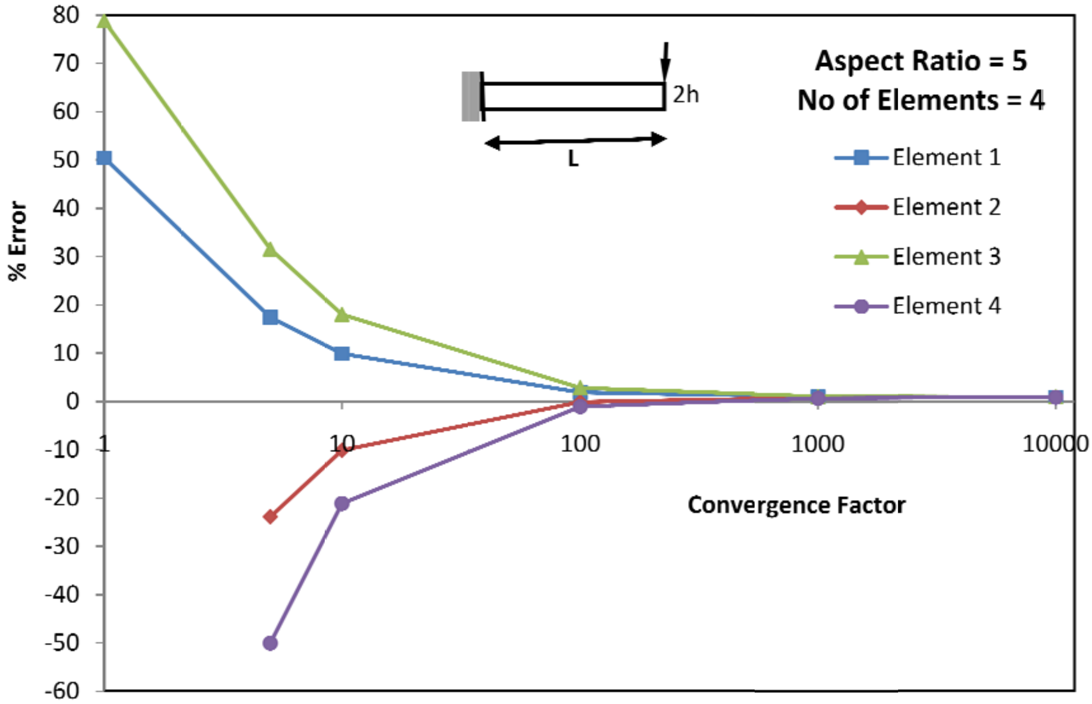


Fig-2_CB_AR_5_NEL_4

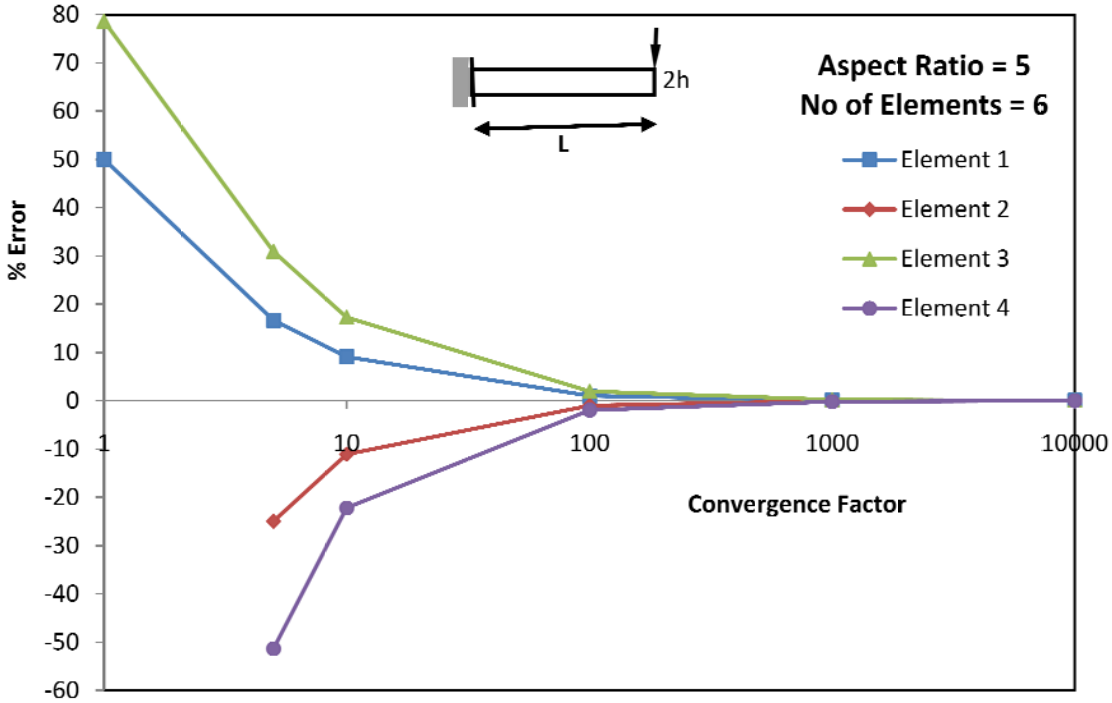


Fig-3_CB_AR_5_NEL_6

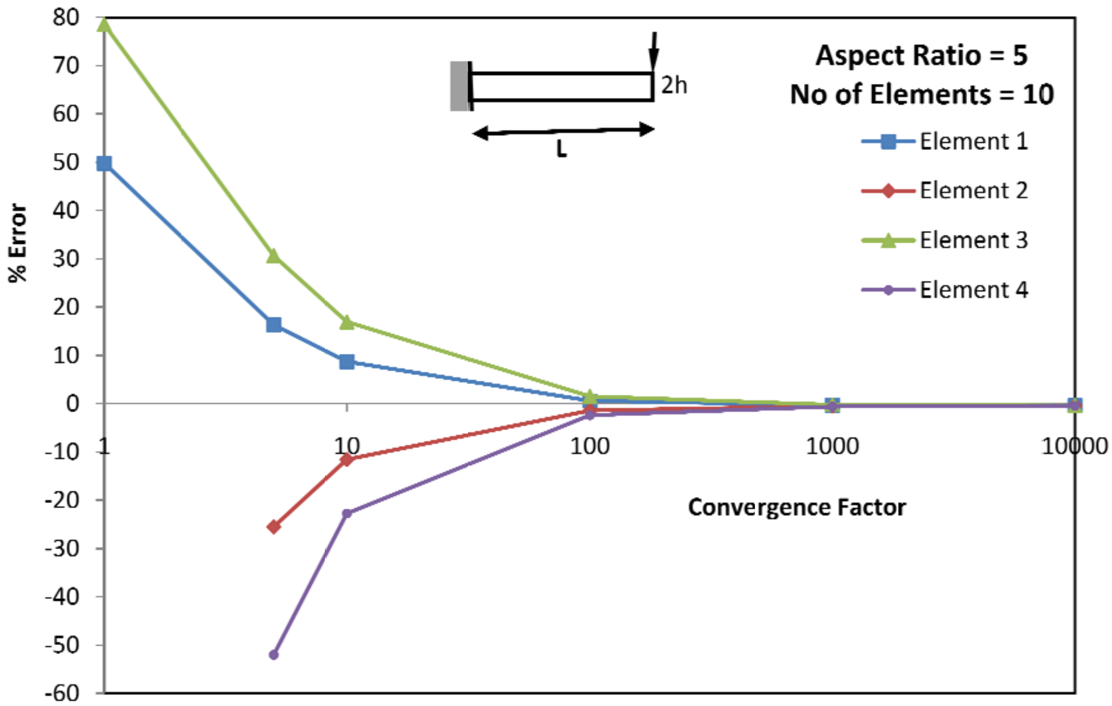


Fig-4_CB_AR_5_NEL_10

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 10

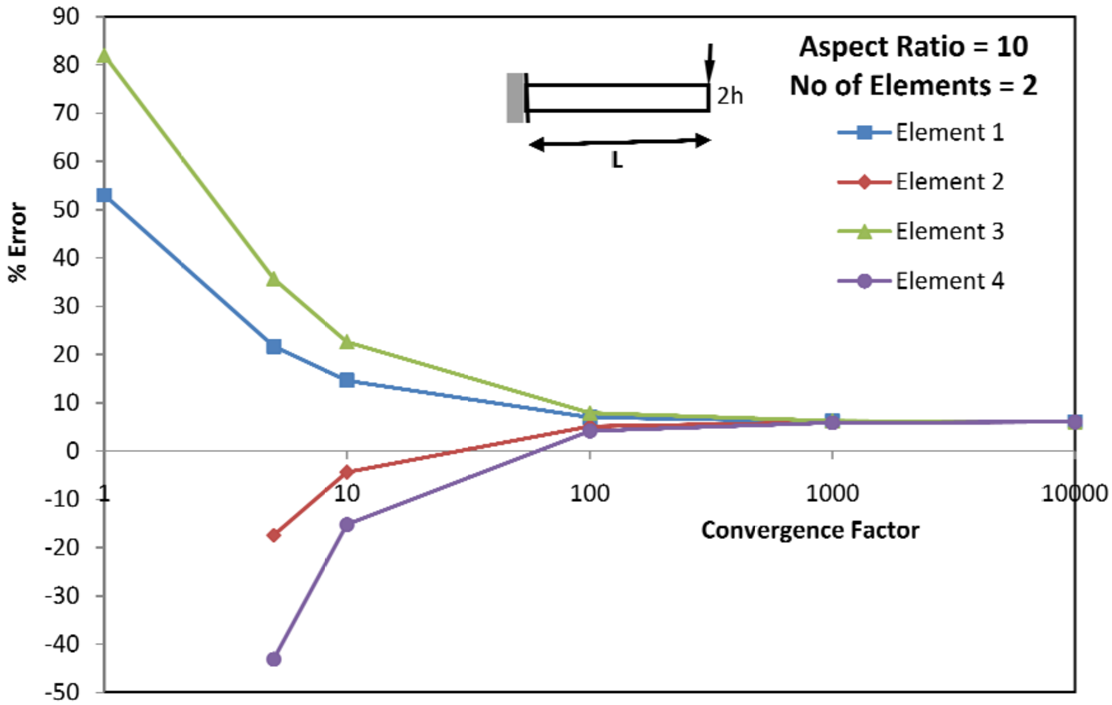


Fig-1_CB_AR_10_NEL_2

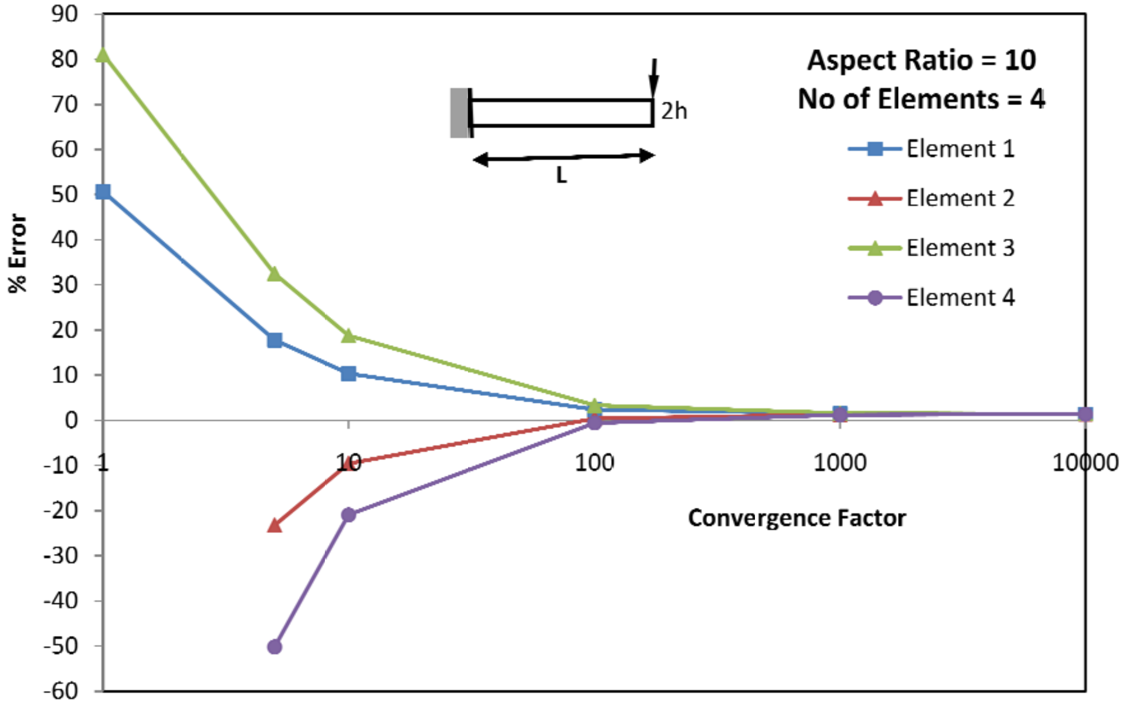


Fig-2_CB_AR_10_NEL_4

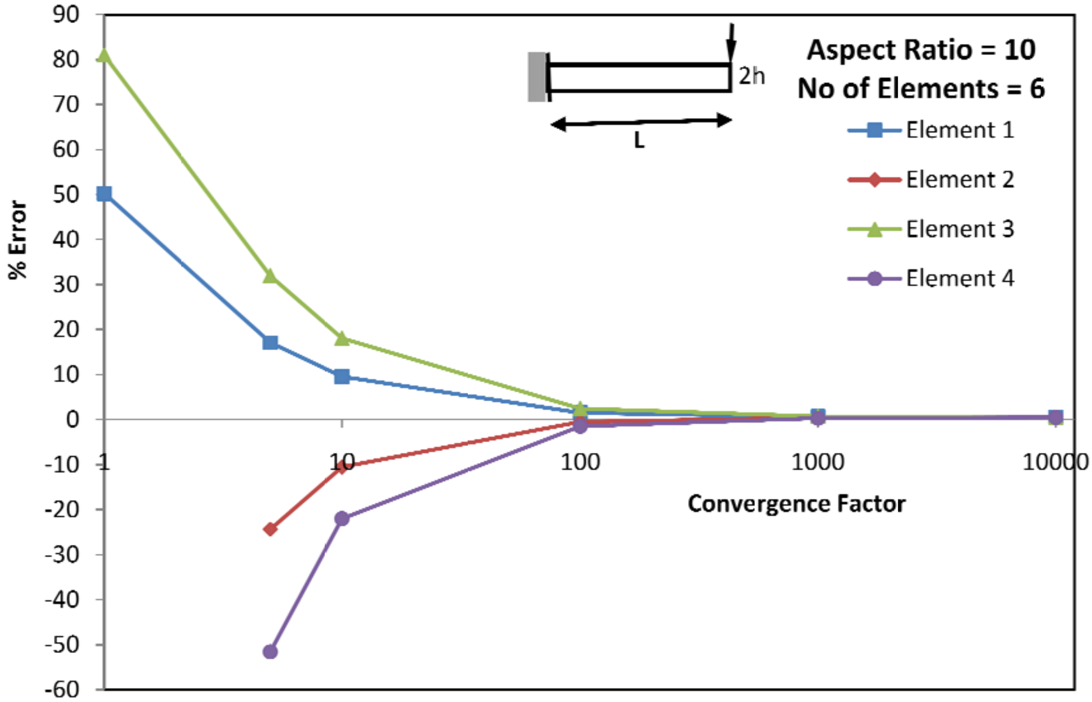


Fig-3_CB_AR_10_NEL_6

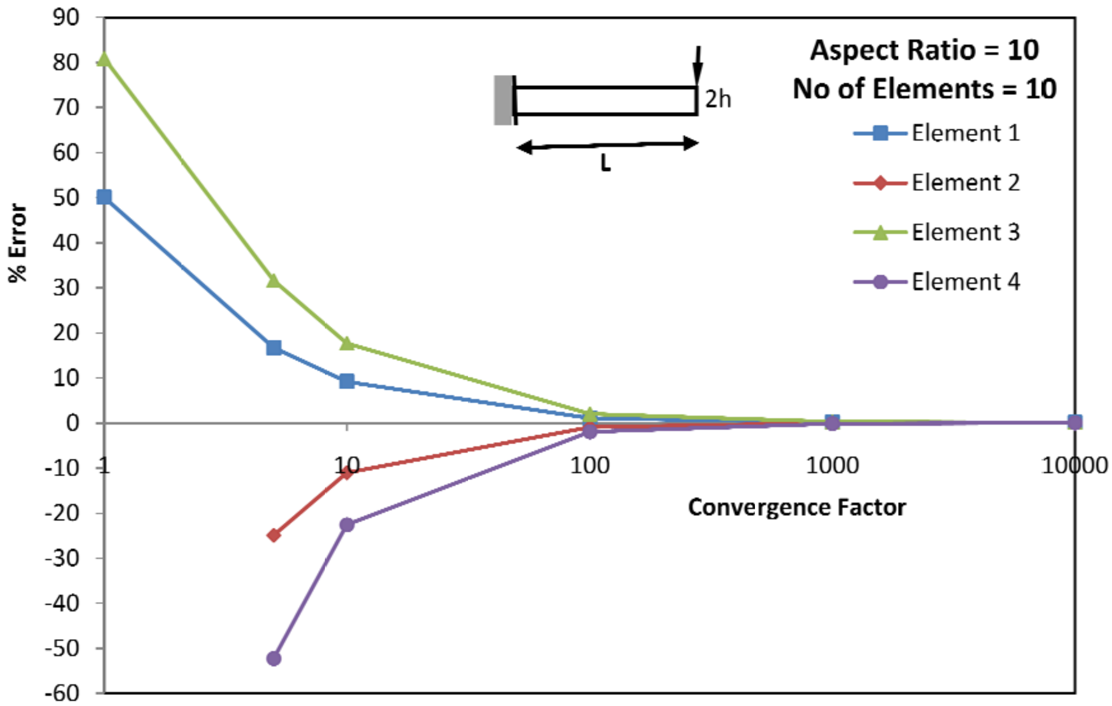


Fig-4_CB_AR_10_NEL_10

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 100

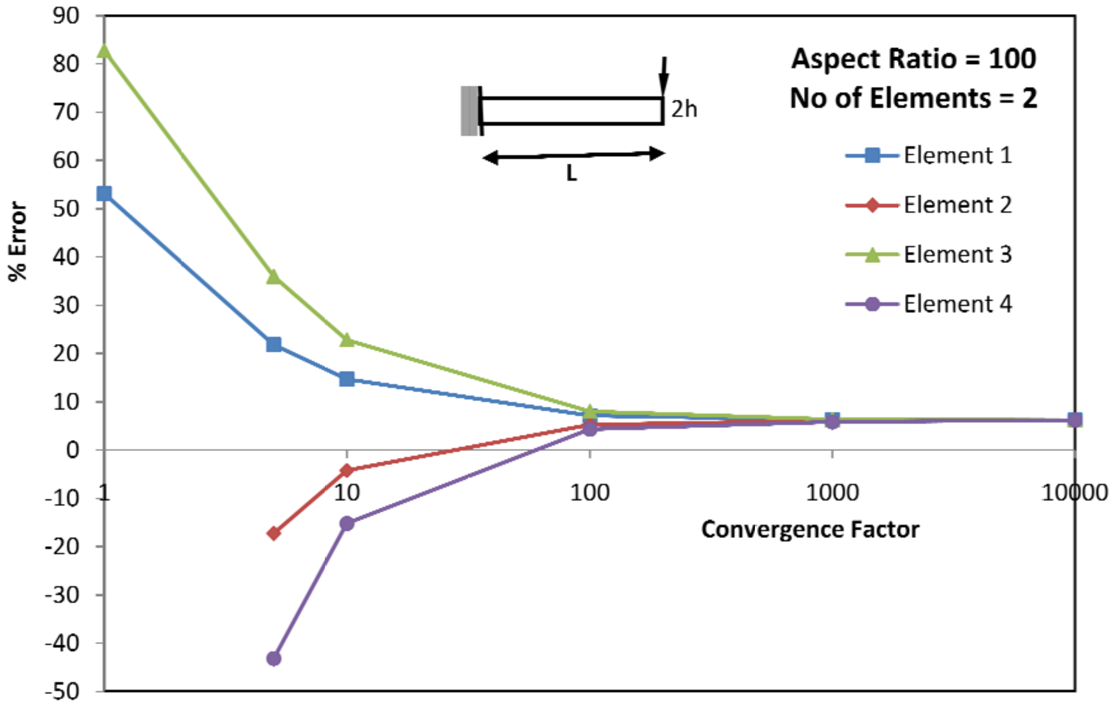


Fig-1_CB_AR_100_NEL_2

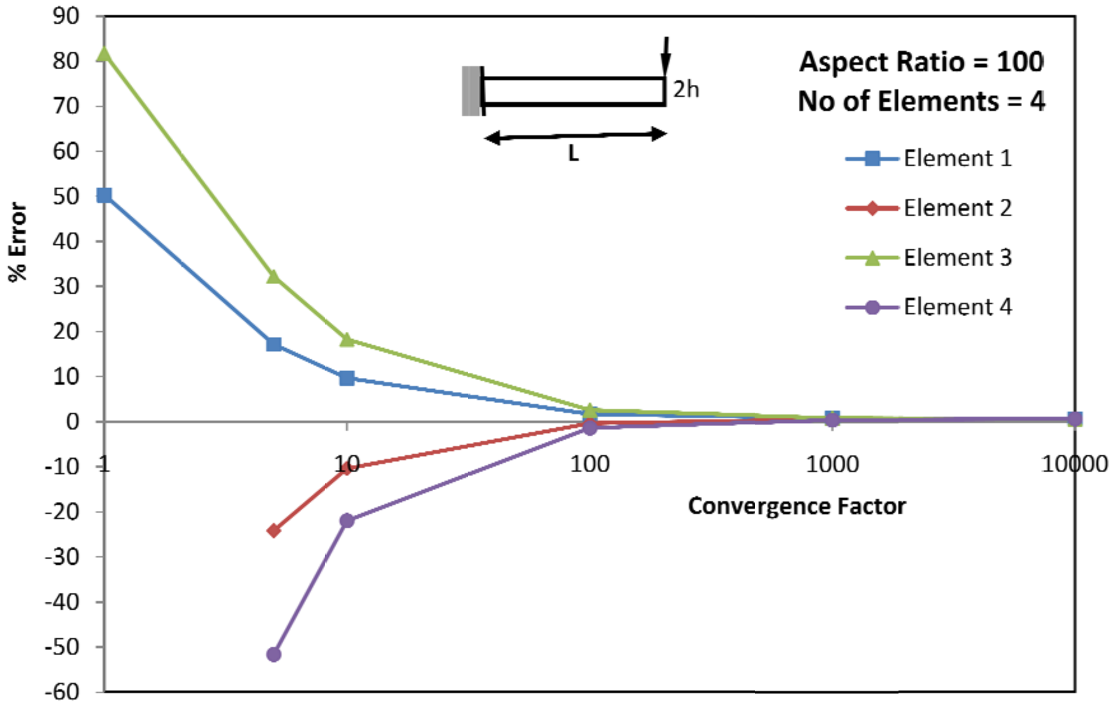


Fig-2_CB_AR_100_NEL_4

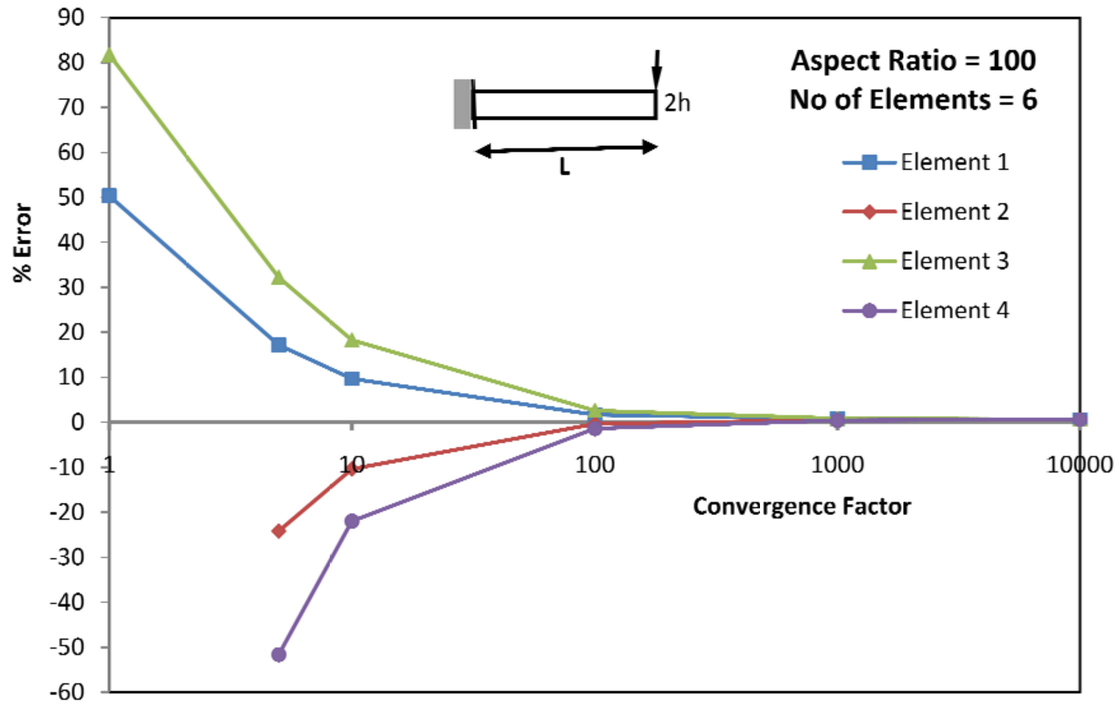


