February 7, 2007

ES 242r

Problem Set #1

Due by submission to website on February 22, 2007

1. **Post an entry to iMechanica to explain to your teaching staff and classmates why you are taking this class.**

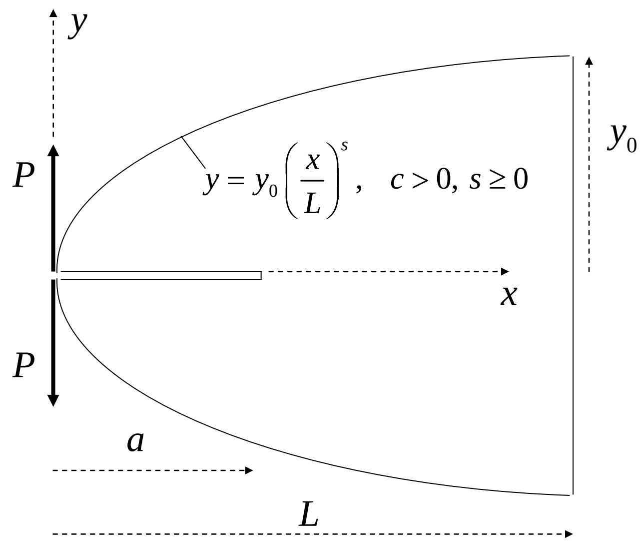
Please add the following 4 tags (case sensitive): ES 242r, fracture mechanics, Spring 2007, students. In this way the entry will be automatically delivered to the teaching staff and classmates via their RSS readers. Please avoid using any tags featured in the header of iMechanica.

Include the following items in your entry:

- Where you are currently enrolled (Harvard, MIT, Nebraska, others?)
- Your prior courses in solid mechanics.
- Your undergraduate major and where you where enrolled.
- What might be your strength and weakness related to this course.
- Your research group if you already belong to one (please give a link to the web page of your group).
- Your likely research direction(s).
- How do you think fracture mechanic will help you in your research?
- How do you think fracture mechanics will contribute to your education in general?

As you can imagine, there is no right or wrong answers to this assignment. You will be given credit for effort and thoughtfulness. As with all entries in iMechanica, you can edit it as many times as you like. You can also comment on each other's entries. The intention of this "problem" is for us to get to know each other better and to give the instructors a clearer sense of your background.

2. Suppose you want to design a mode I test specimen for fracture testing which has the property that K is independent of crack length a under prescribed load/thickness P. Such specimens are particularly useful in fatigue crack growth testing and in corrosion crack testing. Consider the family of specimens shown in the figure below. Use beam theory to estimate the compliance and derive the formulas for G and K. Then identify the value of s such that K is independent of a. (For beams of slowly varying thickness, $M = \bar{E}I(x)w''$, where $I(x)$ is the x-dependent moment of inertia.)



3. The small scale yielding curve of a moderately high strength aluminum alloy is given by

$$K_R(\Delta a) = K_C\left(1 + \frac{\Delta a}{d}\right)$$

where $K_C = 30 MPa\,m^{1/2}$ and $d = 5mm$. Assume small scale yielding applies and analyze the history of crack length, L, versus applied load (σ^∞ or P), including the instability point, for each of the two following problems.

(a) A crack of length $L = 2a$ in an infinite sheet or plate loaded remotely by σ^∞ normal to the crack. Assume the initial crack length is $L_0 = 20mm$.

(b) A crack of length L in a double cantilever beam specimen loaded by load/thickness, P. The initial length of the crack is again $L_0 = 20mm$ and the height of each beam arm is $5mm$.

Present the results in the form of plots. In part (a), what is the maximum value of the initial crack length, L_0, such that the crack would be unstable at the first increment of crack growth? In part (a) give a rough estimate of the minimum yield stress such that the history can sensibly regarded as small scale yielding.

