

Numerical Results for shear-lock free finite elements based on Mindlin-Reissner plate and Timoshenko beam theories

Plate Elements

| | |
|--|--|
| N x N | Mesh Size (Full Plate) |
| Exact Solution | Srinivasa Rao and AK Rao and Theory of Plates and Shells |
| CF | Convergence Factor |
| ASR | Aspect Ratio |
| MR_FE_1 MR_FE_2 MR_FE_3 MR_FE_4 | Finite Elements based on Mindlin-Reissner theory using new shape functions |
| Material: | $E=1.092E06$; $\nu = 0.3$; |
| Geometry: | Square plate $a=10.0$; $h=2.0, 1.4, 1.0, 0.5$; |
| Type of plate | Simply supported (SS) |
| Load: | UDL=10.0 |

Timoshenko beam elements

| | |
|--|---|
| N | Number of Elements (Full beam) |
| Exact Solution | Theory of Elasticity |
| CF | Convergence Factor |
| ASR | Aspect Ratio |
| Timo_FE_1 Timo_FE_2 Timo_FE_3 Timo_FE_4 | Finite Elements based on Timoshenko beam theory using new shape functions |
| Material: | 29,000; $\nu = 0.3$; |
| Geometry: | $L=5, 10, 25, 100, 200, 400$; $h=1.0$; $b=1.0$; |
| Load: | Concentrated Load $q=100.0$ and UDL=10.0 |

Higher order beam elements (capable of accurately predicting three dimensional stresses) based on the higher order shear deformation theories developed by me

| | |
|------------|---------------------------------|
| HFE_1 - | based on Lagrangian polynomials |
| HFE_3 - | based on Lagrangian polynomials |
| FE_NSF_1 - | based on new shape functions |
| FE_NSF_3 - | based on new shape functions |

Advantages of the present new finite elements

1. The primary aim of this research work is to replace the finite elements based on the first order shear deformation theory available in the general purpose finite element packages by these new finite elements.
2. Presently all general purpose finite element packages like MSC-NASTRAN, NX- NASTRAN, ABAQUS, LS-DYNA, and ANSYS use the finite elements based on Timoshenko beam theory, Reissner-Mindlin plate theory and Kirchhoff-Love shell theory (all are called first order shear deformation theories). The drawback of these finite elements is that they can not be used for the analysis of thin structures due to shear lock problem. To eliminate this problem, special integration scheme must be used.

The new finite elements based on the Timoshenko beam theory, Reissner-Mindlin Plate theory developed by me using special shape functions and standard finite element procedure are applied to the analysis of beams and plates. The numerical results show that accurate solution is obtained for less number of elements. The specialties of these finite elements are that (i) thick and thin structures can be analyzed, (ii) No special integration scheme is required, (iii) a new concept, Convergence Factor (CF), is introduced in the formulation of these elements to accelerate convergence, and (iv) Accurate solution is obtained by keeping the number of elements as constant and increasing the value of the CF. Hence, this procedure reduces modelling effort, computational time and increasing accuracy.
3. A comparison study was carried out among the finite elements based on the new shape functions and Lagrangian shape functions using two higher order shear deformation theories developed by me for the analysis of simply supported beam under transverse load. The convergence is achieved faster than that of the finite elements based on the Lagrangian shape functions.
4. Considering the above points, study related to the development of triangular finite elements based on the new shape functions is in progress.
5. Compare to the development of other thin plate finite elements like Discrete Kirchhoff Theory (DKT) finite element and Mixed Interpolated Tensorial Component (MITC4) element, the development of these finite elements is very simple.
6. Since these new shape functions are very effective in accurately predicting displacements, strains and stresses, finite elements based on these shape functions can be developed for various applications, for example, multiscale modelling, crack propagation and gradient-enhanced damage models.
7. It satisfies Partition of unity condition.
8. It satisfies Kronecker Delta condition and hence imposition of essential boundary condition is not a problem.
9. The last two conditions can not be achieved in Isogeometric Analysis.