Fig-3_CB_AR_100_NEL_6

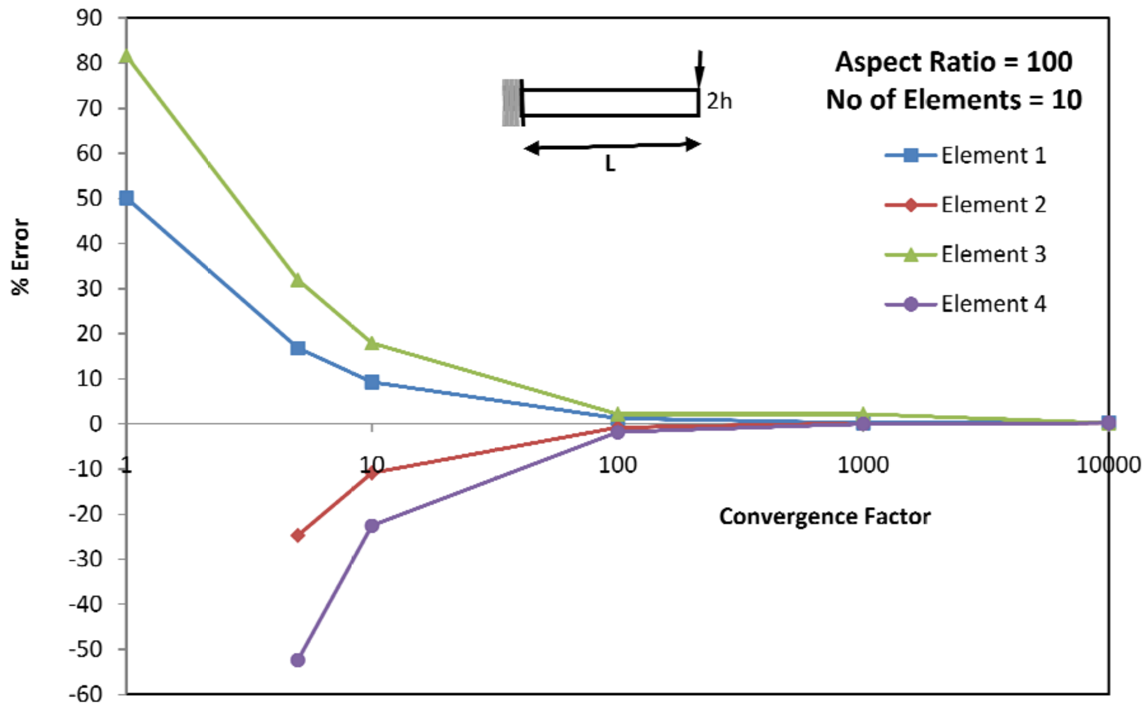


Fig-4_CB_AR_100_NEL_10

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 400

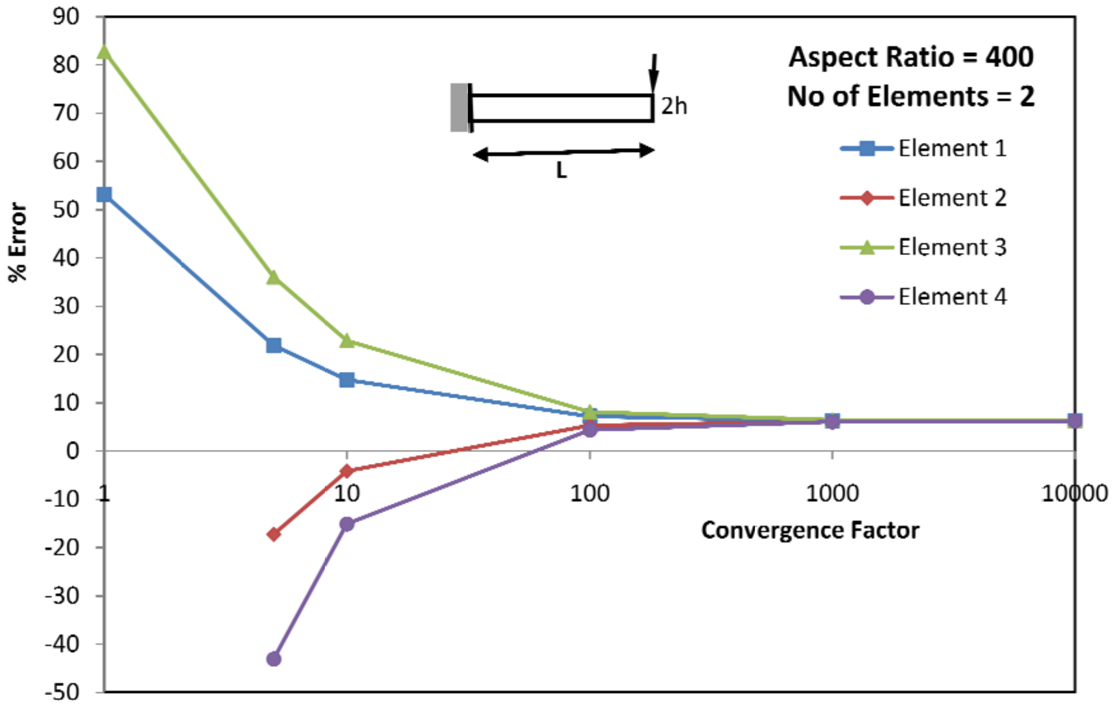


Fig-1_CB_AR_400_NEL_2

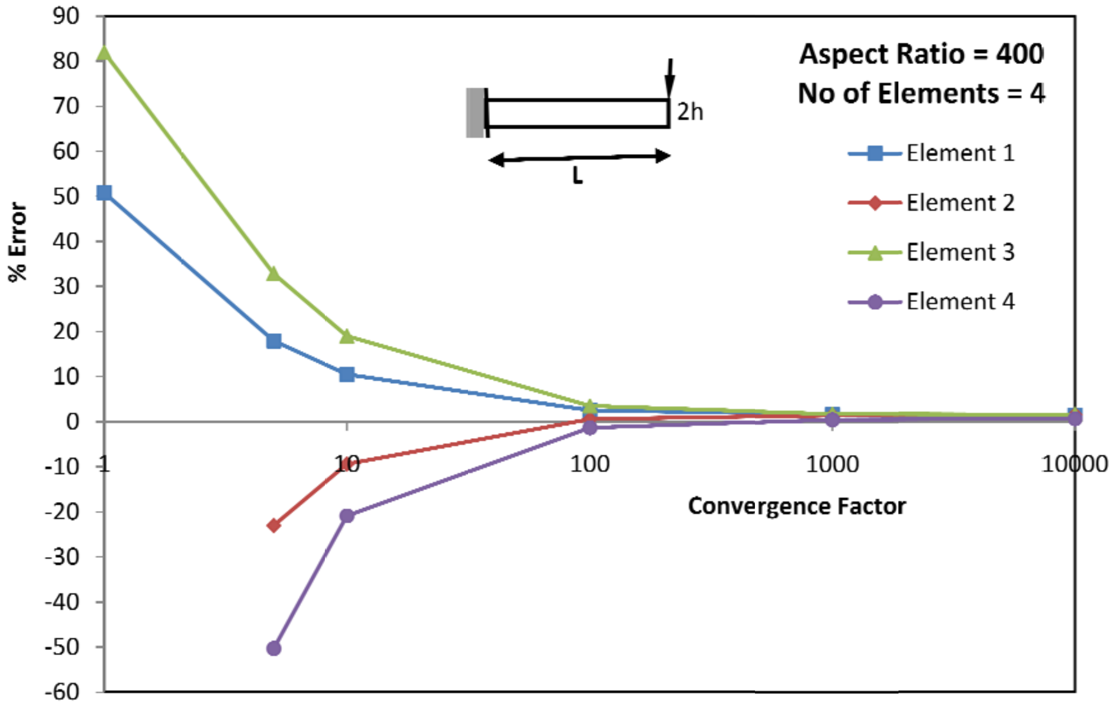


Fig-2_CB_AR_400_NEL_4

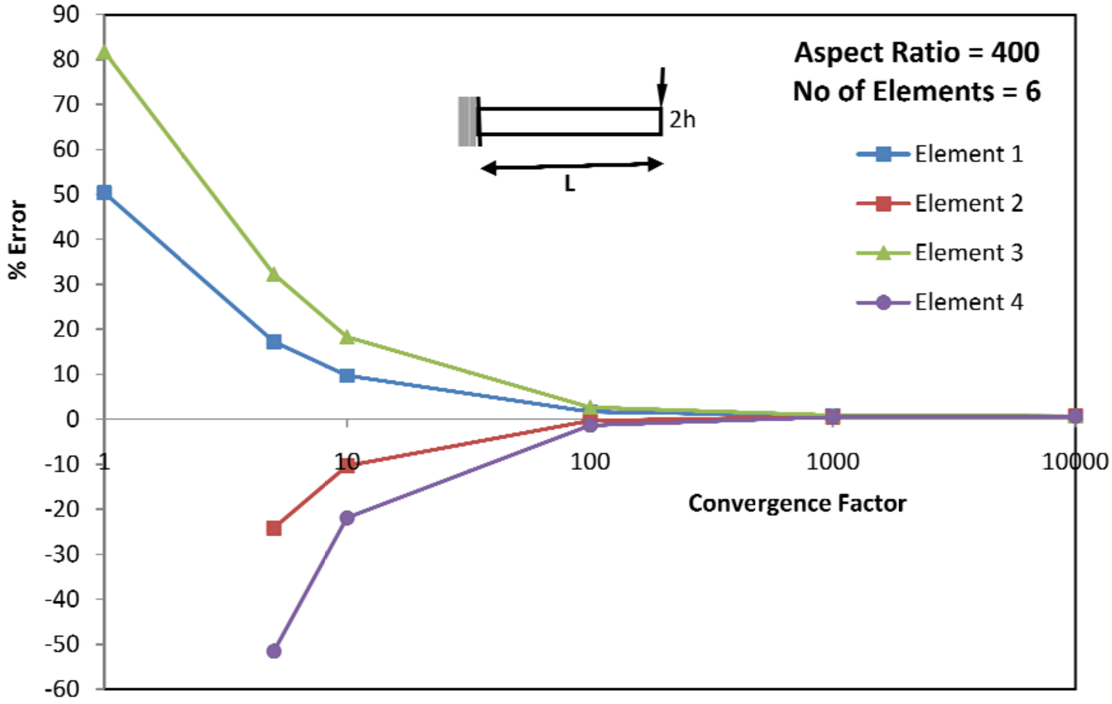


Fig-3_CB_AR_400_NEL_6

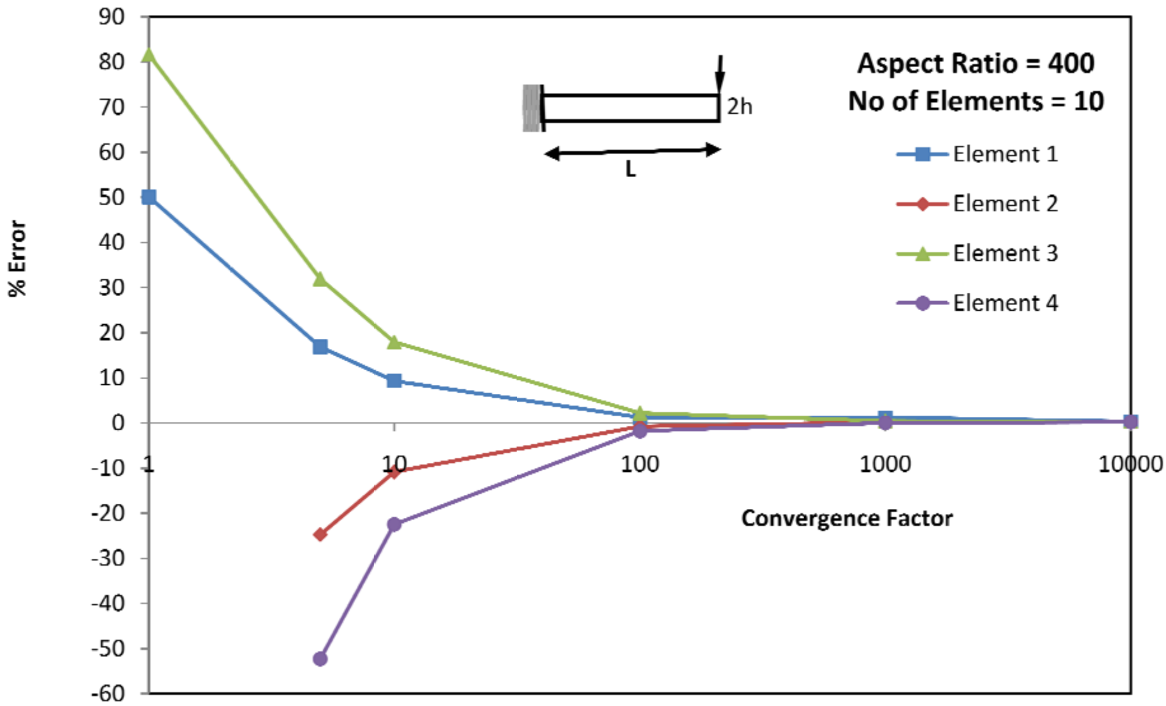
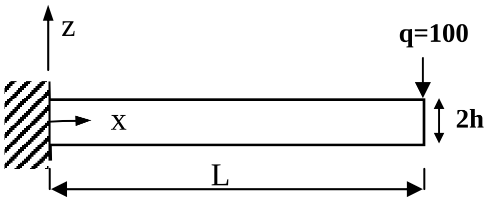


Fig-4_CB_AR_400_NEL_10

Comparison among the finite elements (1) based on Lagrangian shape functions using Higher order deformation theories developed by me (2) based on new shape functions using Higher order deformation theories and (3) based on new shape functions using Timoshenko beam theory