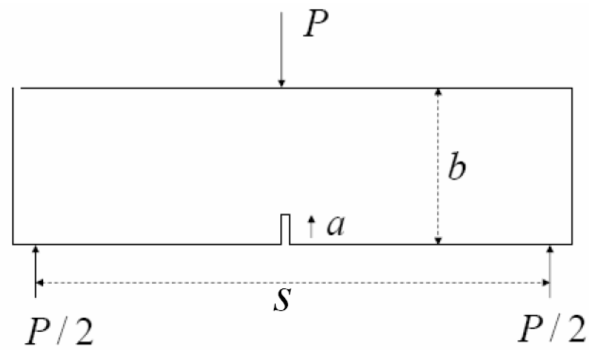
4. We have derived results for the energy release rate and the stress intensity factor by using results for the compliance in terms of the crack length. The process can be reversed. That is, given a result for the stress intensity factor in terms of crack length, one can derive the compliance in terms of the crack length. Consider the 3-point bend specimen shown, and derive a formula for the compliance, $C = \Delta / P$, for $s/b = 4$ given the following (taken from Tada, et al.).

$$K = \sigma\sqrt{\pi a}F(a/b), \quad \sigma = 6M/b^2, \quad M = Ps/4$$

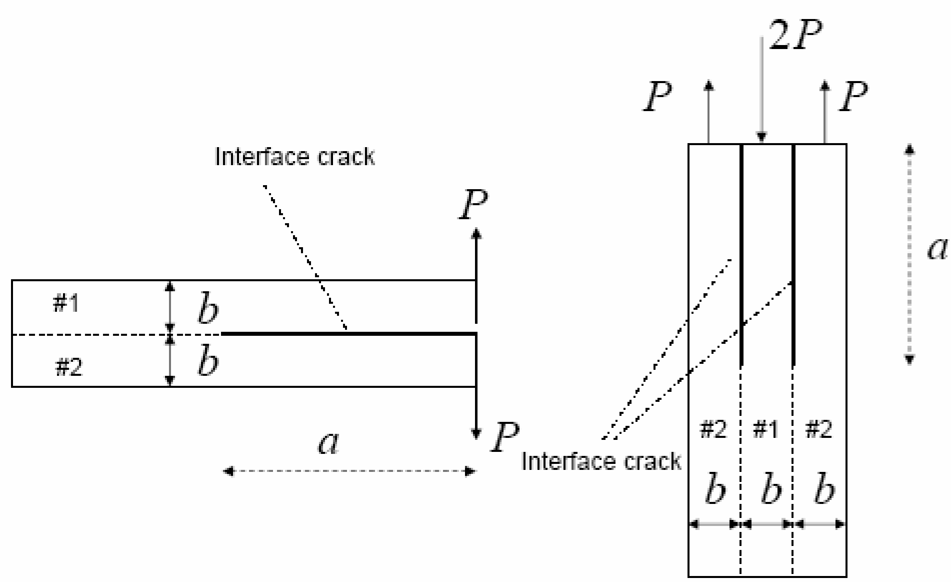
where, for $s/b = 4$,

$$F(x) = \frac{1}{\sqrt{\pi}} \frac{1.99 - x(1-x)(2.15 - 3.93x + 2.7x^2)}{(1+2x)(1-x)^{3/2}} \quad (x = a/b)$$

(Tada, et al. claim $F(x)$ is accurate to within 0.5% for all x.) The material is isotropic. Plot $(C - C_0)\bar{E}$ as a function of a/b where C_0 is the compliance for $a/b = 0$.



5. An important problem in the design of composites is the debonding (or delamination) by cracking along the interface between two materials or laminates. Consider the double-cantilever specimen depicted below where material 1 has Young's modulus and Poisson's ratio, E_1 and ν_1, and material 2 has E_2 and ν_2. Derive the energy release rate, G, for crack advance along the interface. (Hint, as in class, use beam theory to compute the compliance.)



6. As a second example of interface fracture, consider the shear-type specimen above made of two different isotropic materials. Assuming the two cracks advance together along the interface, determine G (per crack tip) by assuming that each arm is long ($a >> b$) and that it is in either uniform tension or compression away from the tips.