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 5

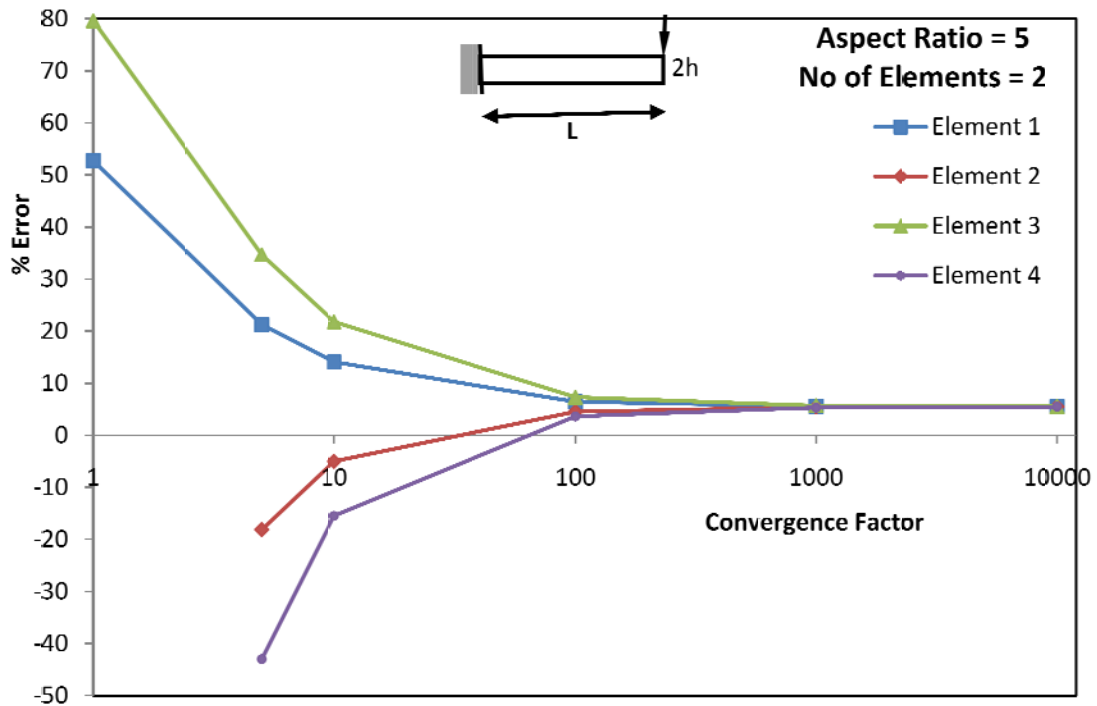


Fig-1_CB_AR_5_NEL_2

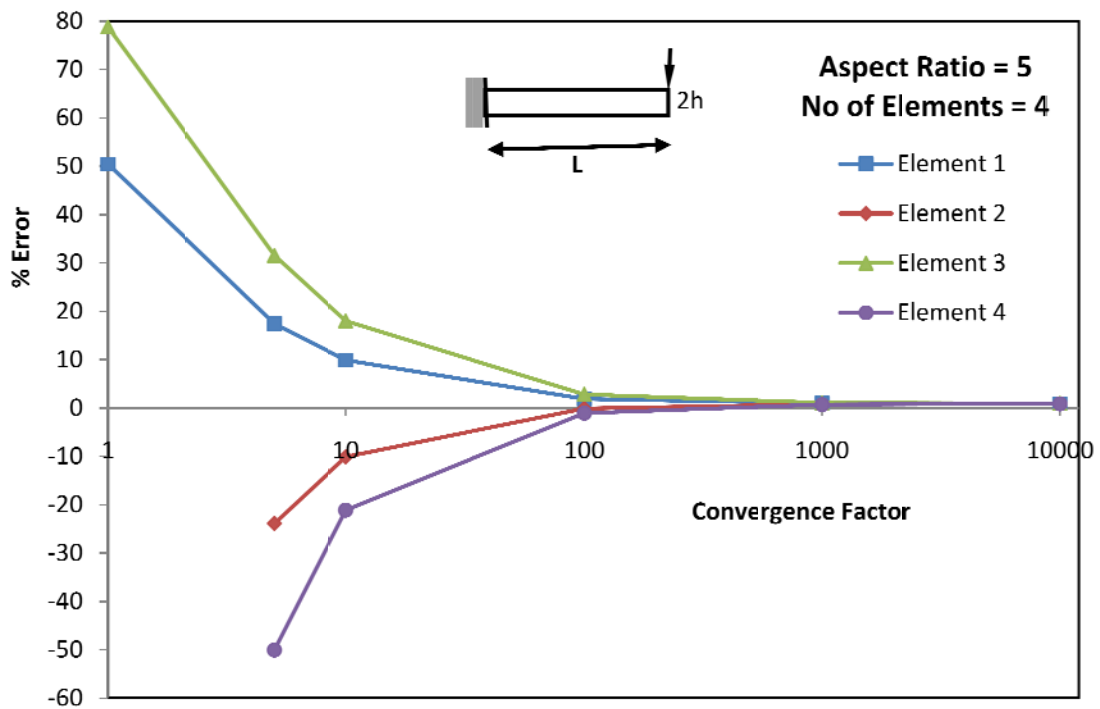


Fig-2_CB_AR_5_NEL_4

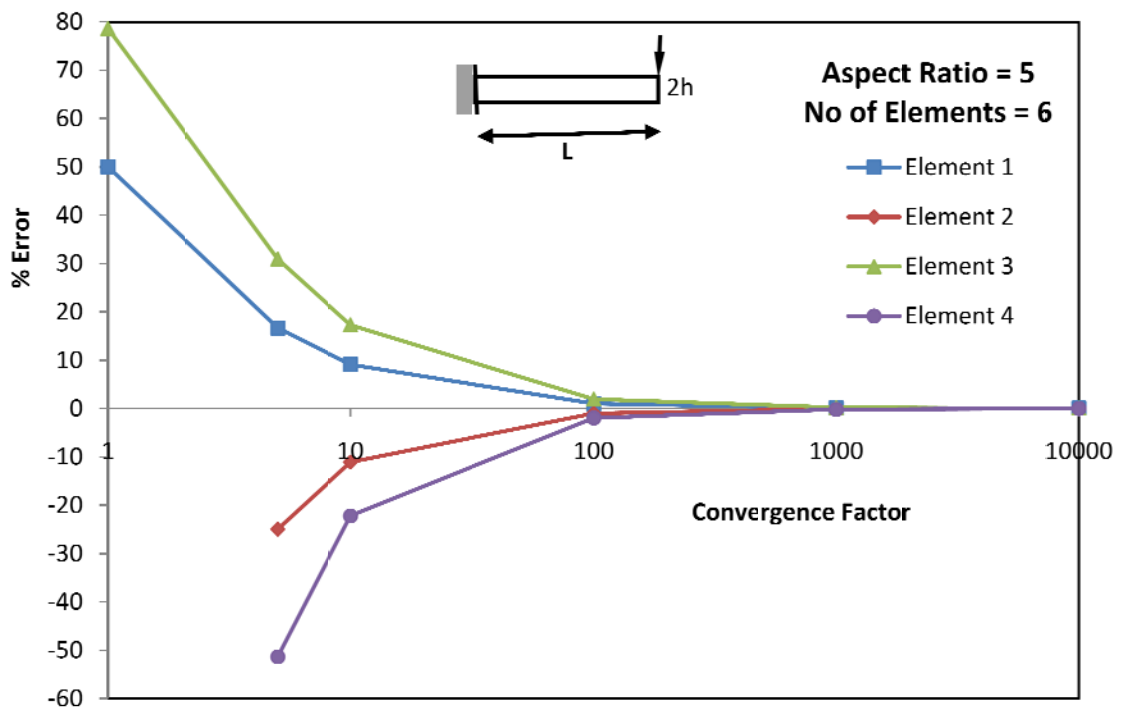


Fig-3_CB_AR_5_NEL_6