	L/2h=160/12						L/2h=80/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**
2	1.8136	1.8136	32.2712 (6 elements with 1000 CF)	32.2391 (6 elements with 1000 CF)	32.2712 (6 elements with 1000 CF)	32.2712 (6 elements with 1000 CF)	0.7867	0.7867	4.1140 (6 elements with 1000 CF)	4.1100 (6 elements with 1000 CF	4.1258 (6 elements with 1000 CF)	4.1218 (6 elements with 1000 CF)
4	6.2233	6.2233					2.0073	2.0073				
8	15.8701	15.8701					3.2797	3.2797				
12	22.2602	22.2602					3.7158	3.7158				
20	28.0410	28.0410					3.9874	3.9874				
24	29.3511	29.3511					4.0381	4.0381				
30	30.5177	30.5177					4.0806	4.0806				
36	31.1911	31.1911					4.1040	4.1040				
Reddy *	32.823						4.1567					
Exact Solution	**32.7844**						**4.1317**					

	L/2h=40/12						L/2h=12/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_2**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_NFE_3**	**FE_NSF_1**	**FE_NSF_3**
2	0.2626	0.2626	0.5366 (6 elements with 1000 CF)	0.5362 (6 elements with 1000 CF)	0.5424 (6 elements with 1000 CF)	0.5419 (6 elements with 1000 CF)	0.02194	0.02194	0.02264 (6 elements with 1000 CF)	0.02264 (6 elements with 1000 CF)	0.02419 (6 elements with 1000 CF)	0.02418 (6 elements with 1000 CF)
4	0.4302	0.4302					0.02367	0.02367				
8	0.5118	0.5118					0.02414	0.02414				
12	0.5305	0.5305					0.02423	0.02423				
20	0.5406	0.5406					0.02428	0.02428				
24	0.5424	0.5424										
30	0.5439	0.5439										
36	0.5447	0.5447										
Reddy *	0.54588						0.02393					
Exact Solution	**0.5333**						**0.02052**					

<u>**Simply Supported beam with UDL**</u>

<u>**Error in deflection at the centre of the beam**</u>

<u>**Aspect ratio = 5**</u>

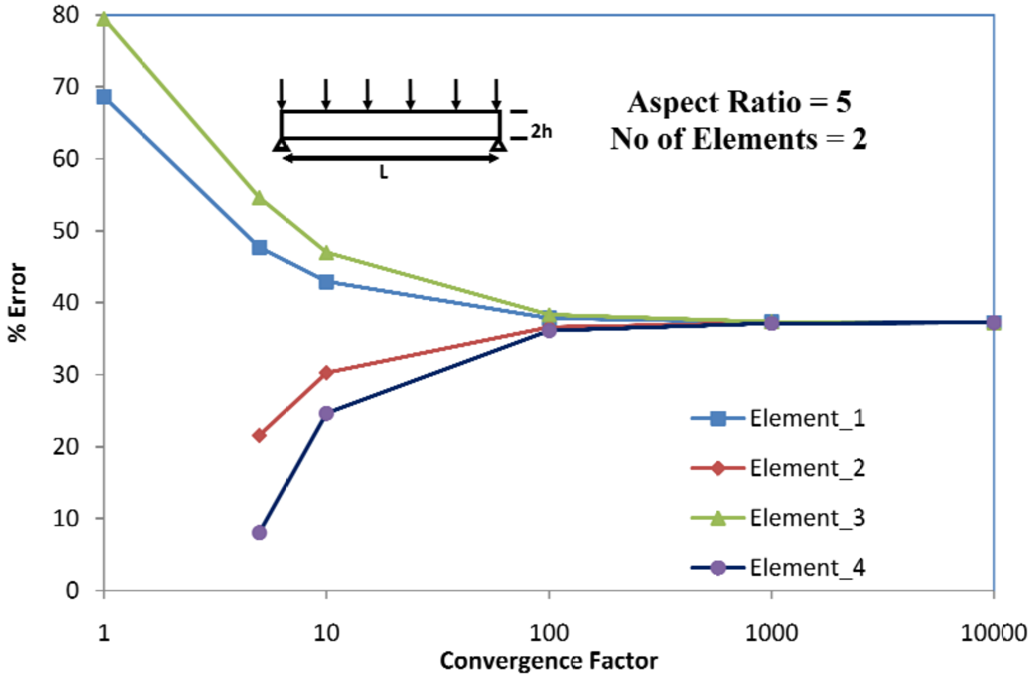


Fig-1_SSB_AR_5_NEL_2

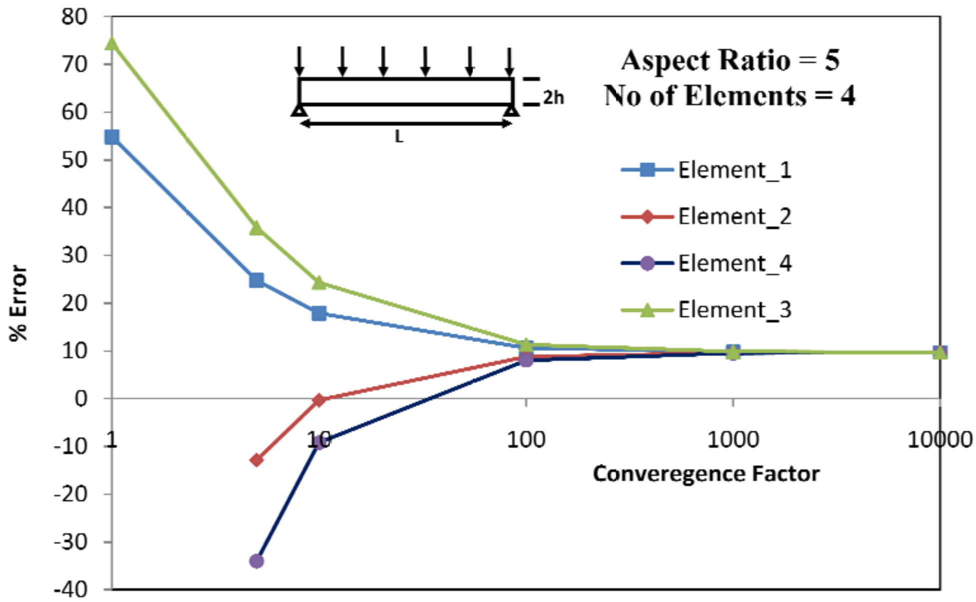


Fig-2_SSB_AR_5_NEL_4

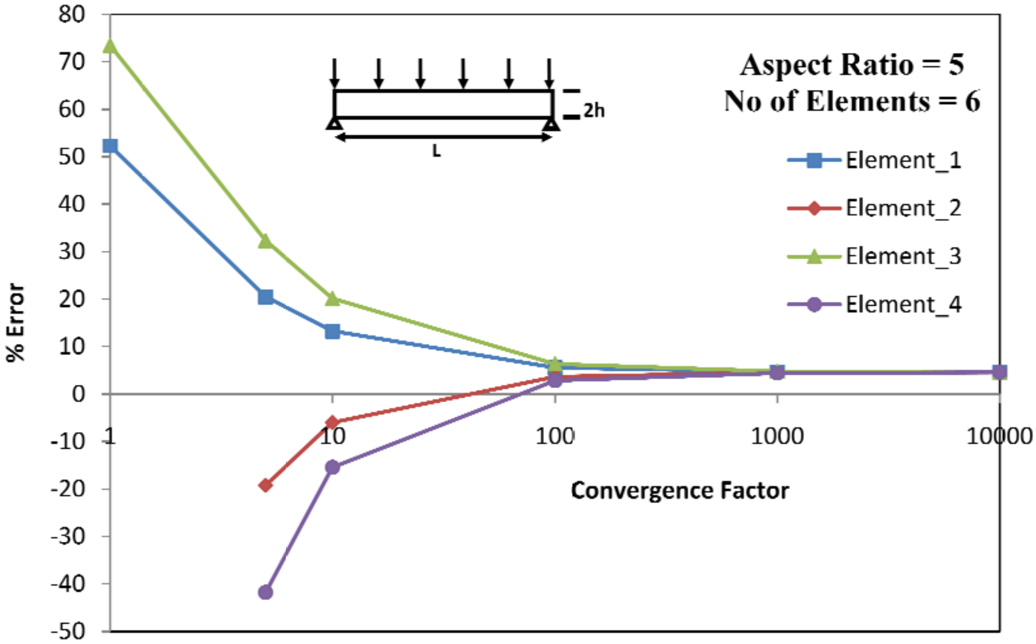


Fig-3_SSB_AR_5_NEL_6

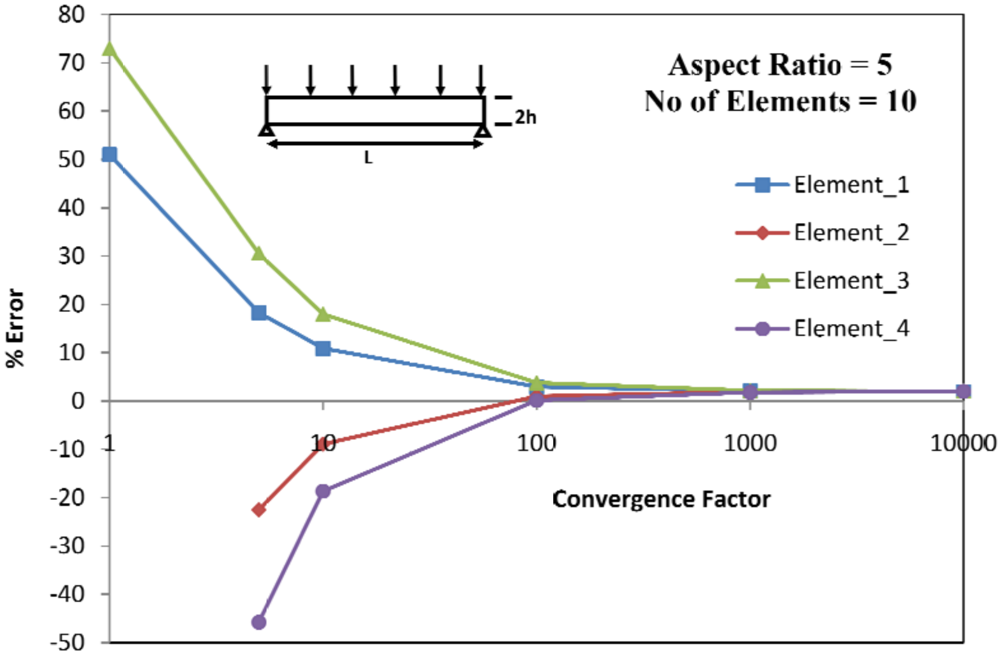


Fig-4_SSB_AR_5_NEL_10

Simply Supported beam with UDL
Error in deflection at the centre of the beam
Aspect ratio = 10

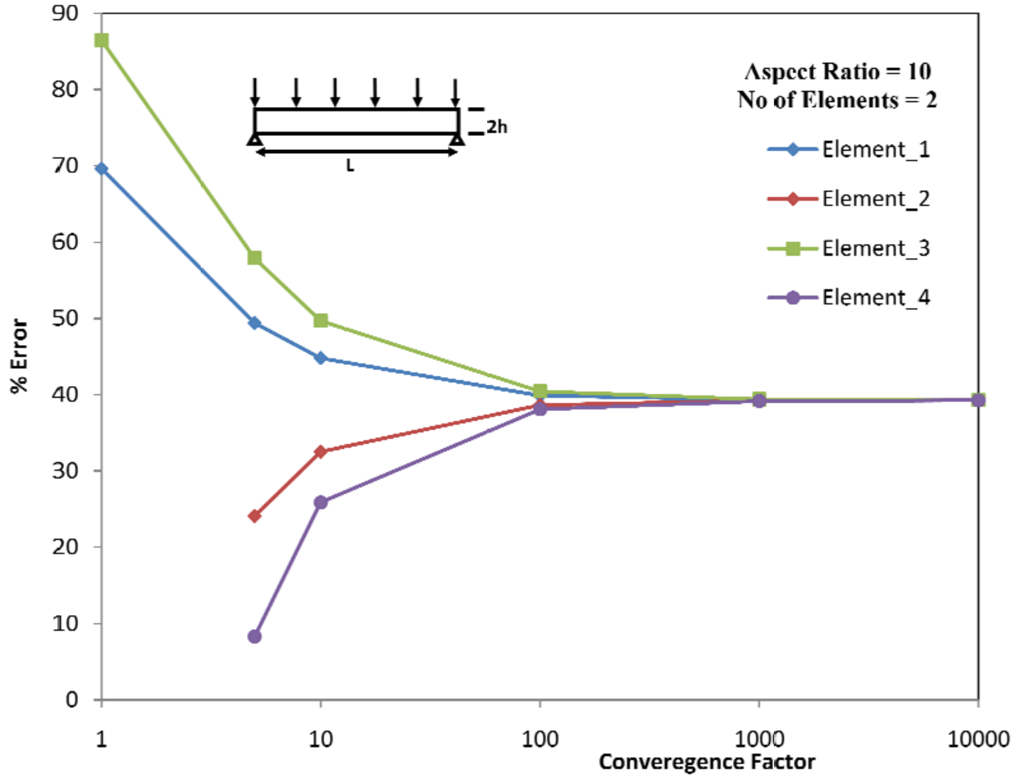


Fig-1_SSB_AR_10_NEL_2

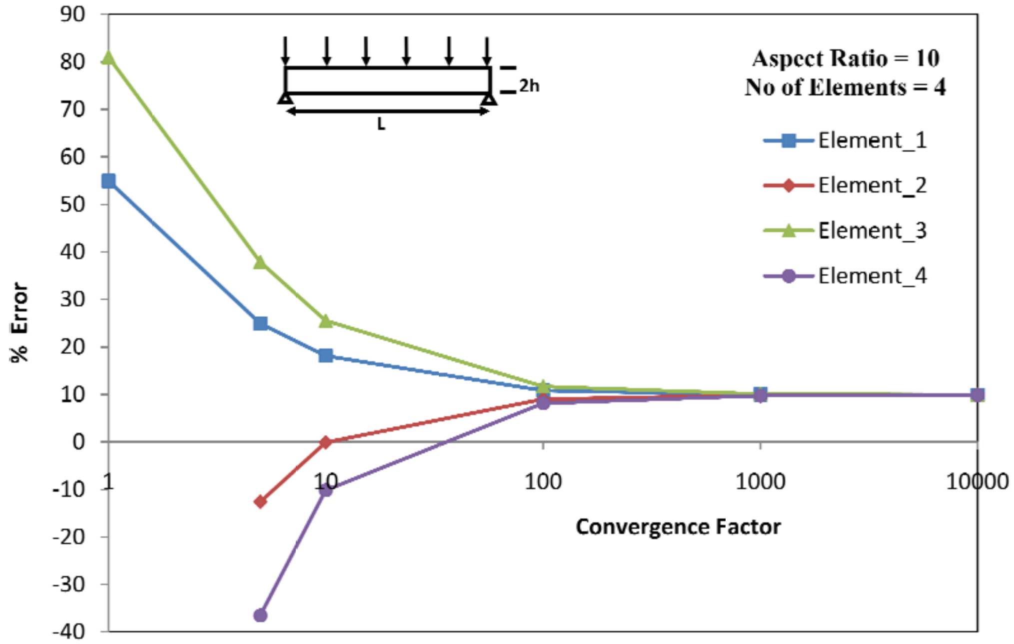


Fig-2_SSB_AR_10_NEL_4

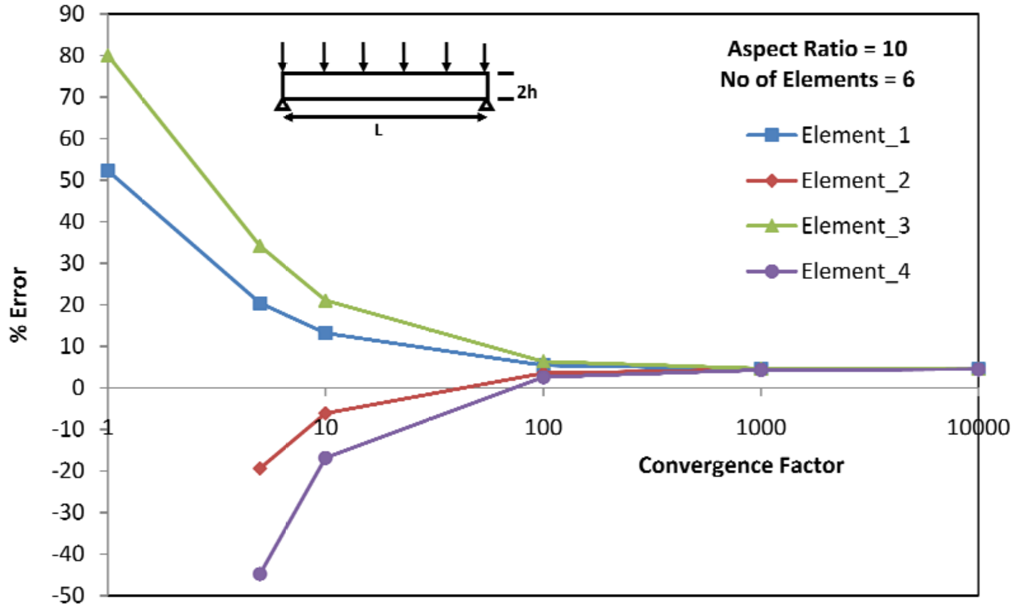


Fig-3_SSB_AR_10_NEL_6

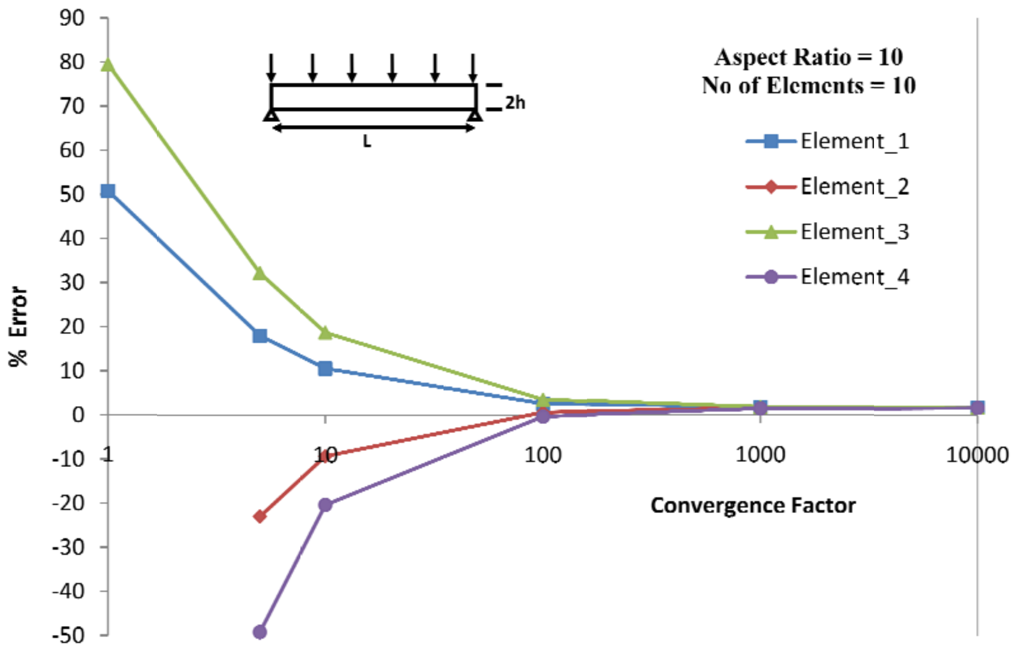


Fig-4_SSB_AR_10_NEL_10

Simply Supported beam with UDL
Error in deflection at the centre of the beam
Aspect ratio = 100

