Relation Between Bending Moment, Torsion Moment and Inertia Moments for a Beam

The Fundamental Law of Rotational Dynamics known as Kinematic Moment Theorem is

$$\sum \vec{M}_{ext}(A) = \vec{\delta}^{i}\,(A)$$

where

$$\vec{M}_{ext}(A) = \begin{cases} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{cases}$$

M_{Ay} and M_{Az} are the Bending Moments
and M_{Ax} is the Torsion Moment
$\vec{\delta}^{i}(A)$ is the Dynamic Moment with Respect to $R_i(A, \vec{x}_A, \vec{y}_A, \vec{z}_A)$
For a Symmetric General Beam Loaded in Bending in $(XAZ) -$ Plane and $(XAY) -$ Plane and Torsion along (AX) axis

$$\vec{\delta}^{i}(A) = \frac{d^{i}\left(\vec{\mu}^{i}(A)\right)}{dt} + \vec{V}^{i}(A) \wedge m\,\vec{V}^{i}(G)$$

If $A = G$ or $\vec{V}^{i}(A) = \vec{0}$ then $\vec{V}^{i}(A) \wedge m\,\vec{V}^{i}(G) = \vec{0}$:
The Point A Must Belong to the Solid Beam in these Formula

$$\vec{\mu}^{i}(A) = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_y \end{Bmatrix} = \begin{Bmatrix} I_x\,\dot{\theta}_x \\ I_y\,\dot{\theta}_y \\ I_z\,\dot{\theta}_z \end{Bmatrix}$$

$$\dot{\theta}_k = \frac{d\theta_k}{dt}$$

$I_x = J = \frac{1}{2}mR^2$, $I_y = \frac{1}{4}mR^2$ and $I_z = \frac{1}{4}mR^2$ for a Circular Beam with a Constant Section
If the Derivative is in the same Projection Reference One Gets

$$\vec{\delta}^{i}(A) = \begin{Bmatrix} I_x\,\ddot{\theta}_x \\ I_y\,\ddot{\theta}_y \\ I_z\,\ddot{\theta}_z \end{Bmatrix}$$

$$\ddot{\theta}_k = \frac{d\dot{\theta}_k}{dt}$$

The Differential Equations are then $M_{Ax} = I_x\,\ddot{\theta}_x$, $M_{Ay} = I_y\,\ddot{\theta}_y$ and $M_{Az} = I_z\,\ddot{\theta}_z$