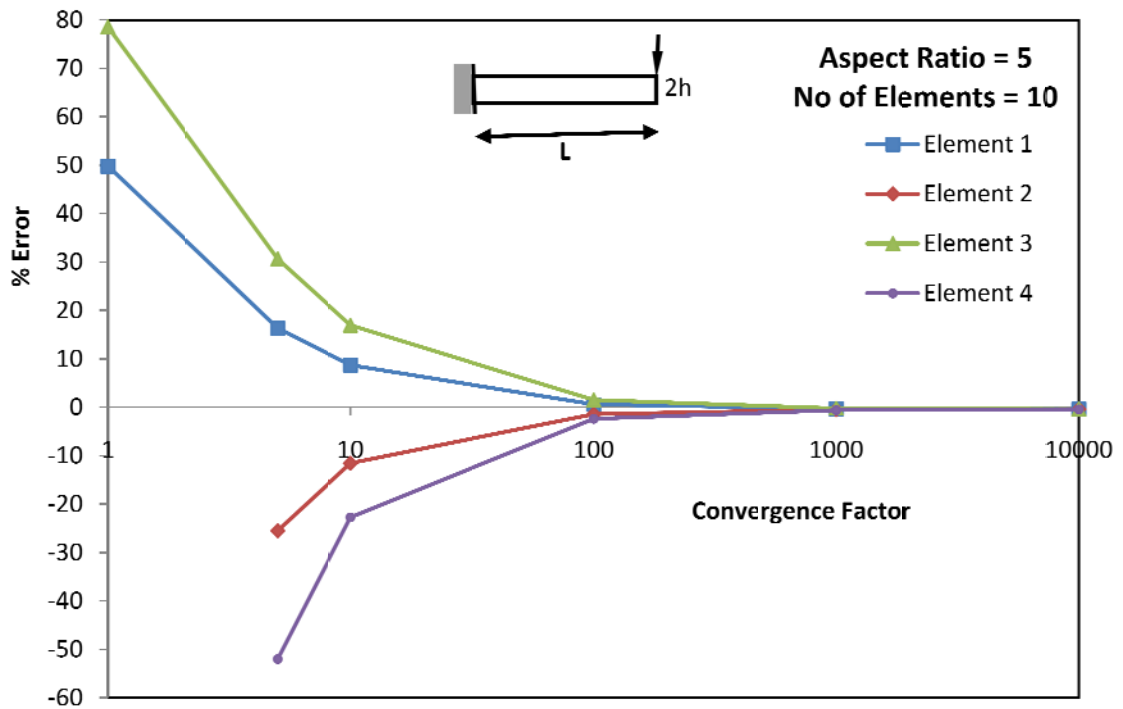


Fig-4_CB_AR_5_NEL_10

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 10

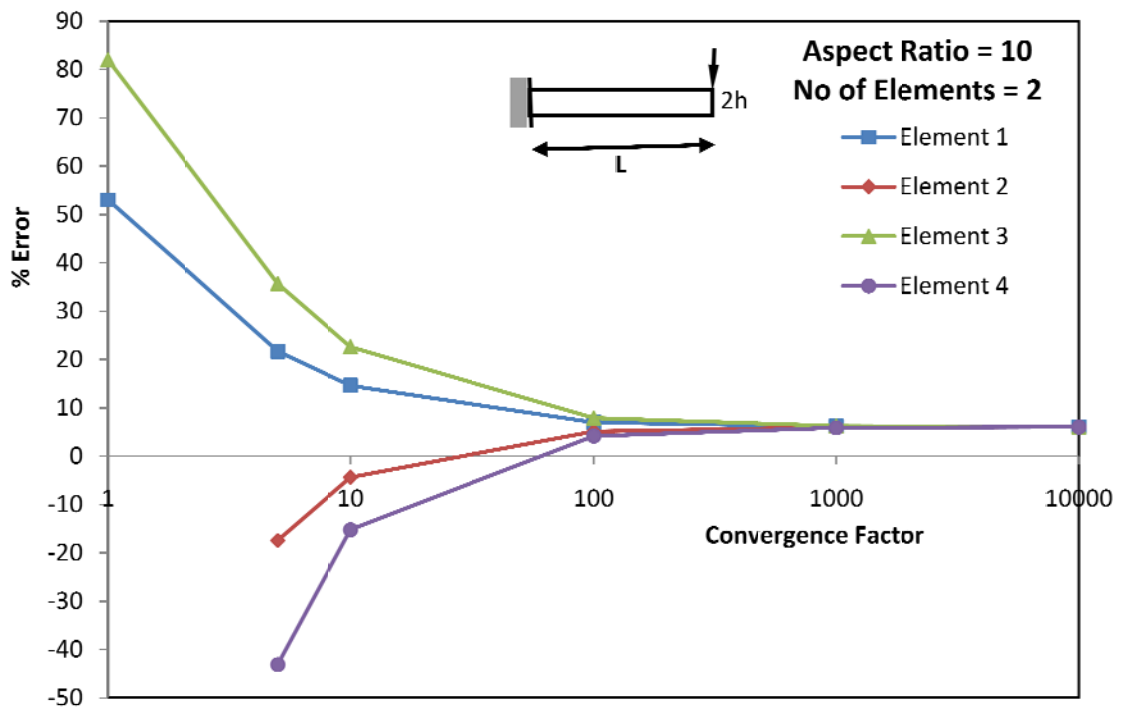


Fig-1_CB_AR_10_NEL_2

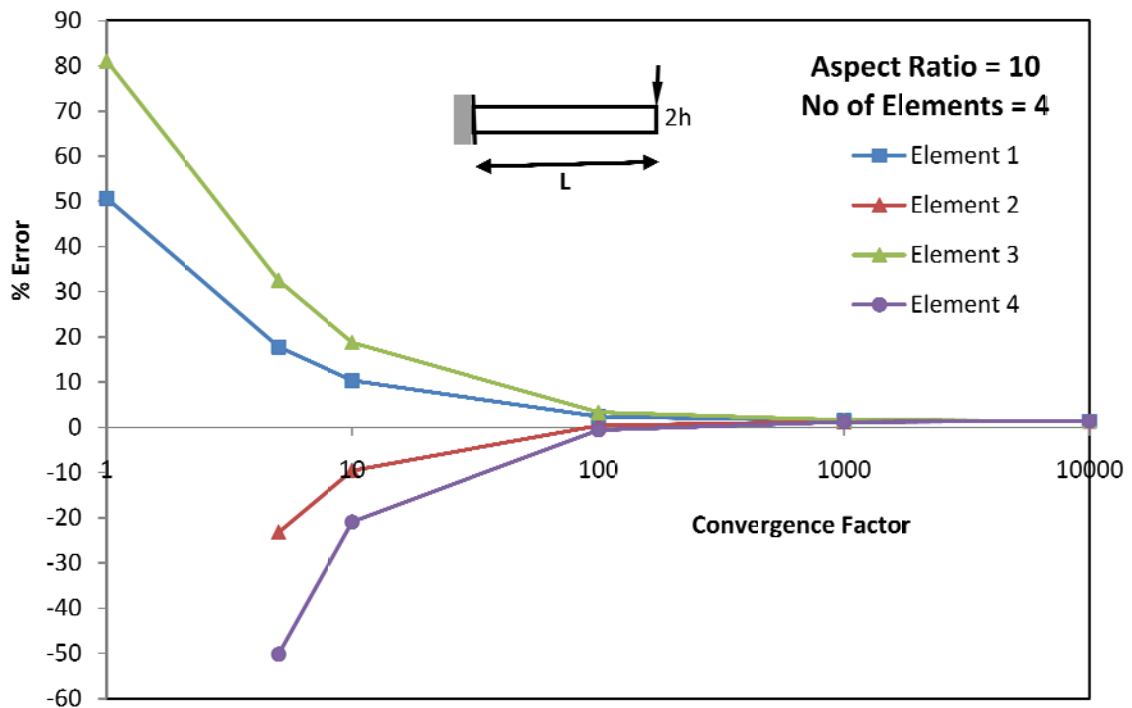


Fig-2_CB_AR_10_NEL_4

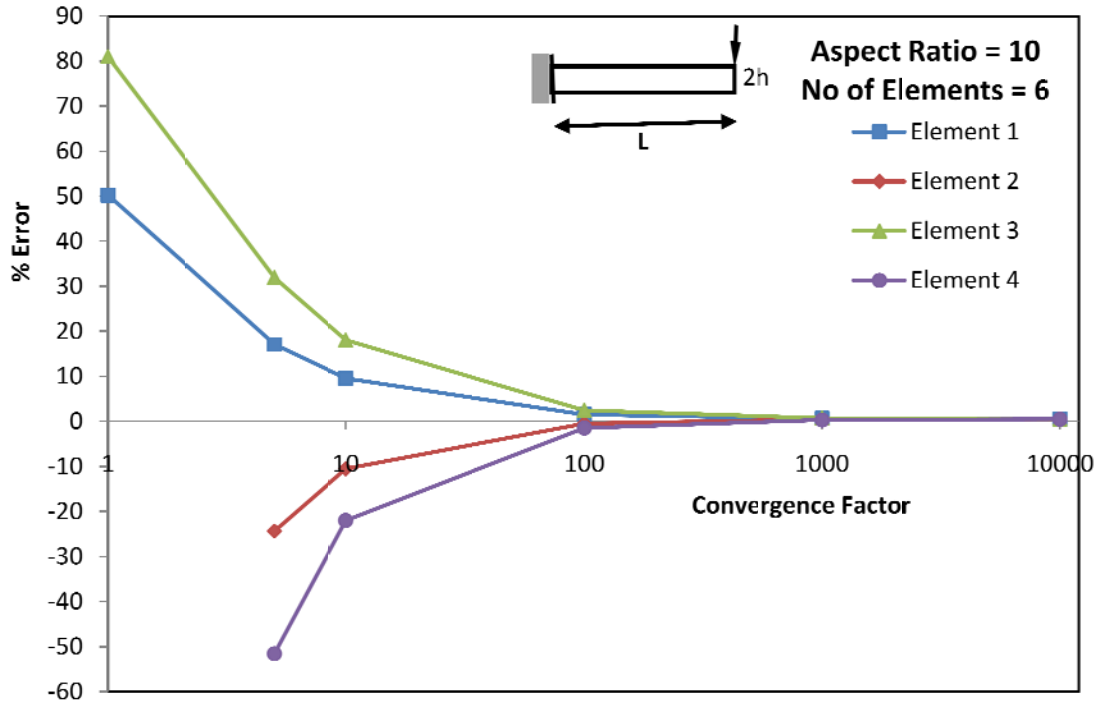