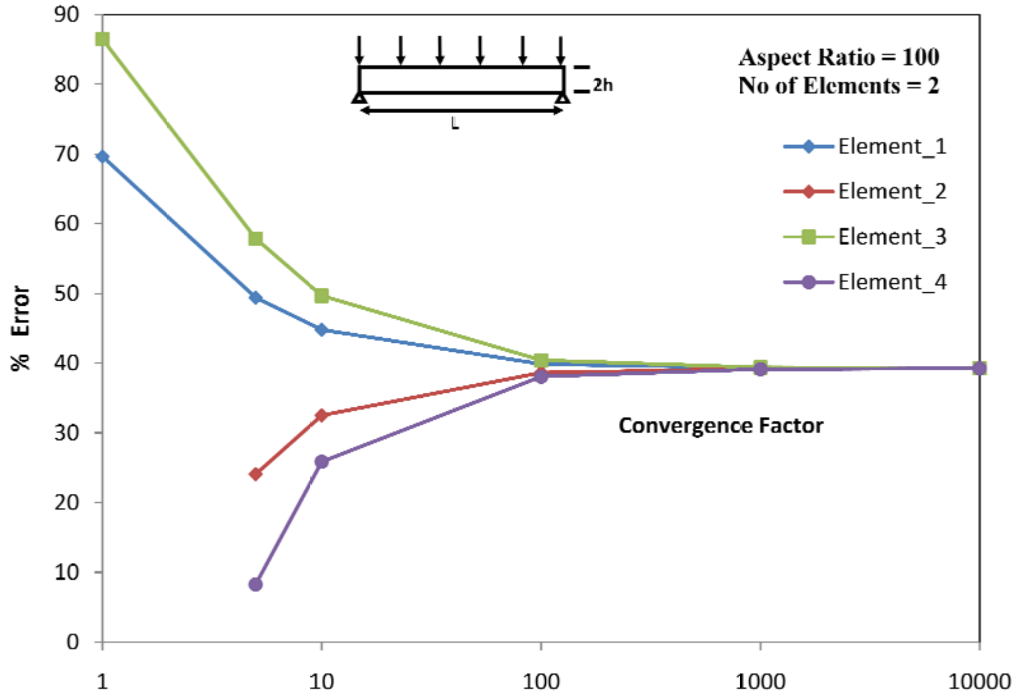


Fig-1_SSB_AR_100_NEL_2

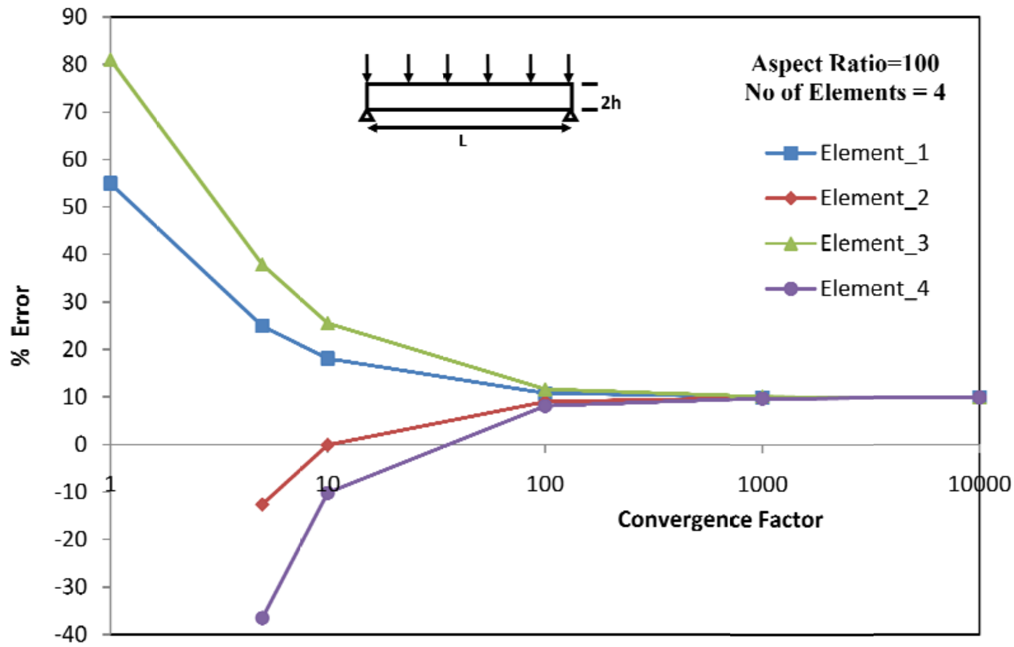


Fig-2_SSB_AR_100_NEL_4

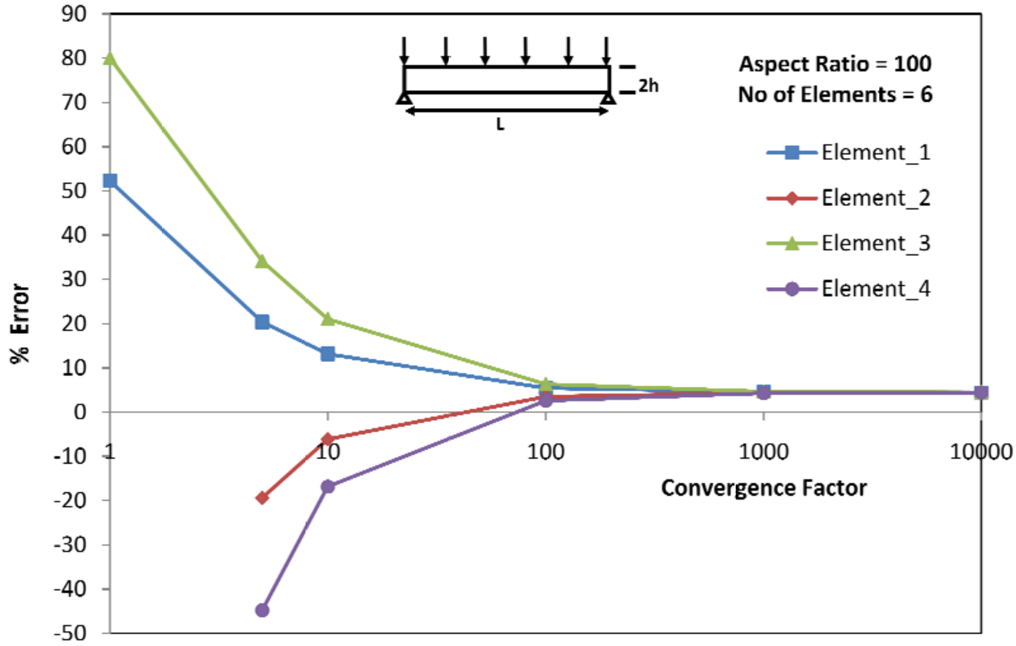


Fig-3_SSB_AR_100_NEL_6

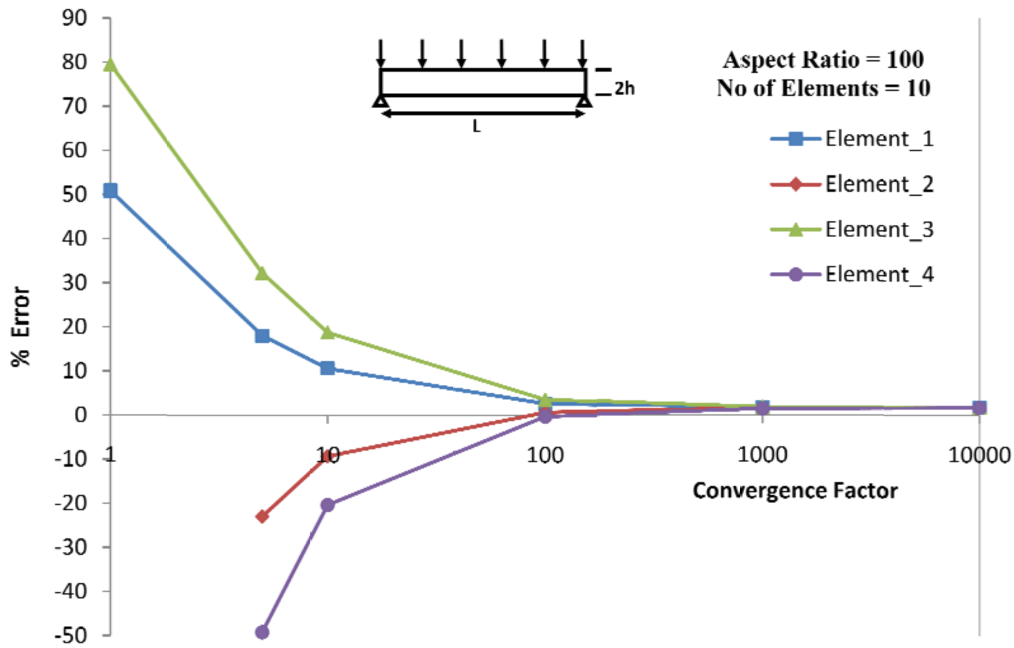


Fig-4_SSB_AR_100_NEL_10

<u>**Simply Supported beam with UDL**</u>
<u>**Error in deflection at the centre of the beam**</u>
<u>**Aspect ratio = 400**</u>

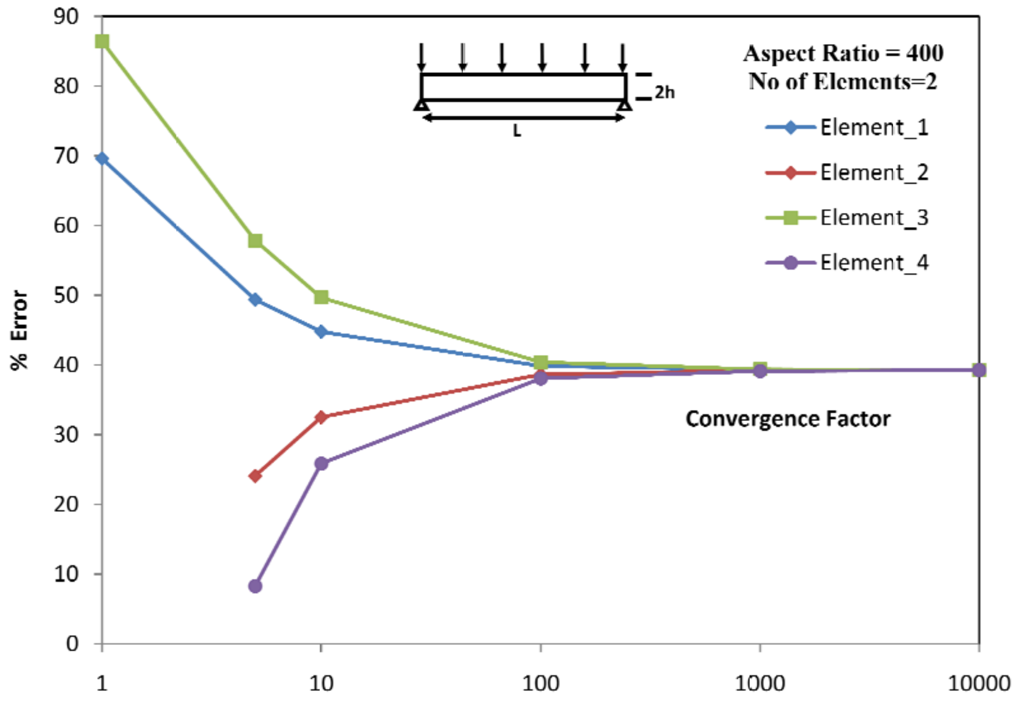


Fig-1_SSB_AR_400_NEL_2

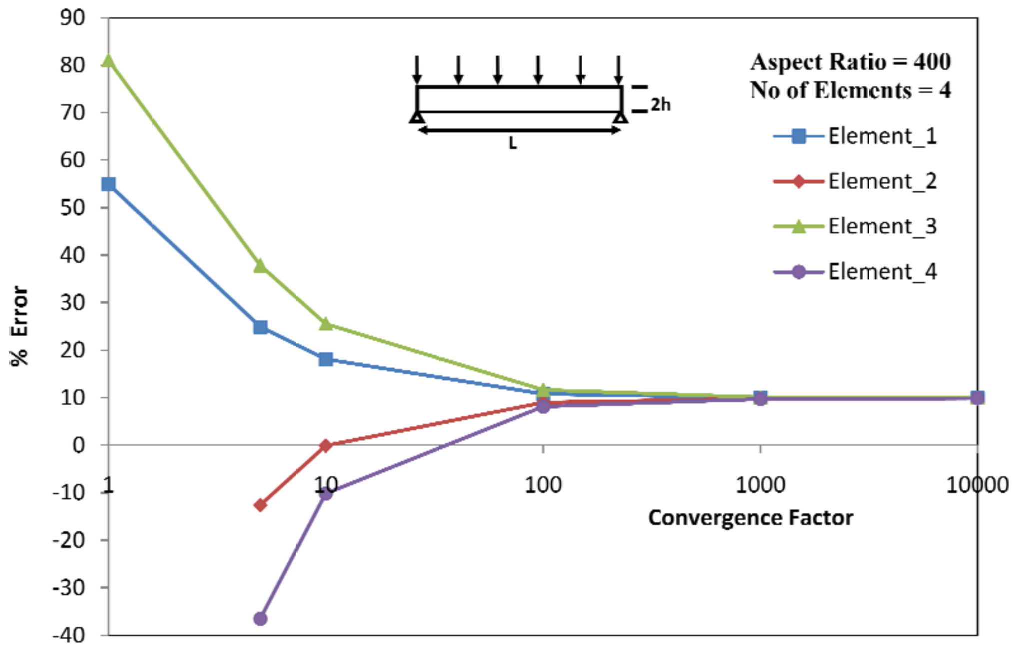


Fig-2_SSB_AR_400_NEL_4

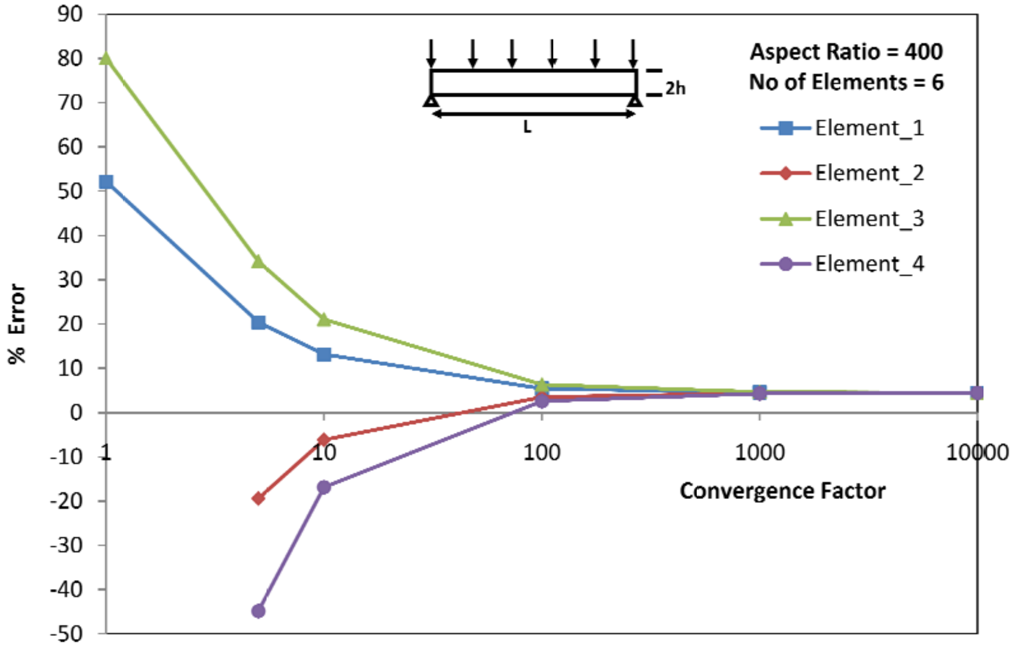


Fig-3_SSB_AR_400_NEL_6

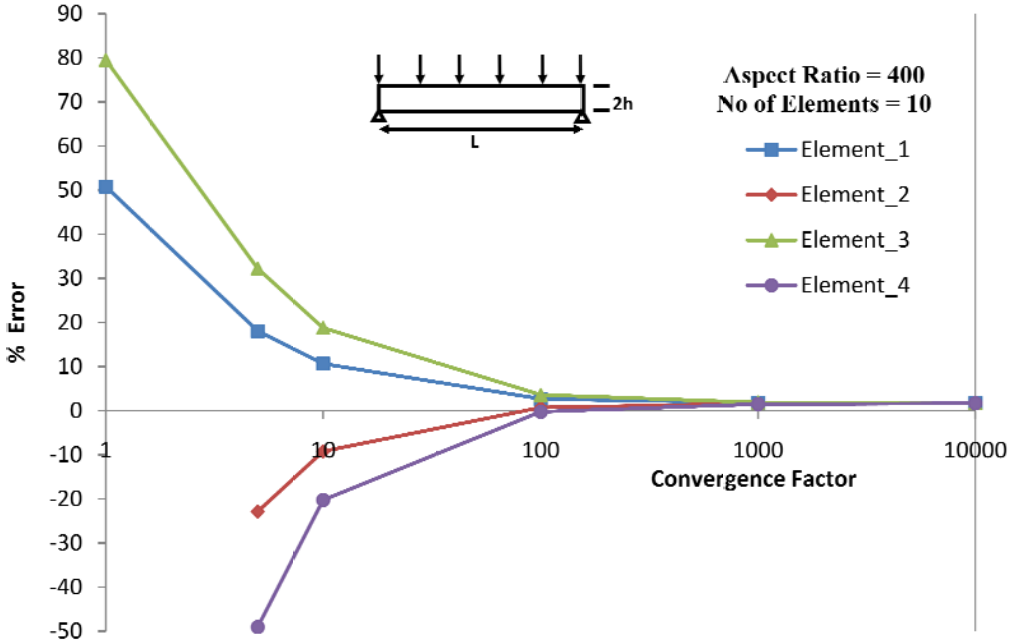
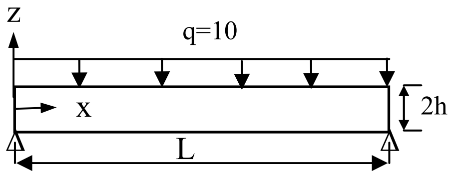


Fig-4_SSB_AR_400_NEL_6

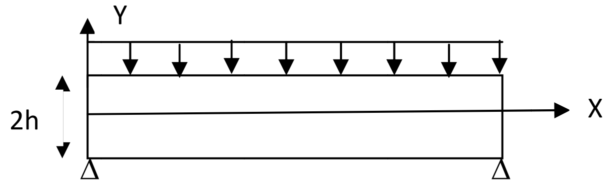
Comparison among the finite elements (1) based on Lagrangian shape functions using Higher order deformation theories developed by me (2) based on new shape functions using Higher order deformation theories and (3) based on new shape functions using Timoshenko beam theory



	L/2h=160/12						L/2h=80/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**
2	0.8254	0.8254	20.326 (10 elements with 1000 CF)	20.306 (10 elements with 1000 CF)	20.374 (10 elements with 1000 CF)	20.354 (10 elements with 1000 CF)	0.1849	0.1849	1.3151 (10 elements with 1000 CF)	1.3139 (10 elements with 1000 CF)	1.3271 (10 elements with 1000 CF	1.3259 (10 elements with 1000 CF
4	3.6431	3.6431					0.6046	0.6046				
8	9.8570	9.8570					1.0482	1.0482				
12	13.9791	13.9791					1.2009	1.2009				
20	17.7108	17.7108					1.2960	1.2960				
24	18.5568	18.5568					1.3138	1.3138				
30	19.3102	19.3102					1.3287	1.3287				
36	19.7452	19.7452					1.3369	1.3369				
Reddy *	20.717						1.3486					
Exact Solution	**20.6892**						**1.3408**					

	L/2h=40/12						L/2h=12/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**
2	0.03475	0.03475	0.09339 (10 elements with 1000 CF)	0.09333 (10 elements with 1000 CF)	0.09634 (10 elements with 1000 CF)	0.09628 (10 elements with 1000 CF)	0.0016486	0.0016486	0.001979 (10 elements with 1000 CF)	0.001980 (10 elements with 1000 CF)	0.002230 (10 elements with 1000 CF)	0.002210 (10 elements with 1000 CF)
4	0.07202	0.07202					0.0021069	0.0021069				
8	0.09064	0.09064					0.0022298	0.0022298				
12	0.09491	0.09491					0.0022529	0.0022529				
20	0.09723	0.09723					0.0022649	0.0022649				
24	0.09764	0.09764					-----------	-----------				
30	---------	---------					-----------	-----------				
36	---------	---------					-----------	-----------				
Reddy *	0.09770						0.002220					
Exact Solution	**0.09576**						**0.002082**					

Stress Results for Simply Supported Beam



Length =5.0, width=1.0, thickness=1.0, Maximum Bending Stress at the center of the beam

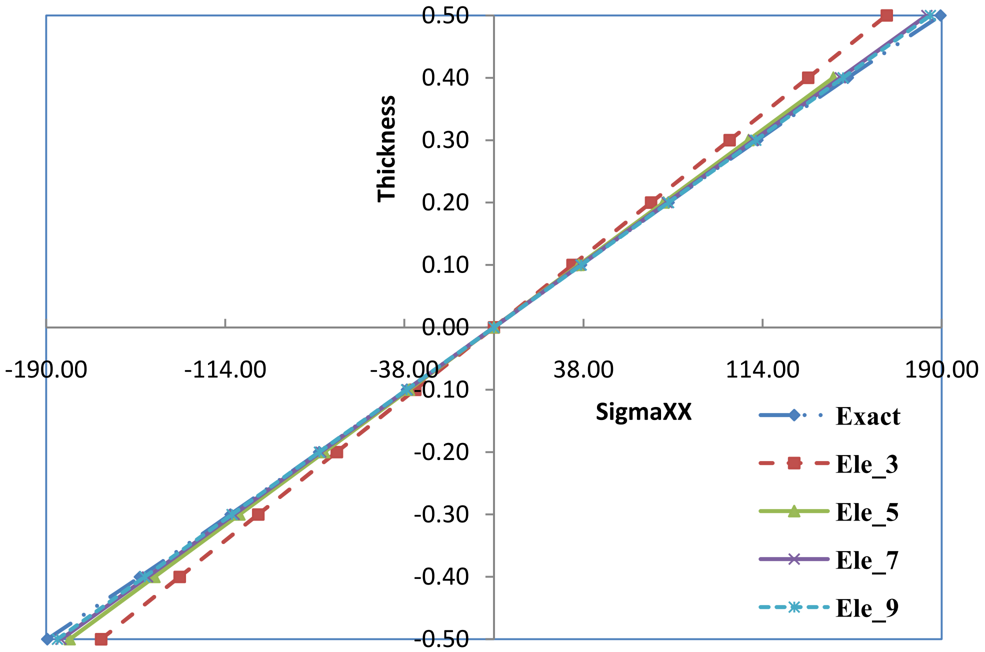
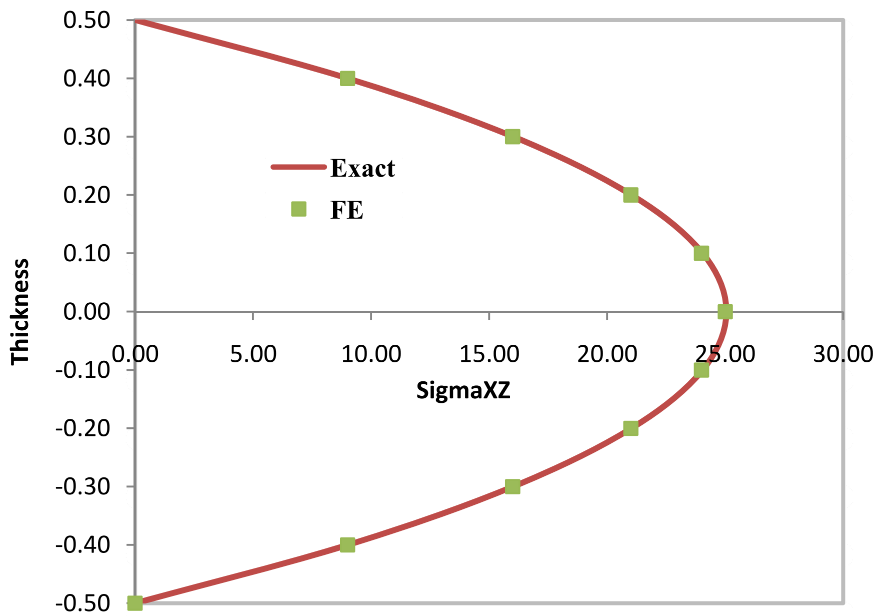


Fig.1 SS_ASR_5_Bending_Stress



Shear stress calculation = 0.83

Fig.2 SS_ASR_5_Shear_Stress

Length =10.0, width=1.0, thickness=1.0
Maximum Bending Stress at the center of the beam

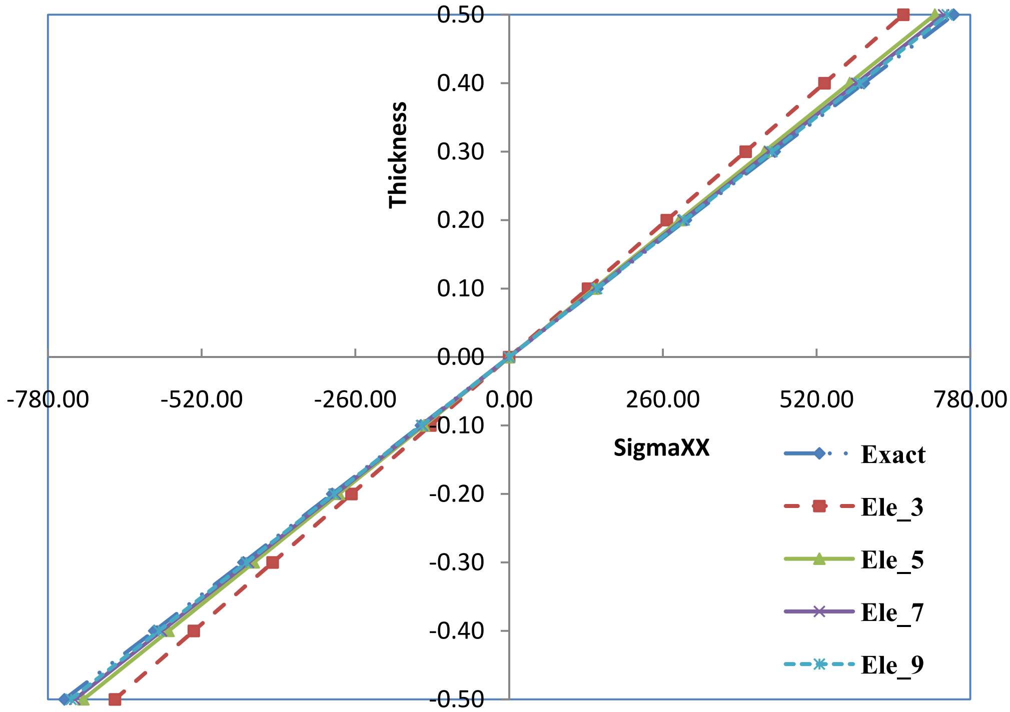
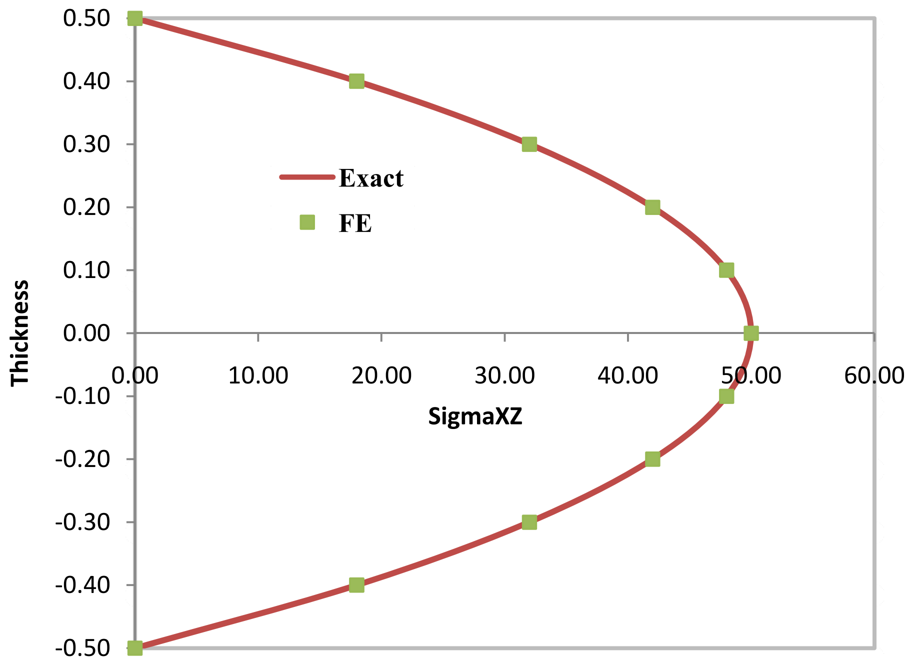


Fig.3 SS_ASR_10_Bending_Stress



Shear stress calculation = 1.666

Fig.4 SS_ASR_10_Shear_Stress

Length =200.0, width=1.0, thickness=1.0
Maximum Bending Stress at the center of the beam

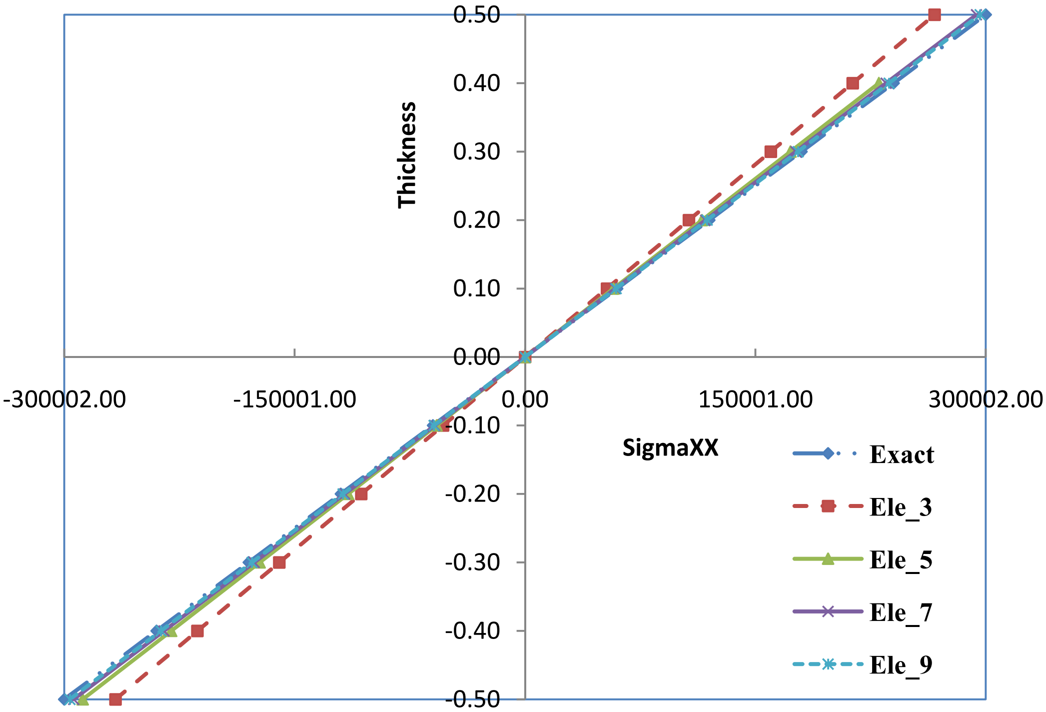
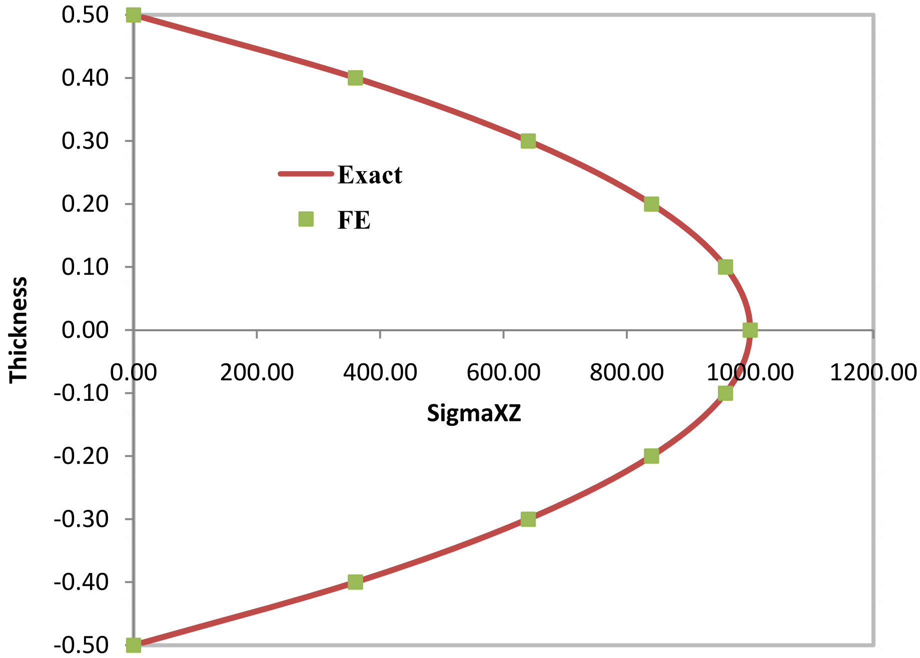


Fig.5 SS_ASR_200_Bending_Stress



Shear stress calculation = 33.33

Fig.6 SS_ASR_200_Shear_Stress

Length =400.0, width=1.0, thickness=1.0
Maximum Bending Stress at the center of the beam

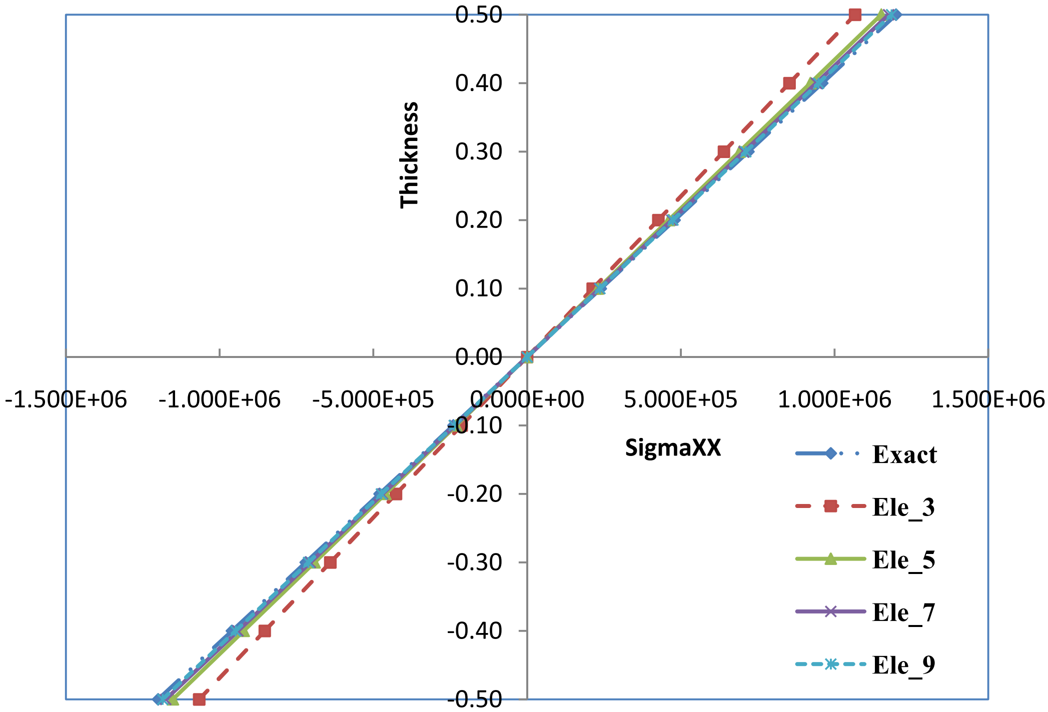
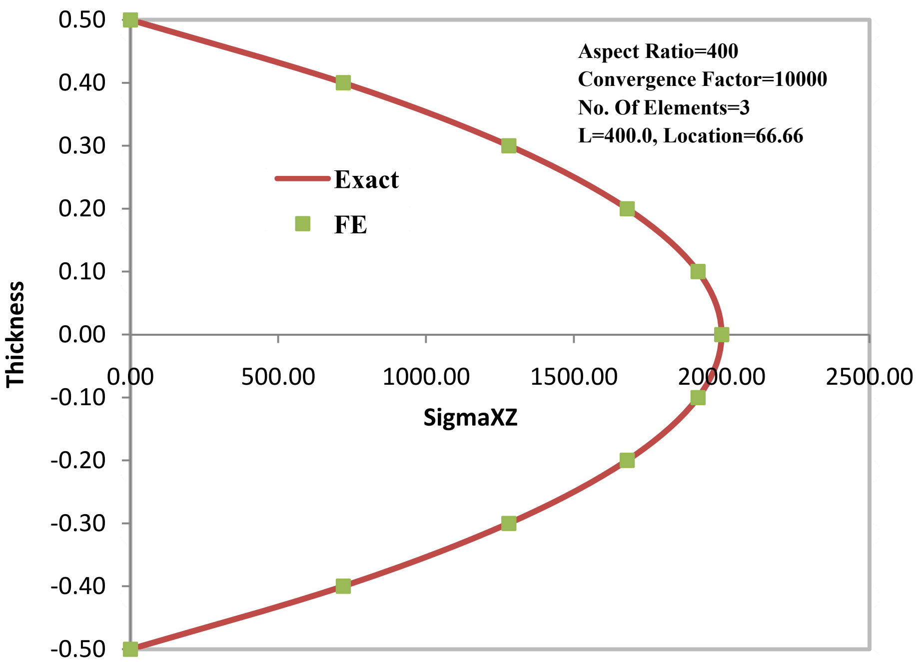