Fig-3_CB_AR_10_NEL_6

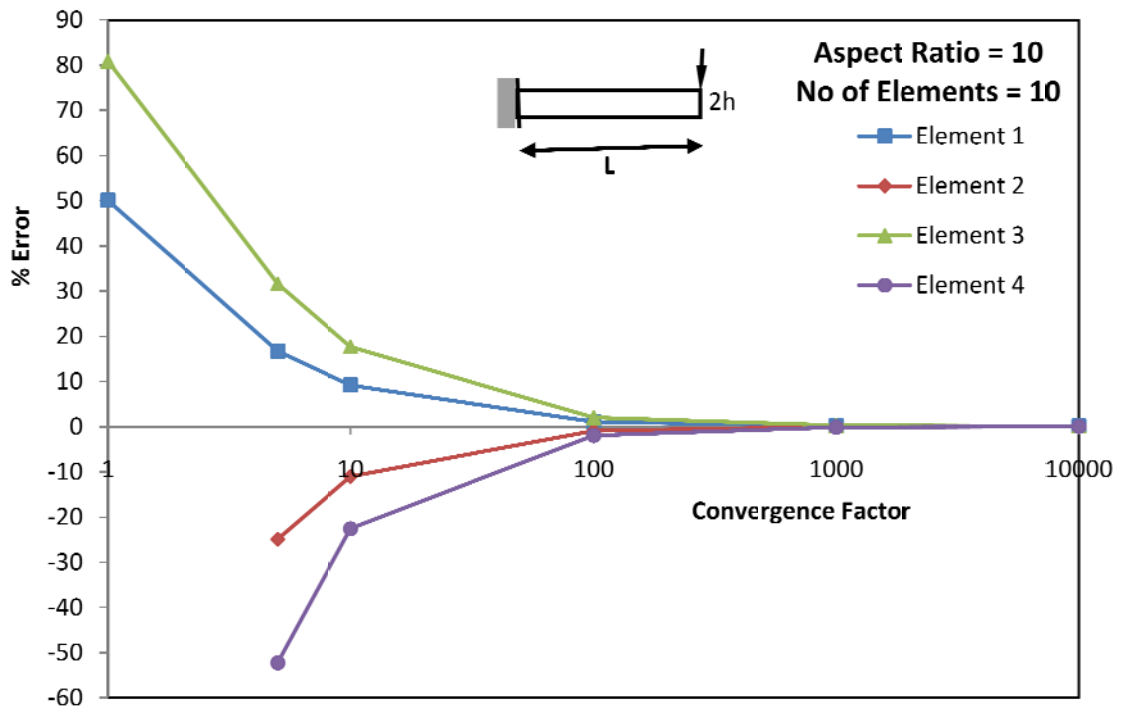


Fig-4_CB_AR_10_NEL_10

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 100

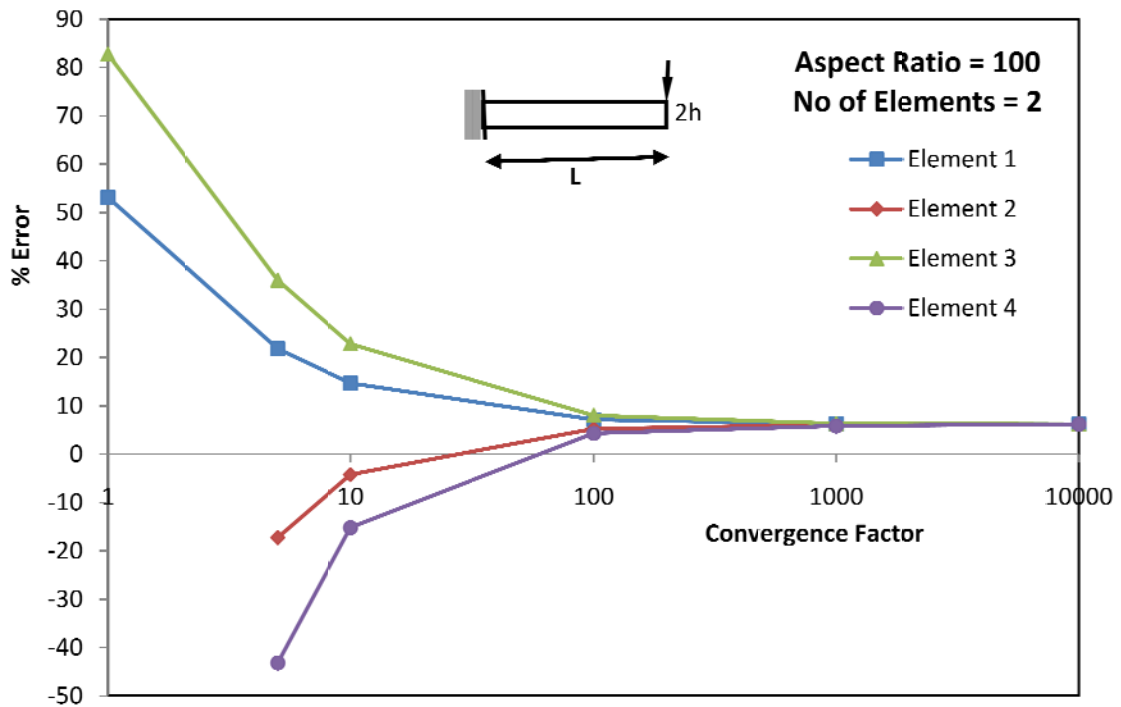


Fig-1_CB_AR_100_NEL_2

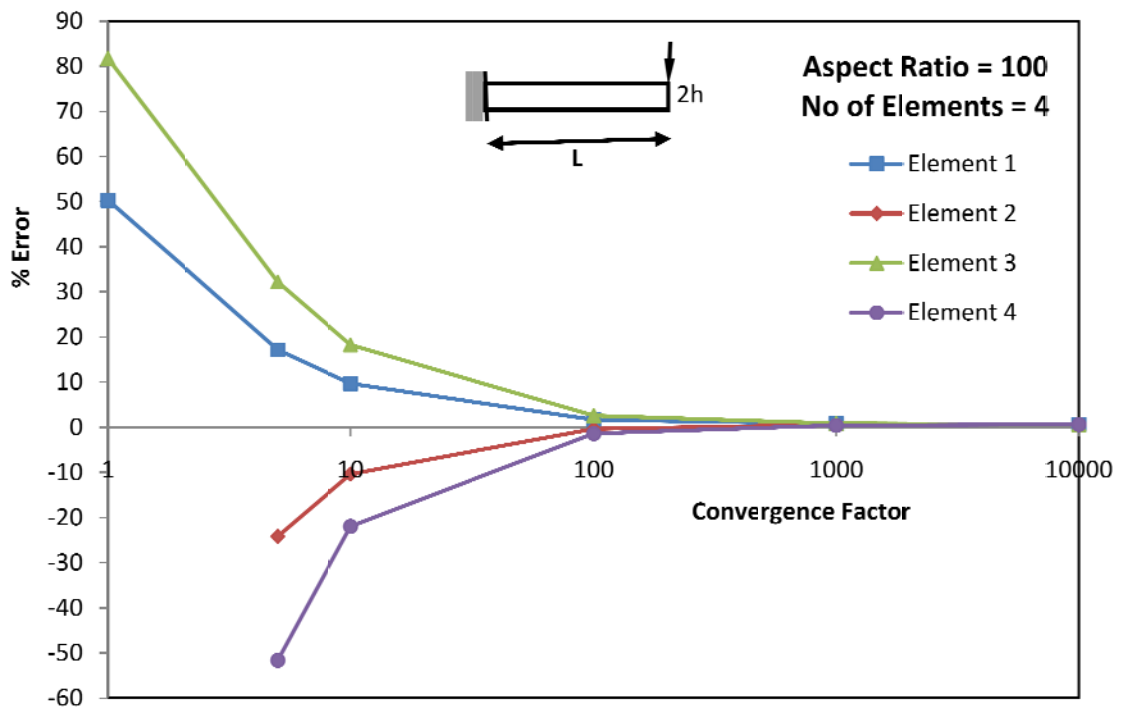