Fig.7 SS_ASR_400_Bending_Stress



Shear stress calculation = 66.66

Fig.8 SS_ASR_400_Shear_Stress

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 5

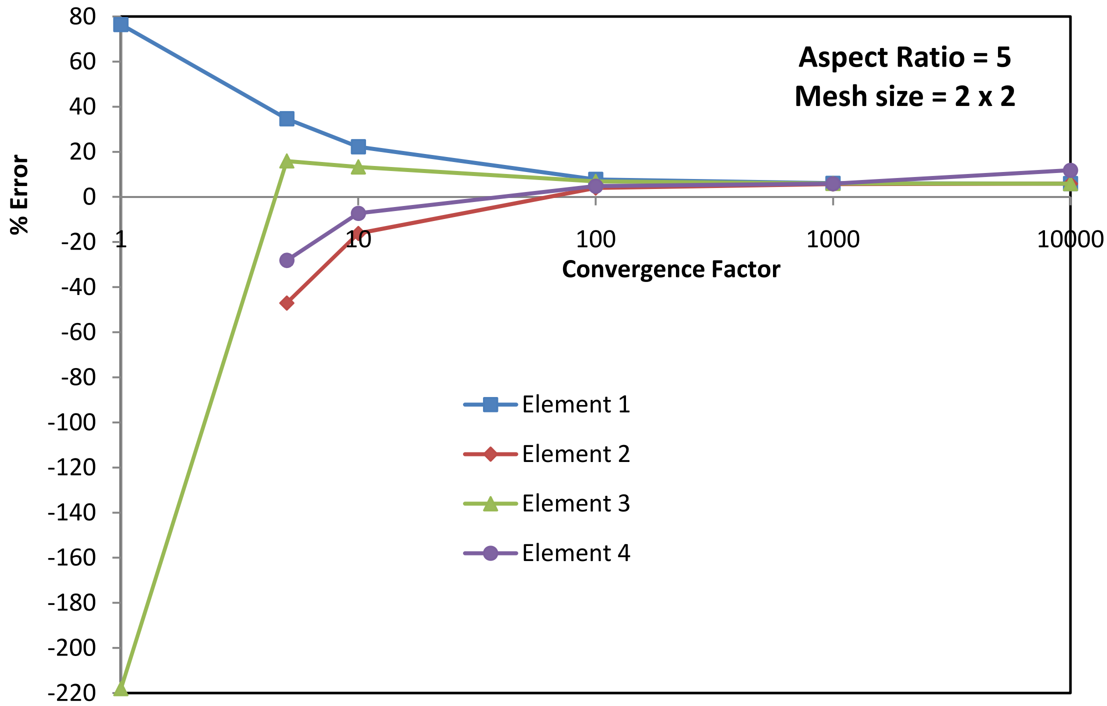


Fig-1_SSP_AR_5_NEL_2x2

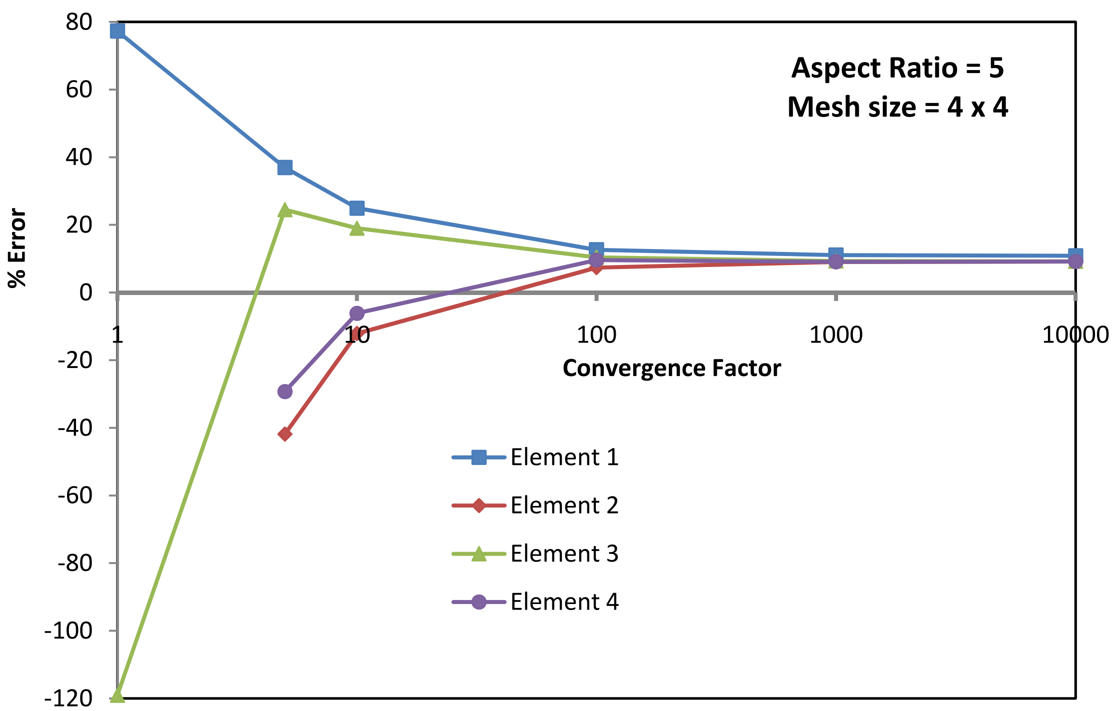


Fig-2_SSP_AR_5_NEL_4x4

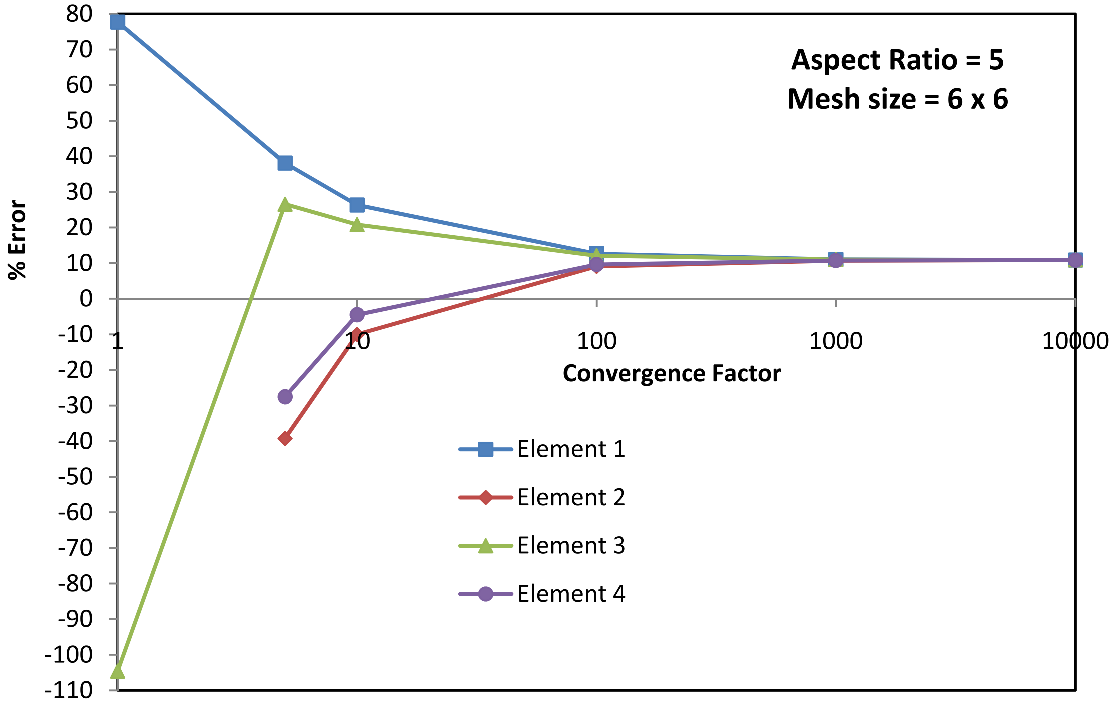


Fig-3_SSP_AR_5_NEL_6x6

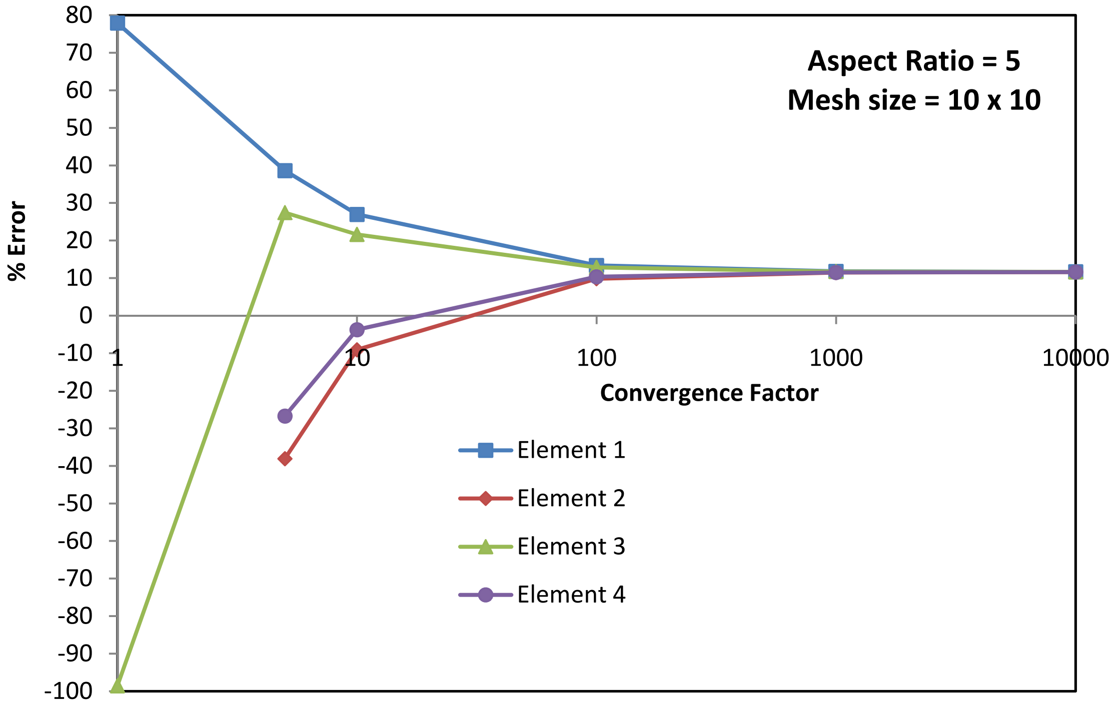


Fig-4_SSP_AR_5_NEL_10x10

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 7.14

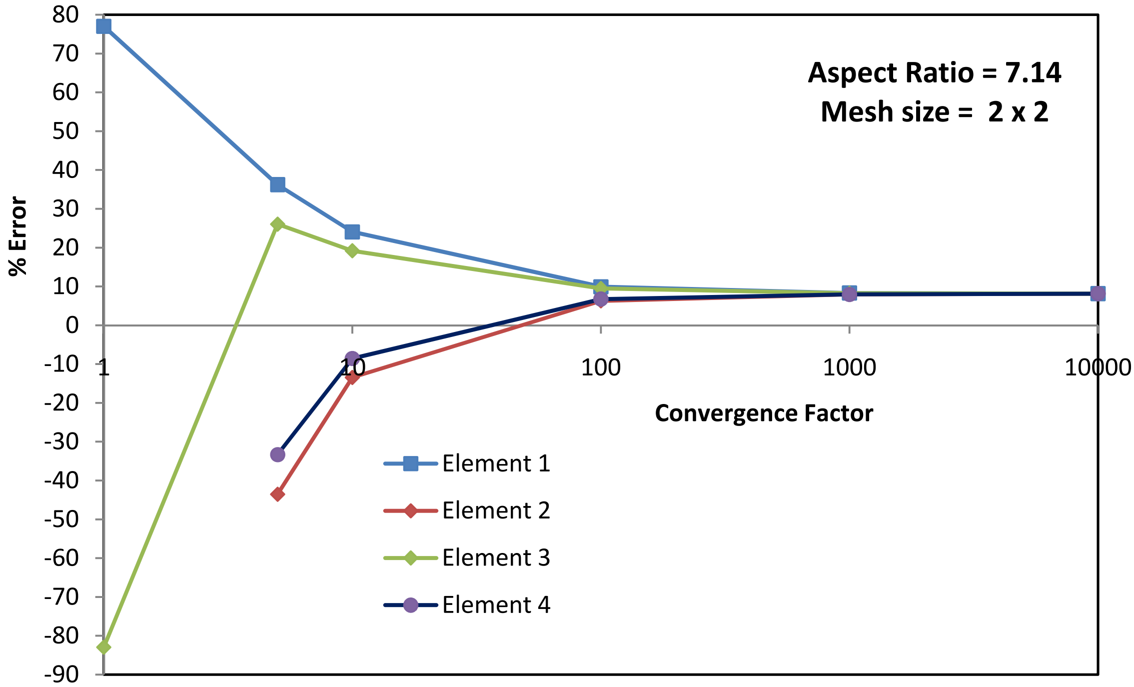


Fig-1_SSP_AR_7.14_NEL_2x2

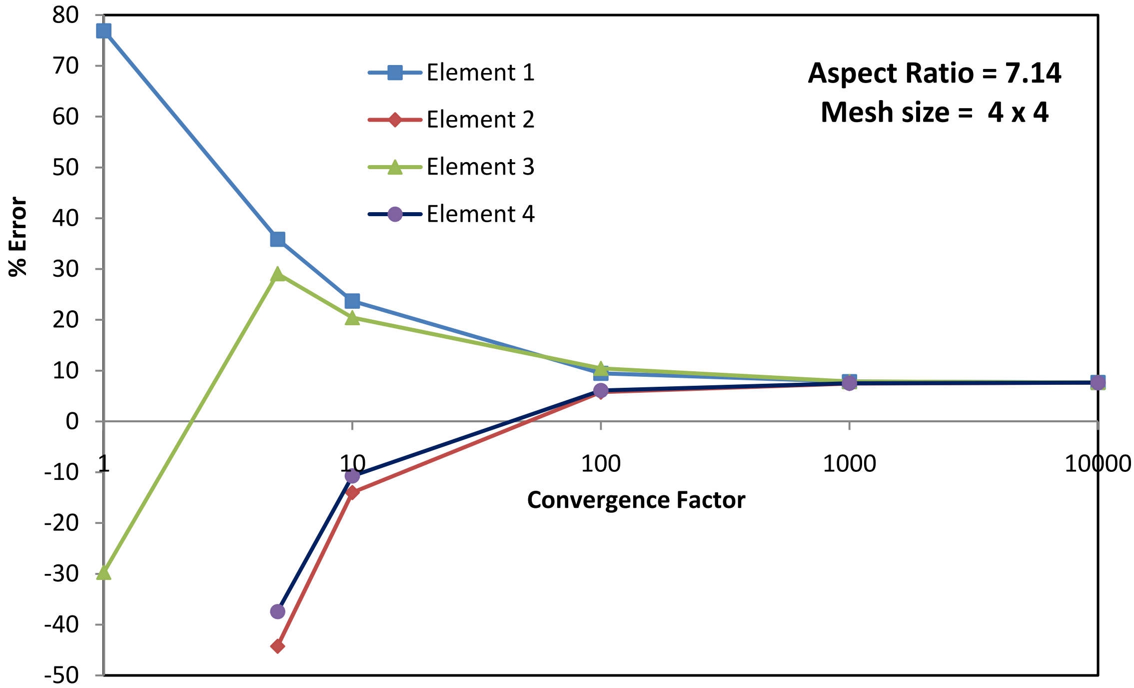


Fig-2_SSP_AR_7.14_NEL_4x4

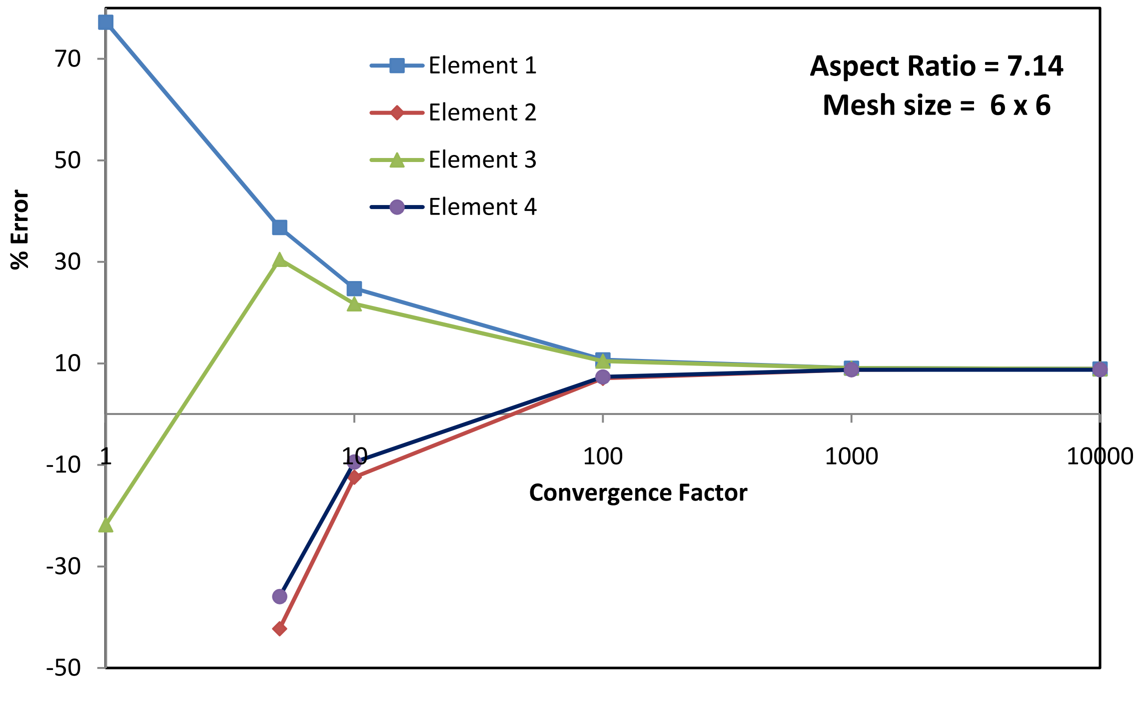


Fig-3_SSP_AR_7.14_NEL_6x6

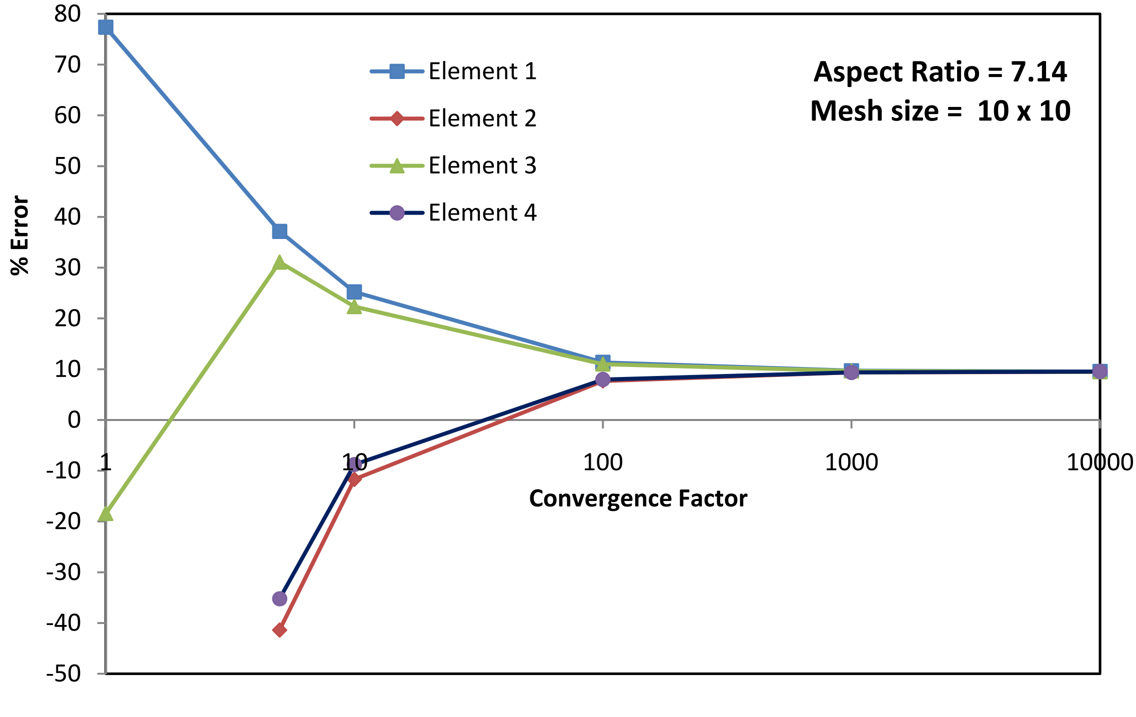


Fig-4_SSP_AR_7.14_NEL_10x10

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 10

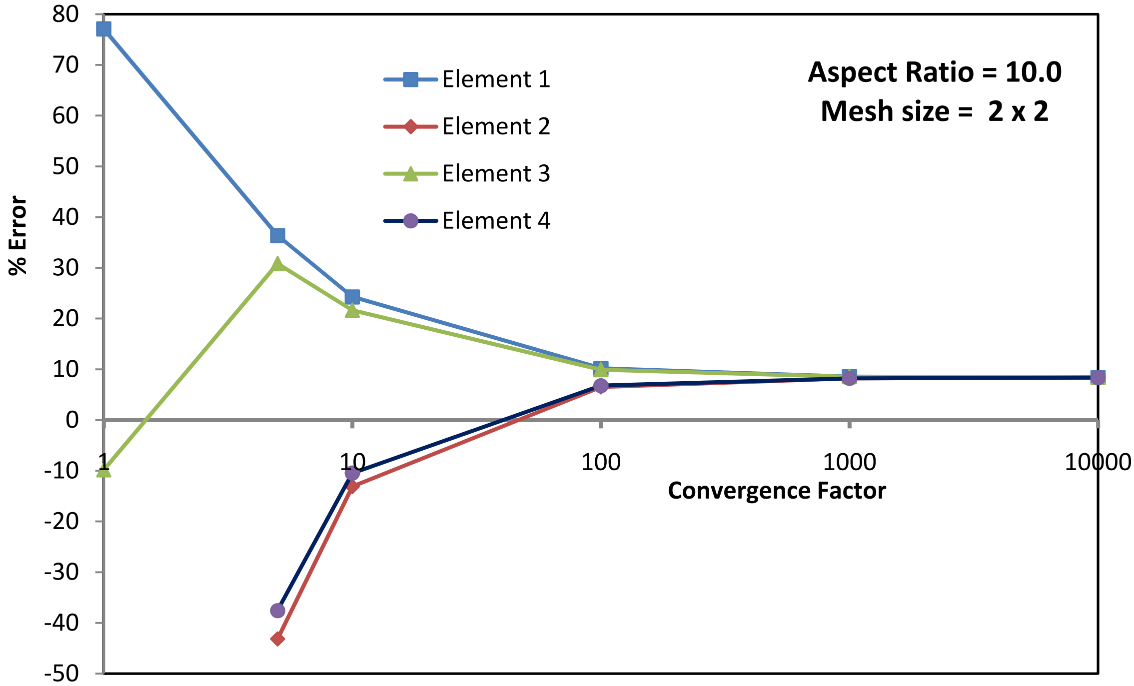


Fig-1_SSP_AR_10_NEL_2x2

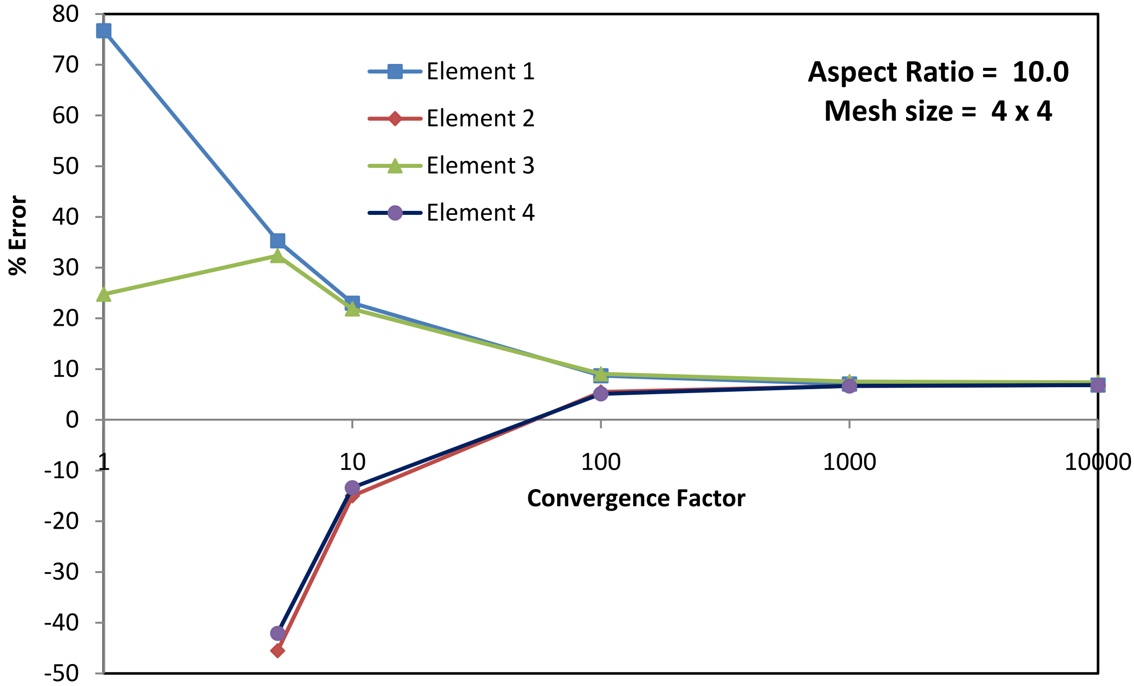


Fig-2_SSP_AR_10_NEL_4x4

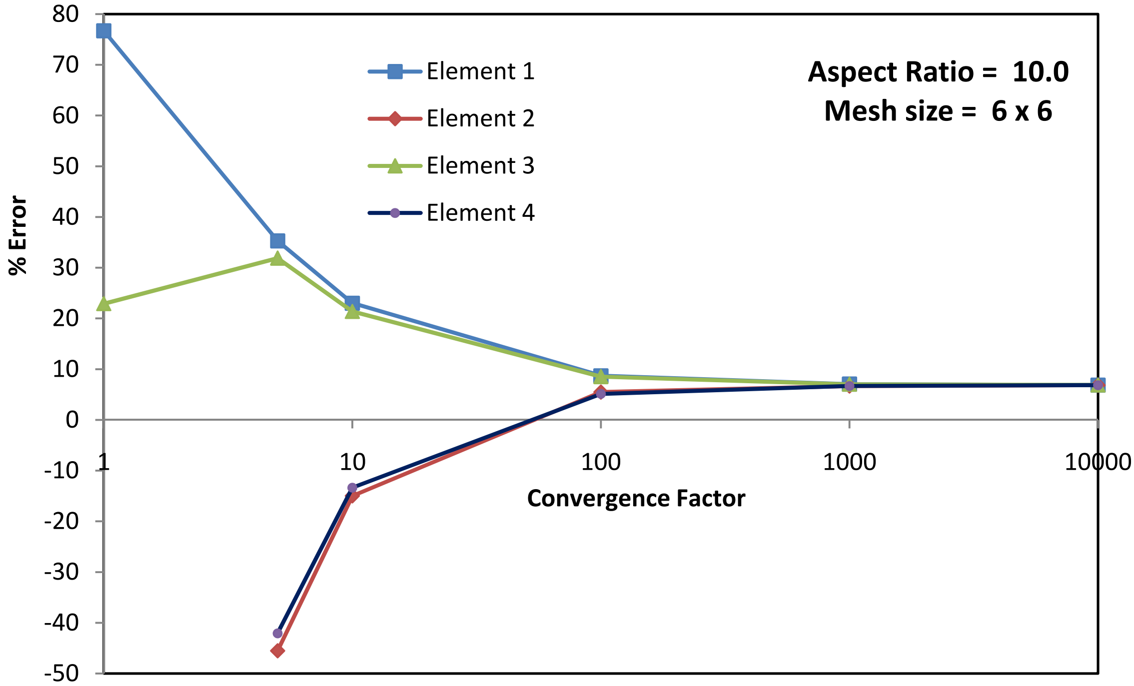


Fig-3_SSP_AR_10_NEL_6x6

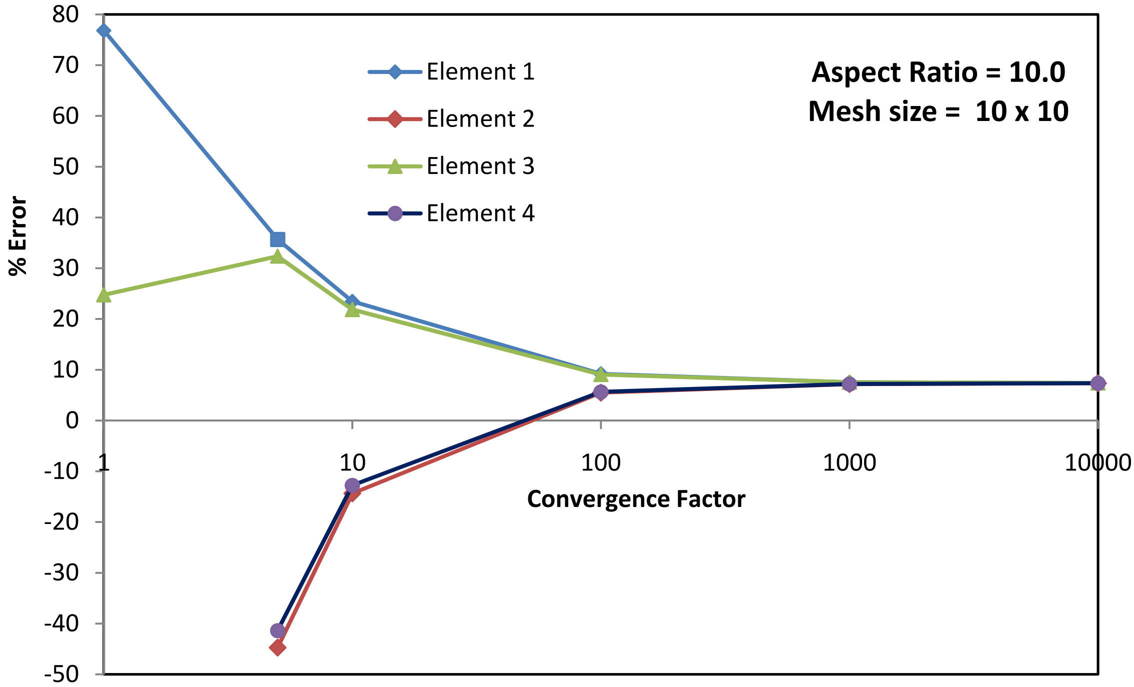


Fig-4_SSP_AR_10_NEL_10x10

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 20

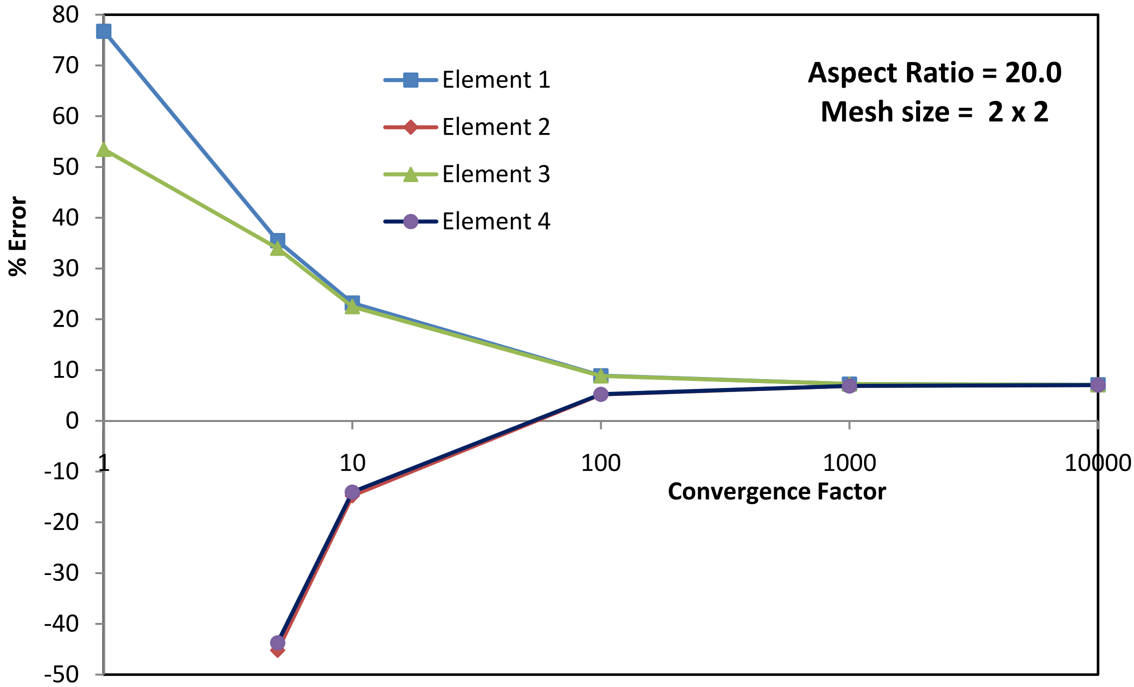


Fig-1_SSP_AR_20_NEL_2x2

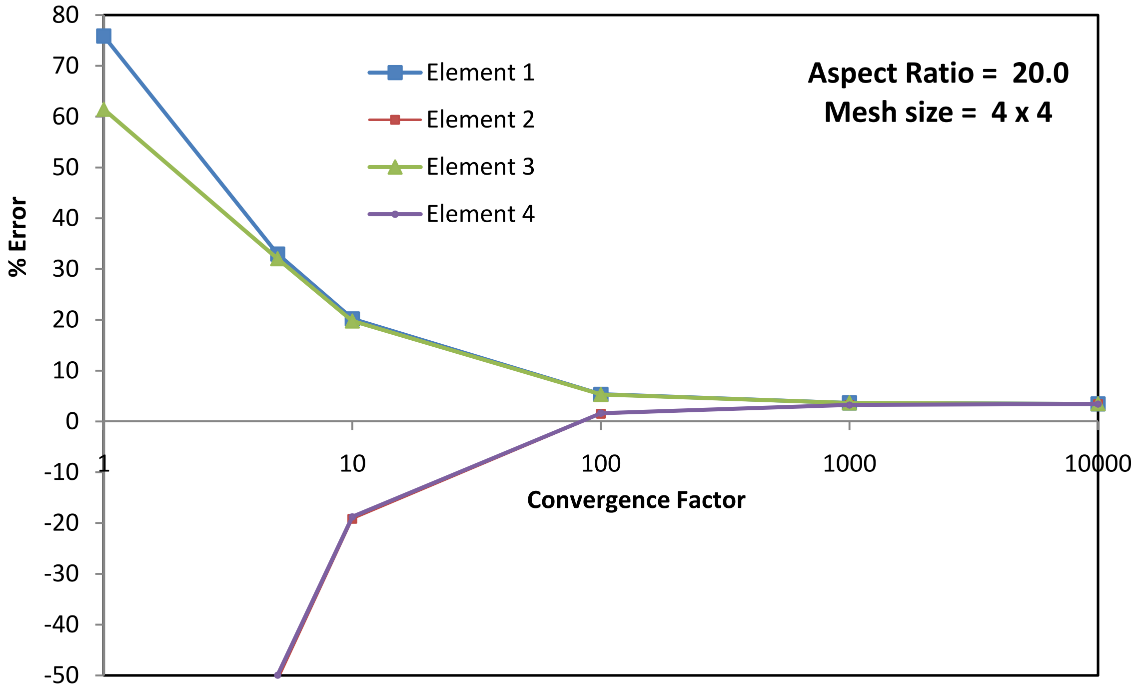


Fig-2_SSP_AR_20_NEL_4x4

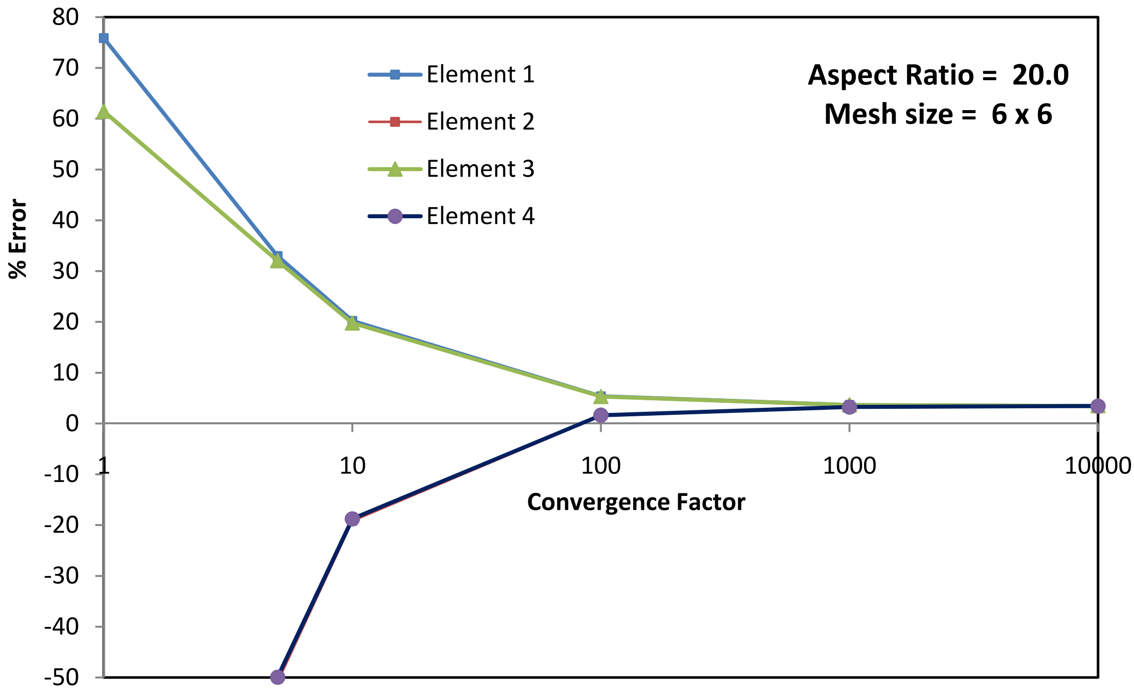


Fig-3_SSP_AR_20_NEL_6x6

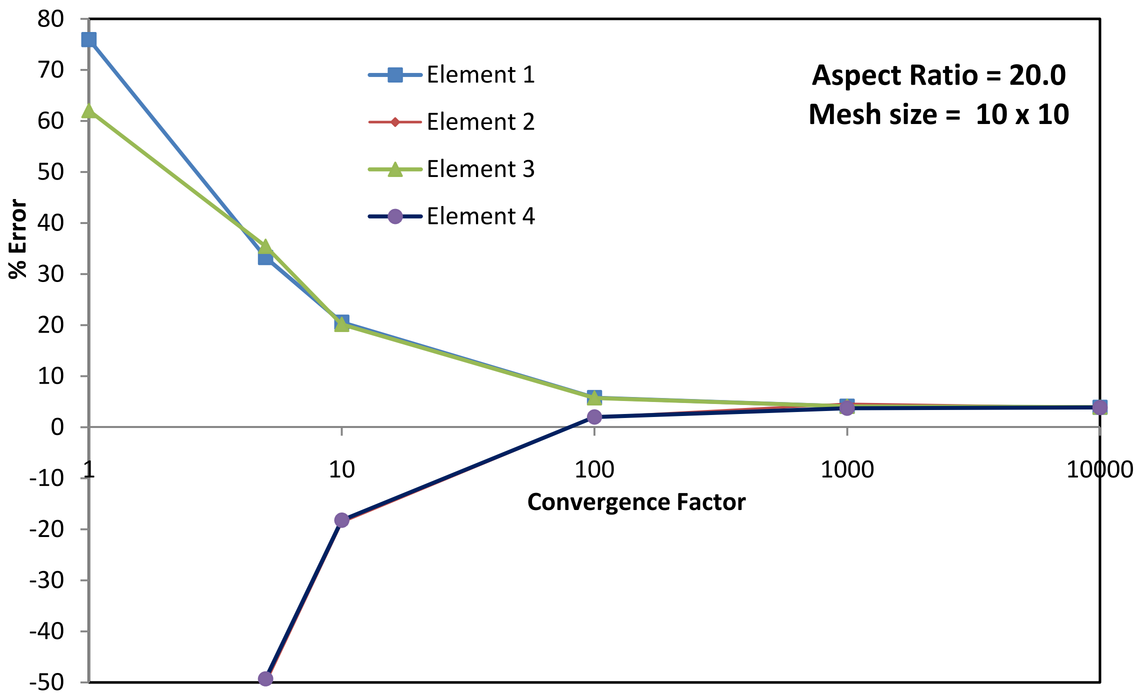
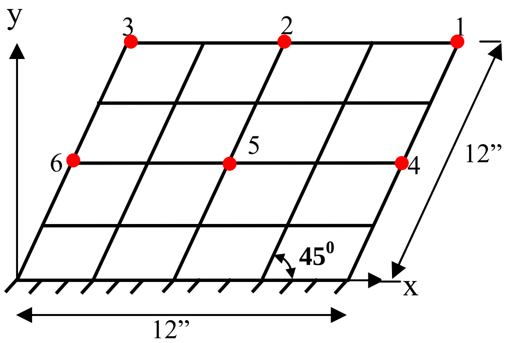