Fig-2_CB_AR_100_NEL_4

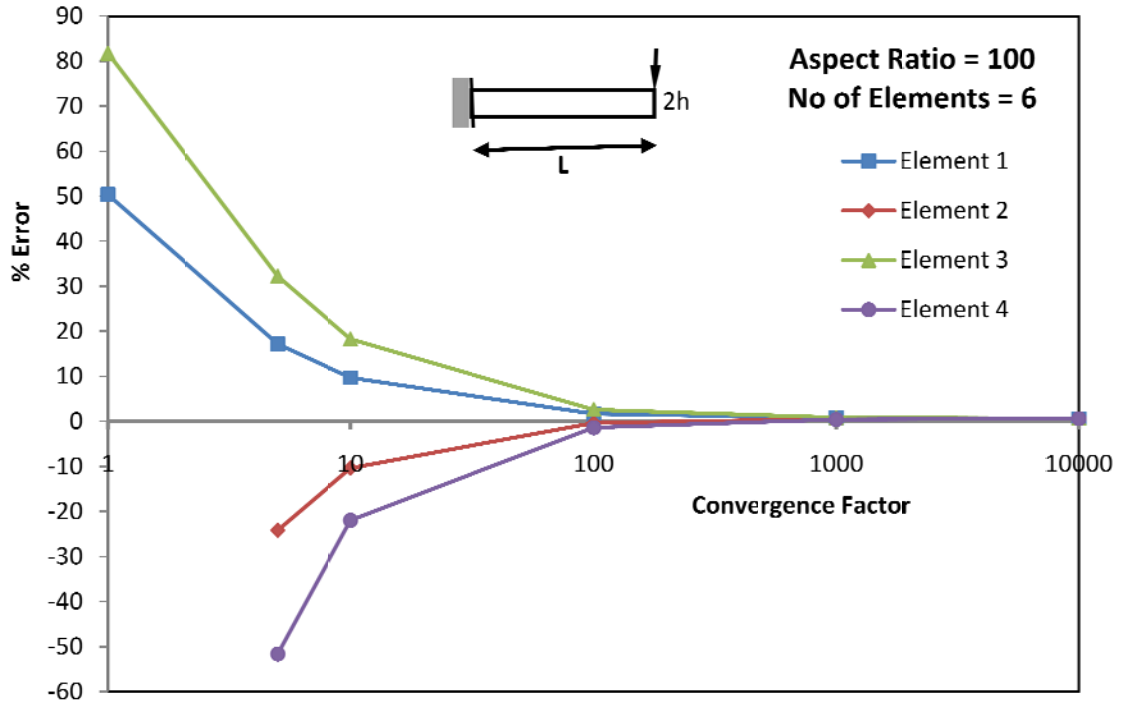


Fig-3_CB_AR_100_NEL_6

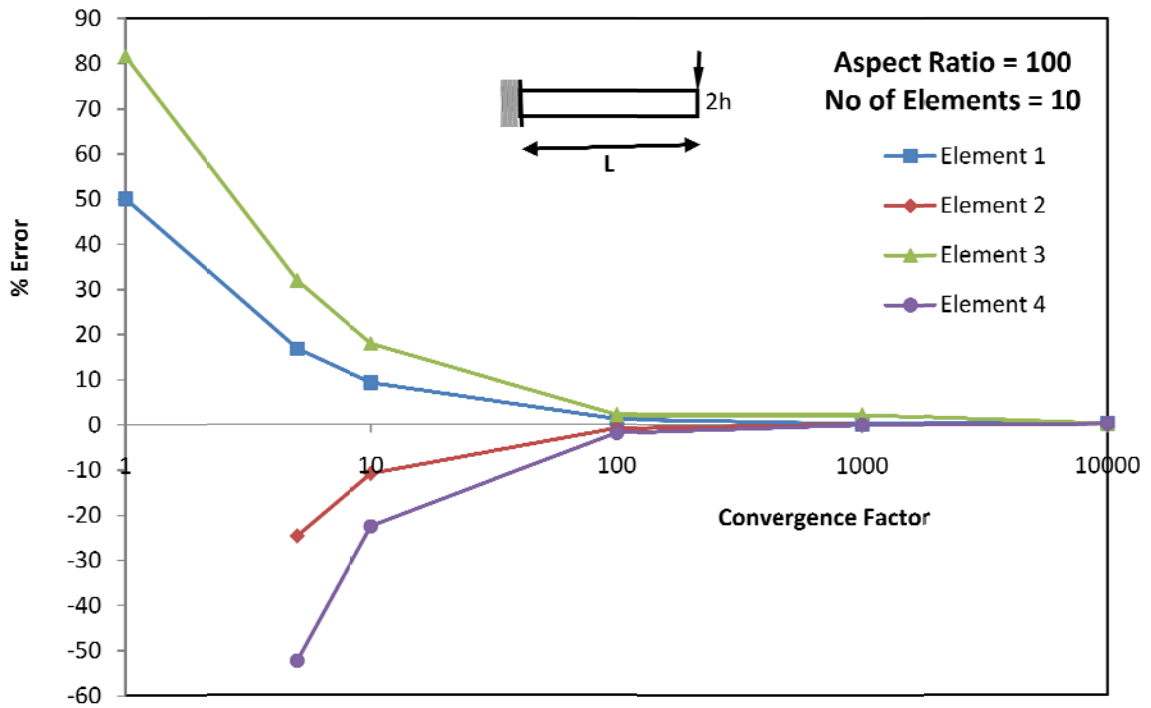


Fig-4_CB_AR_100_NEL_10

Cantilever beam with tip load
Error in deflection at the tip of the beam
Aspect ratio = 400

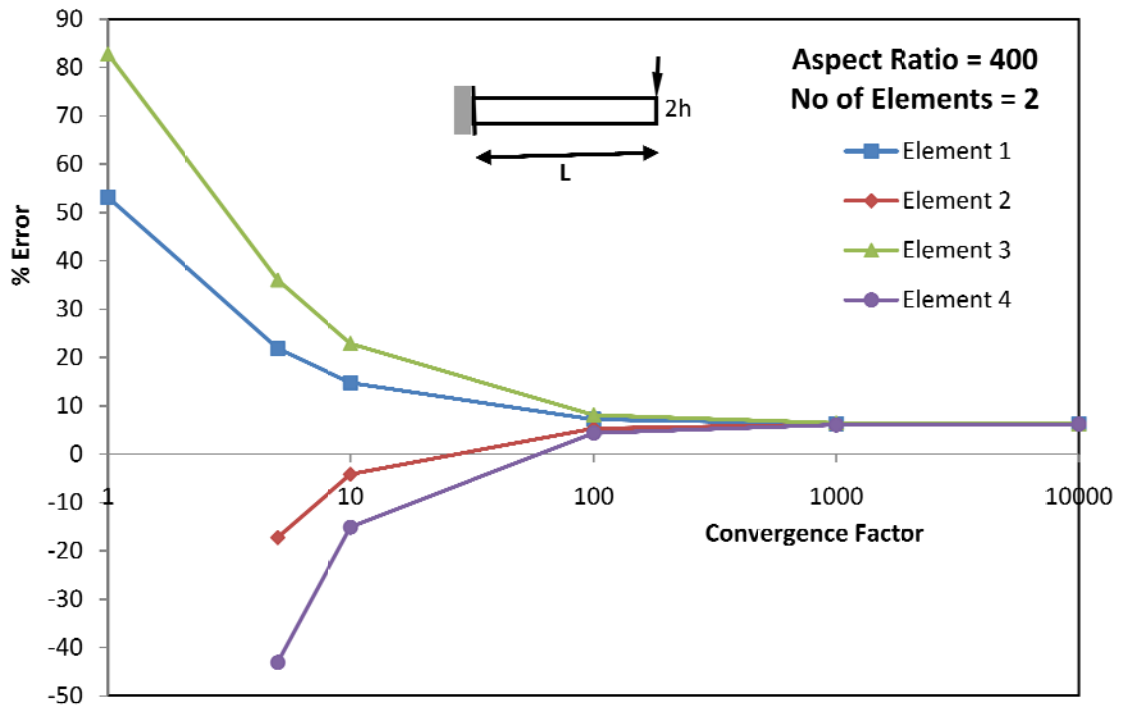


Fig-1_CB_AR_400_NEL_2

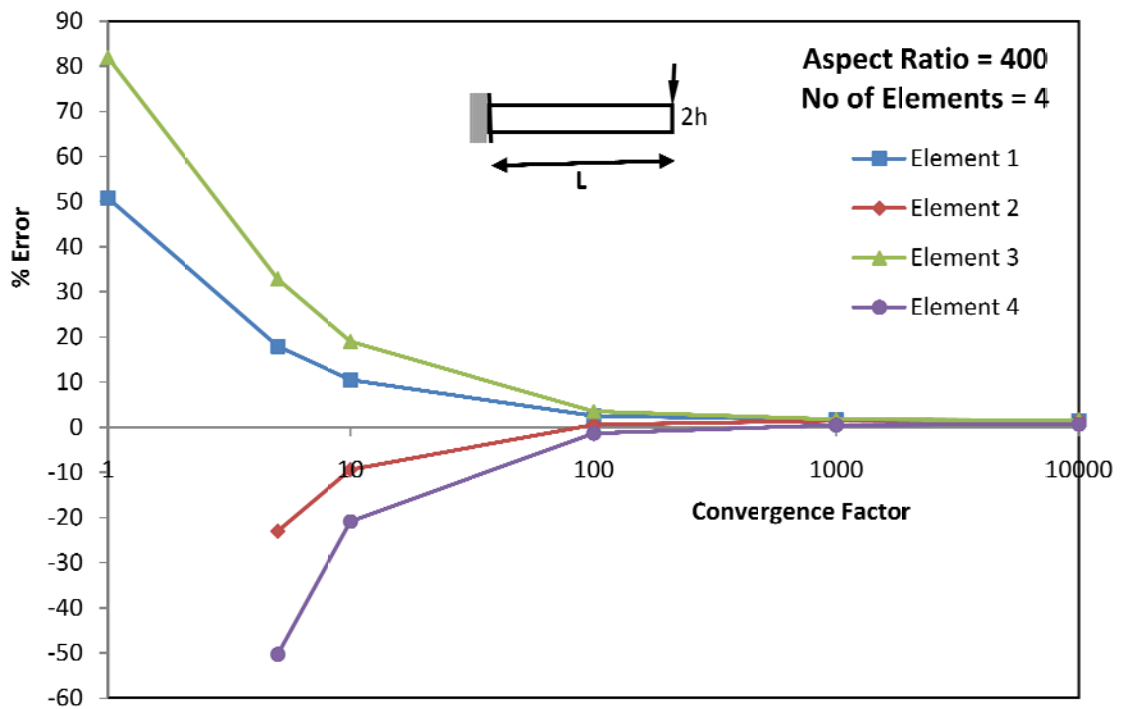


Fig-2_CB_AR_400_NEL_4

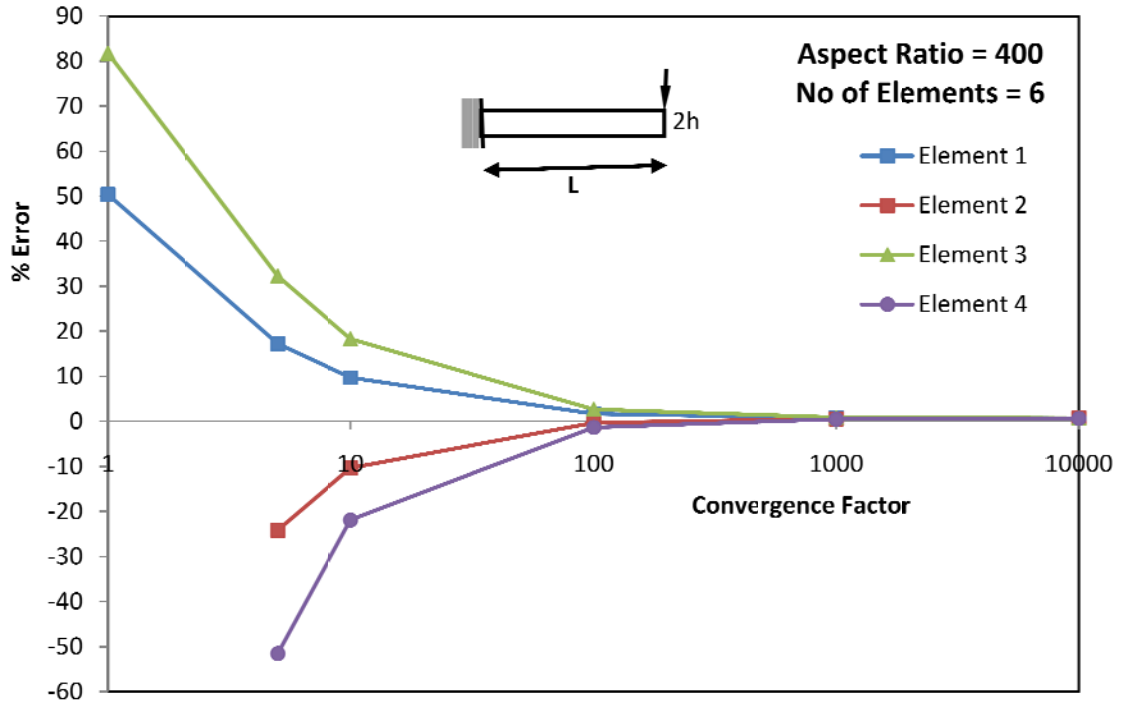


Fig-3_CB_AR_400_NEL_6

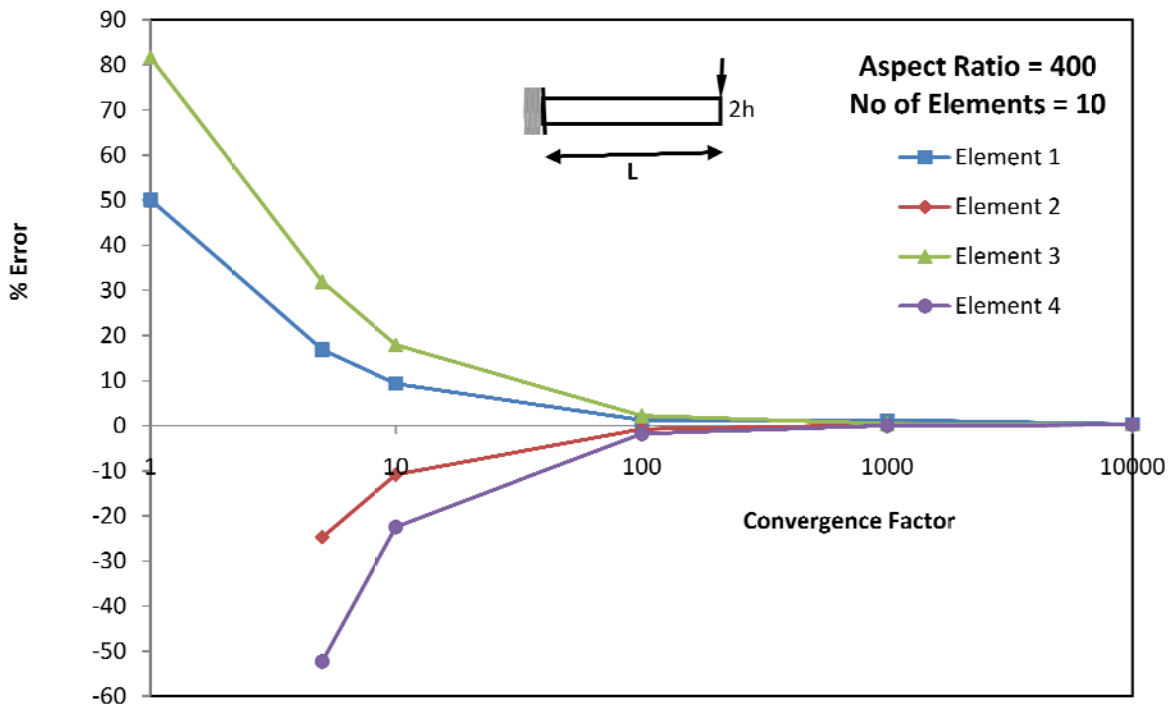
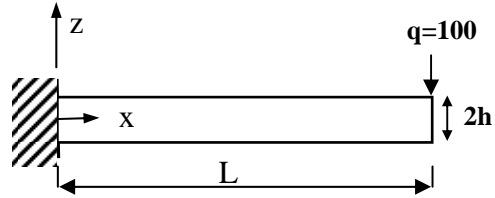


Fig-4_CB_AR_400_NEL_10

Comparison among the finite elements (1) based on Lagrangian shape functions using Higher order deformation theories developed by me (2) based on new shape functions using Higher order deformation theories and (3) based on new shape functions using Timoshenko beam theory



| N | L/2h=160/12 | | | | | | L/2h=80/12 | | | | | |
|-----------------------|----------------|---------|---------------------------|-----------|----------|----------|---------------|--------|-----------|-----------|----------|----------|
| | HFE_1 | HFE_2 | Timo_FE_1 | Timo_FE_3 | FE_NSF_1 | FE_NSF_3 | HFE_1 | HFE_2 | Timo_FE_1 | Timo_FE_3 | FE_NSF_1 | FE_NSF_3 |
| 2 | 1.8136 | 1.8136 | (6 elements with 1000 CF) | 32.2391 | 32.2712 | 32.2712 | 0.7867 | 0.7867 | 4.1140 | 4.1100 | 4.1258 | 4.1218 |
| 4 | 6.2233 | 6.2233 | | | | | 2.0073 | 2.0073 | | | | |
| 8 | 15.8701 | 15.8701 | | | | | 3.2797 | 3.2797 | | | | |
| 12 | 22.2602 | 22.2602 | | | | | 3.7158 | 3.7158 | | | | |
| 20 | 28.0410 | 28.0410 | | | | | 3.9874 | 3.9874 | | | | |
| 24 | 29.3511 | 29.3511 | | | | | 4.0381 | 4.0381 | | | | |
| 30 | 30.5177 | 30.5177 | | | | | 4.0806 | 4.0806 | | | | |
| 36 | 31.1911 | 31.1911 | 4.1040 | 4.1040 | | | | | | | | |
| Reddy * | 32.823 | | | | | | 4.1567 | | | | | |
| Exact Solution | 32.7844 | | | | | | 4.1317 | | | | | |

| N | L/2h=40/12 | | | | | | L/2h=12/12 | | | | | |
|-----------------------|---------------|--------|---------------------------|-----------|----------|----------|----------------|---------|-----------|------------|----------|----------|
| | HFE_1 | HFE_2 | Timo_FE_1 | Timo_FE_2 | FE_NSF_1 | FE_NSF_3 | HFE_1 | HFE_2 | Timo_FE_1 | Timo_NFE_3 | FE_NSF_1 | FE_NSF_3 |
| 2 | 0.2626 | 0.2626 | (6 elements with 1000 CF) | 0.5362 | 0.5424 | 0.5419 | 0.02194 | 0.02194 | 0.02264 | 0.02264 | 0.02419 | 0.02418 |
| 4 | 0.4302 | 0.4302 | | | | | 0.02367 | 0.02367 | | | | |
| 8 | 0.5118 | 0.5118 | | | | | 0.02414 | 0.02414 | | | | |
| 12 | 0.5305 | 0.5305 | | | | | 0.02423 | 0.02423 | | | | |
| 20 | 0.5406 | 0.5406 | | | | | 0.02428 | 0.02428 | | | | |
| 24 | 0.5424 | 0.5424 | | | | | | | | | | |
| 30 | 0.5439 | 0.5439 | | | | | | | | | | |
| 36 | 0.5447 | 0.5447 | | | | | | | | | | |
| Reddy * | 0.54588 | | | | | | 0.02393 | | | | | |
| Exact Solution | 0.5333 | | | | | | 0.02052 | | | | | |