Fig-4_SSP_AR_20_NEL_10x10

Rhombic Plate under UDL

E=10.5e5 psi
υ=0.3
h=0.125 in
q=0.25066 psi

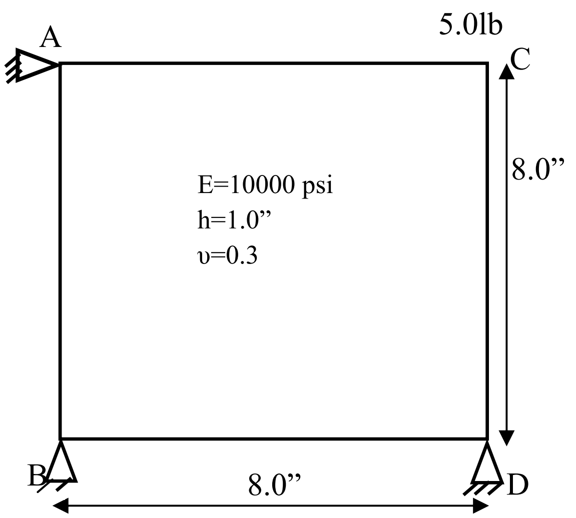


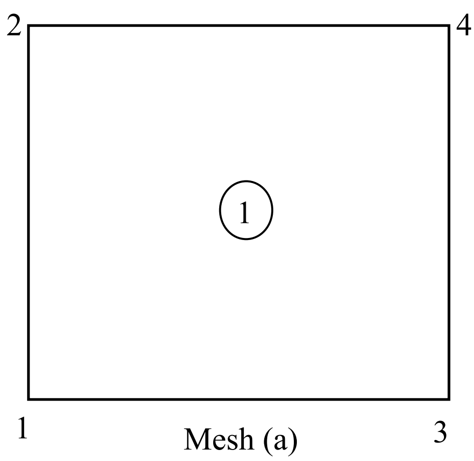
Element	Mesh	DOF	1	2	3	4	5	6
MR_FE_1[1]	4 x 4	75	**0.3132**	**0.2059**	**0.1144**	**0.1255**	**0.05239**	**0.02053**
MR_FE_2[1]	4 x 4	75	**0.3133**	**0.2010**	**0.1144**	**0.1256**	**0.05241**	**0.02054**
MR_FE_3[1]	4 x 4	75	**0.3132**	**0.2059**	**0.1144**	**0.1255**	**0.05239**	**0.02053**
MR_FE_4[1]	4 x 4	75	**0.3133**	**0.2060**	**0.1144**	**0.1256**	**0.05241**	**0.02054**
DKT[2]	4 x 4	75	**0.304**	**0.198**	**0.113**	**0.121**	**0.055**	**0.028**
HSM	4 x 4	75	0.264	0.173	0.100	0.095	0.043	0.021
ACM	8 x 6	189	0.296	0.198	0.114	0.114	0.052	0.020
HCT	8 x 6	189	0.281	0.188	0.111	0.111	0.049	0.018
Experimental Value			**0.297**	**0.204**	**0.121**	**0.129**	**0.056**	**0.022**

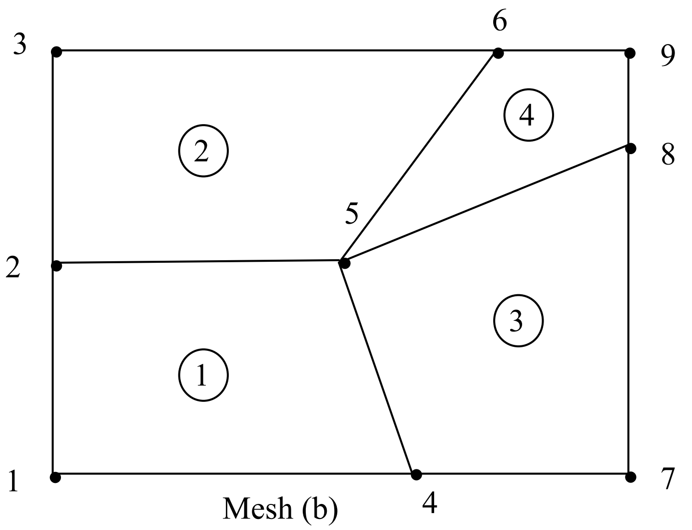
[1] **one point integration;** [2] Three point integration;

DKT – Discrete Krichhoff Theory element , Jean-Louis Batoz et al, A study of three-node triangular plate bending elements, international journal for numerical methods in engineering, Vol. 15, 1771-1812 (1980).

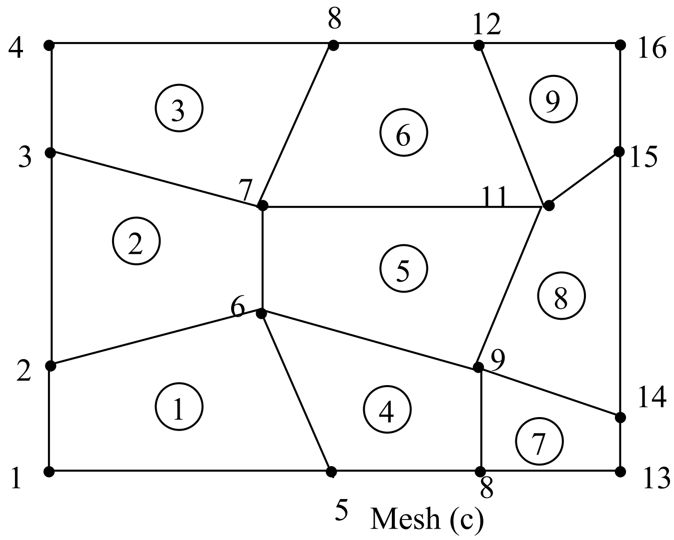
Twisting of a square plate







Mesh (b)



Mesh (c)

Mesh (b)

Node No	1	2	3	4	5	6	7	8	9
X-co	0	0	0	5	5	6.2	8	8	8
Y-co	0	4	8	0	4	8	0	6.2	8

Mesh (c)

Node No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
X-co	0	0	0	0	4	3	3	4	6	6	7	6	8	8	8	8
Y-co	0	2	6	8	0	3	5	8	0	2	5	8	0	1	5	8

Element Type	DKT	HSM	ACM	HCT	Mesh	MR_FE_1	MR_FE_2	MR_FE_3	MR_FE_4
Point C*	0.24960	0.24960	.24972	0.25002	a	0.25740	0.25740	0.25742	0.25739
					b	0.18963	0.18963	0.18945	0.19218
					c	0.19229	0.19229	0.19218	0.19239
0.24960 (Exact thin plate solution)									

* Three point integration

New Numerical algorithm

Currently I am developing an efficient numerical method (based on Newton-Raphson method) to find out the roots of real-valued functions in one variable. The preliminary results for some functions are given below and the refinement of this work is going on. The time achievement is also being examined.

Polynomial functions

* -Two roots can be obtained at the same time. In this case both are the same.

1. Function: $x^3 - 3x^2 - 9x + 27 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
25000	3.000000000	296	-3.000000000	7	-3.000000000	7
15000	2.999999999	623	-3.000000000	7	-3.000000000	7
10000	2.999999999	458	-3.000000000	7	-3.000000000	7
7000	3.000000000	322	-3.000000000	7	-3.000000000	7
500	2.999999999	292	-3.000000000	7	-3.000000000	7
10	3.000000000	122	-3.000000000	7	-3.000000000	7
1	3.000000000	680	---------	--------	3.000000000	126,7
-1	-----	-----	-3.000000000	6,11	------	-----
-5	-3.000000000	6	-3.000000000	7	-3.000000000	7,9
-600	-3.000000000	18	-3.000000000	7	-3.000000000	19,19
-9000	-3.000000000	25	-3.000000000	7	-3.000000000	26,26
-12500	-3.000000000	26	-3.000000000	7	-3.000000000	27,27
-26000	-3.000000000	27	-3.000000000	7	-3.000000000	28,28

2. Function: $x^5 - 11x^4 + 46x^3 - 90x^2 + 81x - 27 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
98765	3.000009907	1475	1.000000000	2,2	1.000000000	635
3995	2.999989503	1251	1.000000000	2,2	1.000000000	635
987	--------	---------	1.000000000	2,2	1.000000000	635
195	2.999982054	1233	1.000000000	2,2	1.000000000	635
-95	0.999999999	547	1.000000000	2,2	1.000000000	198
-987	1.000000009	1522	1.000000000	2,2	0.999999999	70
-9865	0.999999999	368	1.000000000	2,2	1.000000013	326
-98765	1.000000012	95	1.000000000	2,2	1.000000020	94

3. Function: $x^3 - x - 1.0 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
126779	1.324717957	33	1.324717957	5,24	1.324717957	6,17
87765	1.324717957	33	1.324717957	5,24	1.324717957	6,17
8725	1.324717957	28	1.324717957	5,24	1.324717957	6,17
675	1.324717957	26	1.324717957	5,24	1.324717957	6,17
65	1.324717957	15	1.324717957	5,24	1.324717957	6,17
1	1.324717957	6	1.324717957	5,24	1.324717957	6,17
-1	1.324717957	22	1.324717957	5,24	1.324717957	15,19
-107	1.324717957	21	1.324717957	5,24	1.324717957	37,42
-9008	1.324717957	48	1.324717957	5,24	1.324717957	42,50
-77864	1.324717957	77	1.324717957	5,24	1.324717957	62,69

4. Function: $x^3 - 2x^2 - 11x - 12 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
987432	4.000000000	37	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
87546	4.000000000	31	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
576	4.000000000	19	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
61	4.000000000	13	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
1	1.000000000	1	4.000000000 1.000000000	1 10	------	----
-1	4.000000000	2	-3.000000000 1.000000000	6 6	-3.000000000 1.000000000	19 5
-68	-3.000000000	13	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
-743	-3.000000000	19	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
-56732	-3.000000000	30	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
-487472	-3.000000000	35	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5

5. Function $x^3 - 2x + 2 = 0$ **(Special case)**

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
874531	-1.769292354	67	-1.769292354	6,17	----	----
34561	-------	--------	-1.769292354	6,17		
562	-1.769292354	107	-1.769292354	6,17		
43	------	---	-1.769292354	6,17		
2	-1.769292354	10	-1.769292354	6,17		
-3	-1.769292354	7	-1.769292354	6,17		
-71	-1.769292354	14	-1.769292354	6,17		
-8934	-1.769292354	27	-1.769292354	6,17		
-76987	-1.769292354	31	-1.769292354	6,17		

6. Function: $x^2 - 612.0 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
345698	24.7386337537	19	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
14567	24.7386337537	14	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
456	24.7386337537	9	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
5	24.7386337537	8	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
-7	-24.7386337537	7	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	9 9
-654	-24.7386337537	10	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
-19654	-24.7386337537	15	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	8 8
-786542	-24.7386337537	20	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	11 11

7. Function: $(x-1)^3 + 0.512 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
99998.5	1.800000012	34	1.800000012	9,13	1.800000012	17,10
12675.5	1.800000012	29	1.800000012	9,13	1.800000012	17,10
175.5	1.800000012	19	1.800000012	9,13	1.800000012	17,10
1.5	1.800000012	7	1.800000012	9,13	1.800000012	17,7
-0.5	1.800000012	8	1.800000012	5,9	1.800000012	12,20
-4.5	1.800000012	14	1.800000012	5,9	1.800000012	19,19
-453.5	1.800000012	21	1.800000012	5,9	1.800000012	23,25
-15673.5	1.800000012	34	1.800000012	5,9	1.800000012	35,35
-78432.5	1.800000012	38	1.800000012	5,9	1.800000012	39,39

8. Function: $x^3 - x^2 - 10x - 8 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
876234	4.000000000	36	-1.000000000 -2.000000000	6 11	-1.000000000 -2.000000000	1 8
34672	4.000000000	28	-1.000000000 -2.000000000	6 11	-1.000000000 -2.000000000	1 8
764	4.000000000	19	-1.000000000 -2.000000000	6 11	-1.000000000 -2.000000000	1 8
1	-1.000000000	1	-1.000000000 4.000000000	7 8	-1.000000000 -2.000000000	1 8
-1	-1.000000000	1	-1.000000000 -2.000000000	1 7	-1.000000000 -2.000000000	1 8
-467	-2.000000000	20	-1.000000000 -2.000000000	6 11	-2.000000000	21,21
-6542	-2.000000000	26	-1.000000000 -2.000000000	6 11	-2.000000000	27,27
-87321	-2.000000000	33	-1.000000000 -2.000000000	6 11	-2.000000000	34,34

9. Function: $x^3 - 3x^2 + 4 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500000	2.000000000	90	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	6 28
73256	2.000000000	55	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	7 28
235	2.000000000	41	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	7 28
5	2.000000000	76	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	7 28
-10	-1.000000000	10	-1.000000000 2.000000000	6 28	-1.000000000 -1.000000000	11 12
-563	-1.000000000	20	-1.000000000 2.000000000	6 28	-1.000000000 -1.000000000	11 12
-96342	-1.000000000	32	-1.000000000 2.000000000	6 28	-1.000000000 -1.000000000	33 33

10. Function: $x^4 + 3x - 4 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
107890	1.000000000	45	1.000000000	2,8	1.000000000 -1.742959202	2 12
76543	1.000000000	45	1.000000000	2,7	1.000000000 -1.742959202	2 12
546	1.000000000	27	1.000000000	2,8	1.000000000 -1.742959202	2 12
3	1.000000000	9	1.000000000	2,8	1.000000000 -1.742959202	2 12
-2	-1.742959202	6	-1.742959202	5,32	-1.742959202	10,25
-4321	-1.742959202	32	-1.742959202	5,8	-1.742959202	34, >1500
-432678	-1.742959202	49	1.000000000	2,8	-1.742959202	50, >1500

11. Function: $x^3 - 0.165x^2 + 3.993e-04 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
100000	0.146359512	40	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
35678	0.146359512	38	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
453	0.146359512	27	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
1.0	0.146359512	27	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
-15	-4.373708626E-02	19	6.237758018E-02 -4.373708626E-02	5 6	-4.373708626E-02	23,23
-6743	-4.373708626E-02	34	6.237758018E-02 -4.373708626E-02	5 6	-4.373708626E-02	23,23
-98456	-4.373708626E-02	40	6.237758018E-02 -4.373708626E-02	5 6	-4.373708626E-02	35,35

12. Function: $x^3 - 0.03x^2 + 2.4e-04 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
100000	2.661081797E-02	45	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
67543	2.661081797E-02	44	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
723	2.661081797E-02	32	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
2	2.661081797E-02	18	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
-20	-7.952197683E-03	18	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	36 20
-67845	-7.952197683E-03	43	-7.952197642E-03	6,13	-7.952197683E-03	44
-120007	-7.952197683E-03	43	-7.952197642E-03	6,13	-7.952197683E-03	44

13. Function: $2x^3 + 3.5x^2 - 4.21x + 0.546 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
564327	0.700000007	38	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
5637	0.700000007	28	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
74	0.700000007	17	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
0	0.149999999	5	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
-13	-2.600000005	10	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
-2451	-2.600000005	23	0.700000006 0.149999999	8 5	-2.600000005	24,25
-123867	-2.600000005	33	0.700000006 0.149999999	8 5	-2.600000005	24,25

14. Function: $0.004x^3 - 0.2x^2 + 2.45x - 5.4 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
34562	32.363031067	20	32.363031067 2.814202526	12 5	2.814202526	6,8
6534	32.363031067	21	32.363031067 2.814202526	12 5	2.814202526	6,8
31	32.363031067	5	32.363031067 14.822764775	4 6	32.363031067	5,8
0	2.814202526	6	2.814202526 32.363031067	5 12	32.363031067	5,8
-10	2.814202526	8	2.814202526 32.363031067	5 12	2.814202526	8,9
-5678	2.814202526	21	2.814202526 32.363031067	5 12	2.814202526	8,9
-26524	2.814202526	21	2.814202526 32.363031067	5 12	2.814202526	8,9

Transcendental functions

1. Function: $e^{-x} - x = 0$

	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
100000	0.567143290	6	0.567143290	6,7	0.567143290	6,7
8525	0.567143290	6	0.567143290	5,7	0.567143290	6,9
525	0.567143290	6	0.567143290	6,7	0.567143290	6,7
25	0.567143290	6	0.567143290	6,7	0.567143290	6,9
1	0.567143290	5	0.567143290	5,7	0.567143290	6,7
-1	0.567143290	6	0.567143290	5,7	0.567143290	7,10
-20	0.567143290	25	0.567143290	5,7	0.567143290	26,28
-105	0.567143290	110	0.567143290	5,7	0.567143290	111,113
-515	0.567143290	520	0.567143290	5,7	0.567143290	521,523

2. Function: $xe^{x} - 3.0 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500	1.049908899	500	1.049908899	4,16		
150	1.049908899	159	1.049908899	4,16		
11	1.049908899	18	1.049908899	4,16		
0	1.049908899	9	-------	----		
-1	----	-----	----	------		
-10			1.049908899	4,16		
-135			1.049908899	4,16		
-550			1.049908899	4,16		

3. Function: $e^x - 3x = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500	1.512134551	506	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
200	1.512134551	206	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
53	1.512134551	59	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
0	1.512134551	6	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
-24	1.512134551	7	------------	----------------	0.619061286 1.512134551	8 31
-2234	1.512134551	7	0.619061286 1.512134551	5 6	-------------------	----------------
-67456	1.512134551	7	0.619061286 1.512134551	5 6	------------	------------------

4. Function: $e^{x} + x - 2 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500	0.442854401	505	0.442854401	5,9	0.442854401	5,8
150	0.442854401	155	0.442854401	5,9	0.442854401	5,8
2	0.442854401	7	0.442854401	5,9	0.442854401	5,8
0	0.442854401	5	0.442854401	5,9	0.442854401	5,8
-75	0.442854401	8	0.442854401	6,7	0.442854401	5,8
-2350	0.442854401	8	0.442854401	5,9	0.442854401	9,9
-98754	0.442854401	8	0.442854401	5,9	0.442854401	9,9

Conclusion

1. The first algorithm is independent of initial guess.

2. It converges very faster than the other two, that is, the Newton-Raphson method and New Method-2.

3. The modifications carried out in the present two algorithms are very simple and the mathematical expressions used are very short statements involving only arithmetic operations.

4. The performance of the first algorithm is far better than that of the other two and in some cases the performance of the second algorithm is similar to that of the Newton-Raphson method.

5. In some cases, two different roots are simultaneously obtained by these two algorithms. In the case of the NR method, it is not possible.

Future Work

The above work is extended to the analysis of a system of non linear equations in ‘n’ variables to examine the performance of the algorithm.