Simply Supported beam with UDL
Error in deflection at the centre of the beam
Aspect ratio = 5

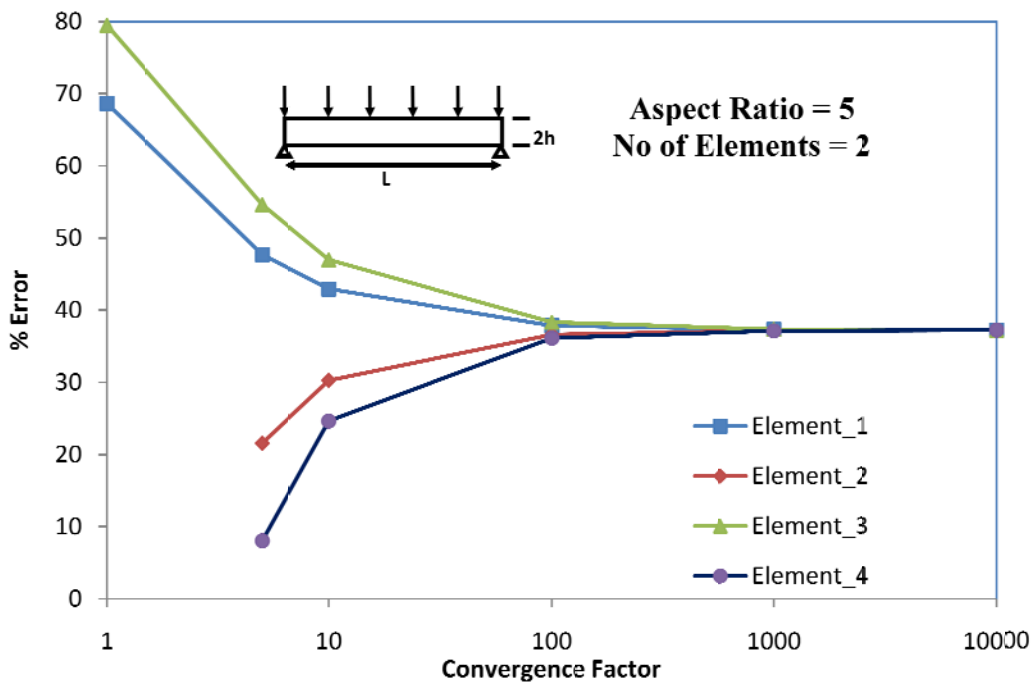


Fig-1_SSB_AR_5_NEL_2

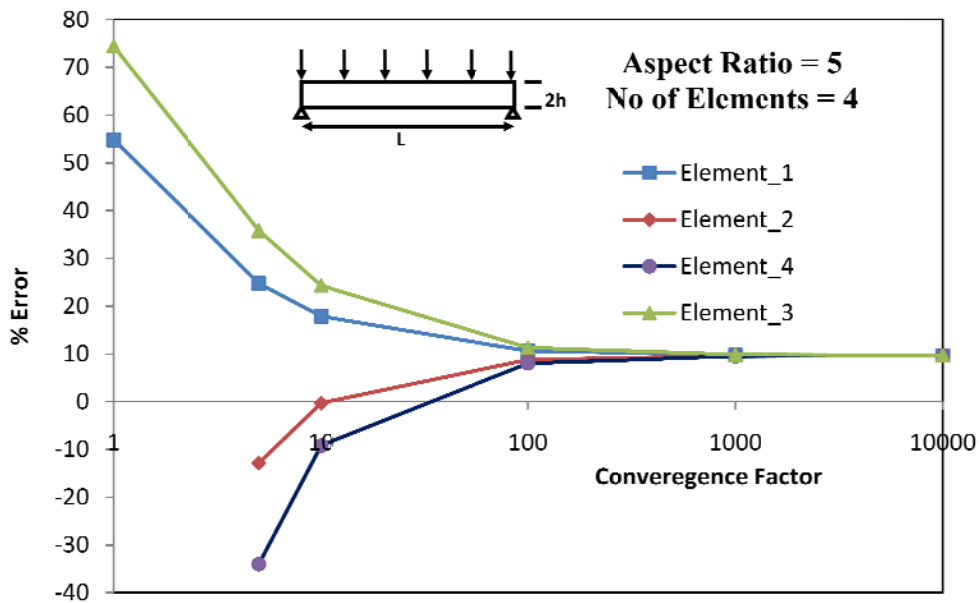


Fig-2_SSB_AR_5_NEL_4

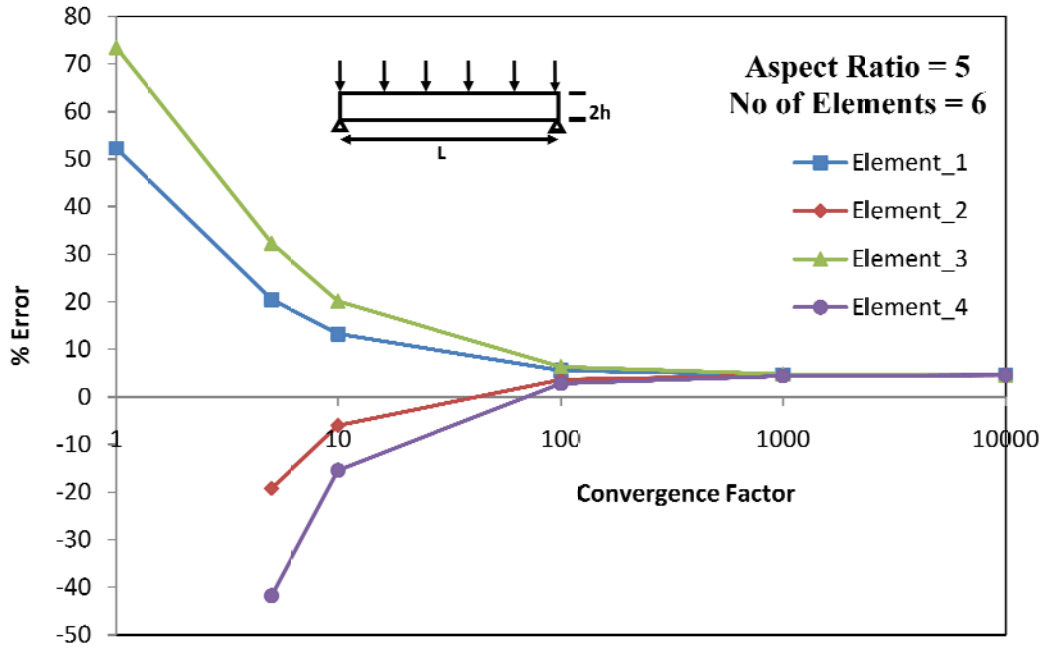


Fig-3_SSB_AR_5_NEL_6

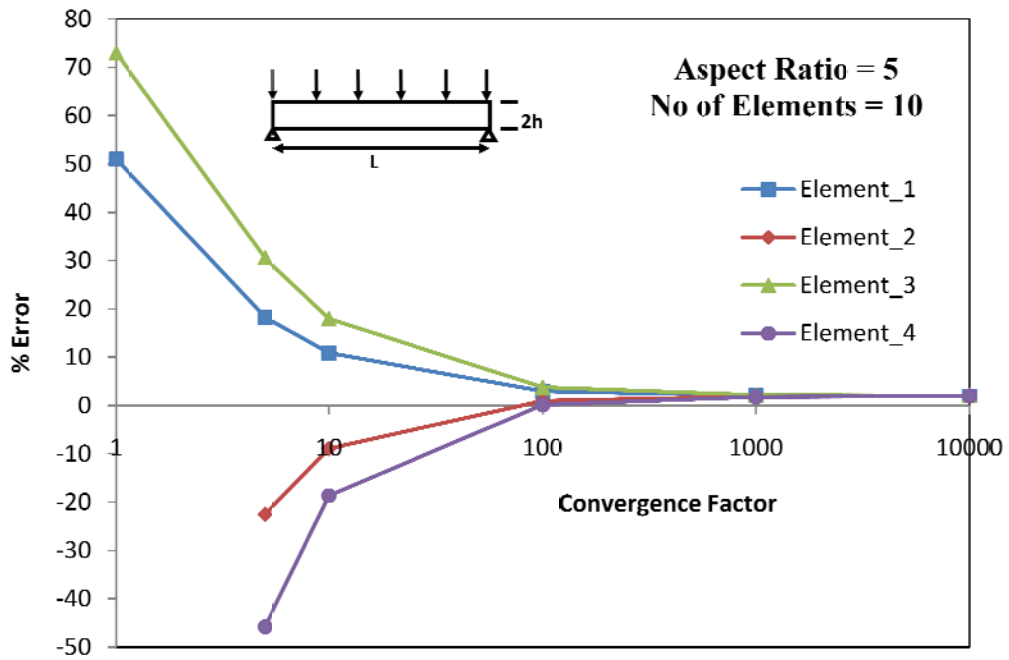


Fig-4_SSB_AR_5_NEL_10

Simply Supported beam with UDL
Error in deflection at the centre of the beam
Aspect ratio = 10

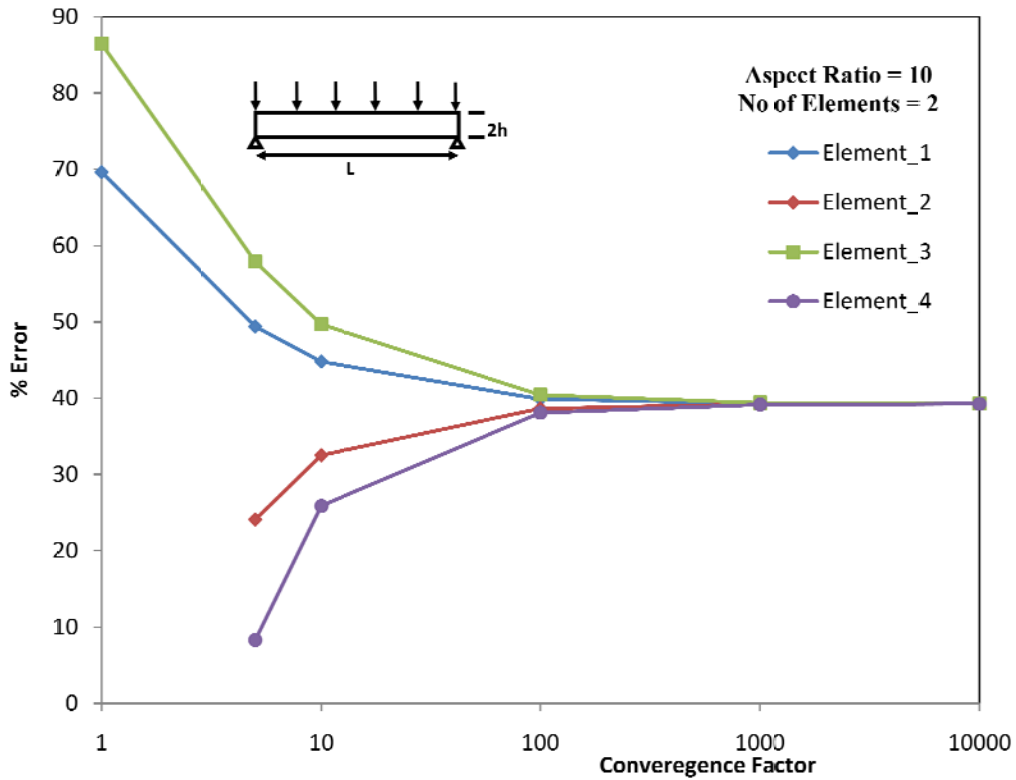


Fig-1_SSB_AR_10_NEL_2

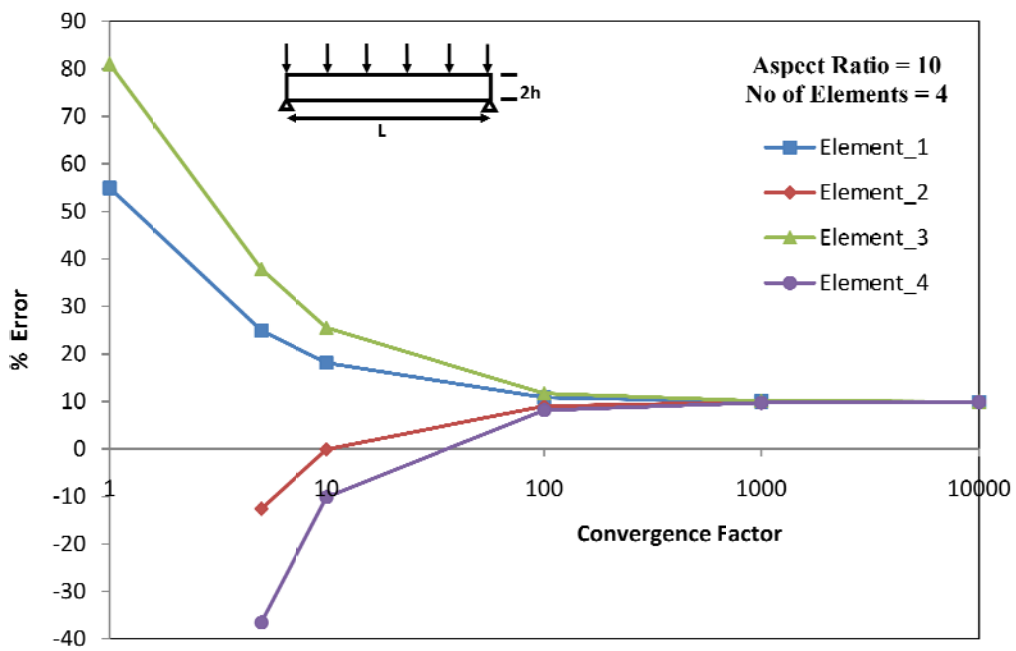


Fig-2_SSB_AR_10_NEL_4

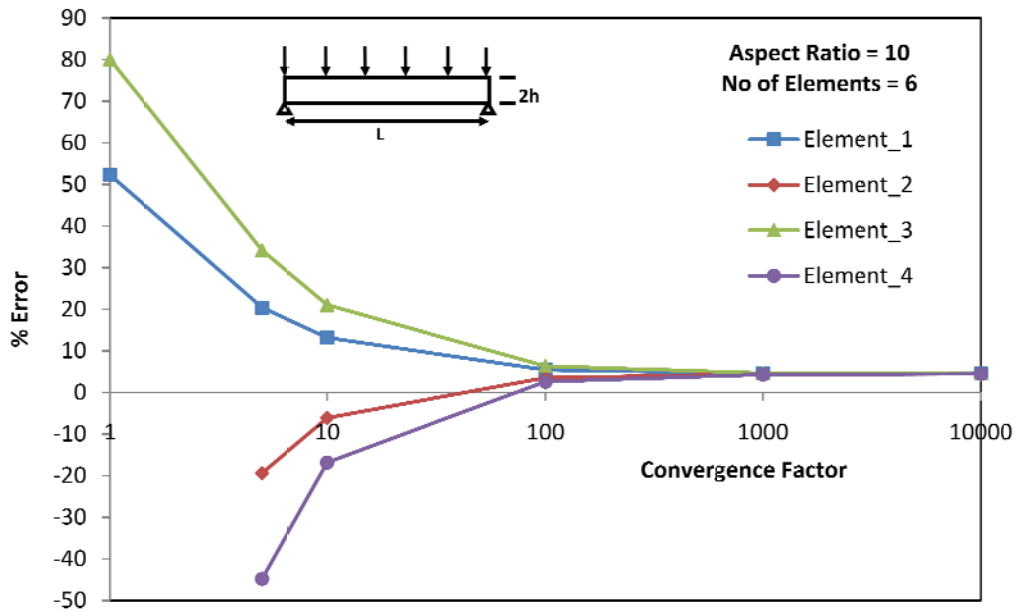


Fig-3_SSB_AR_10_NEL_6

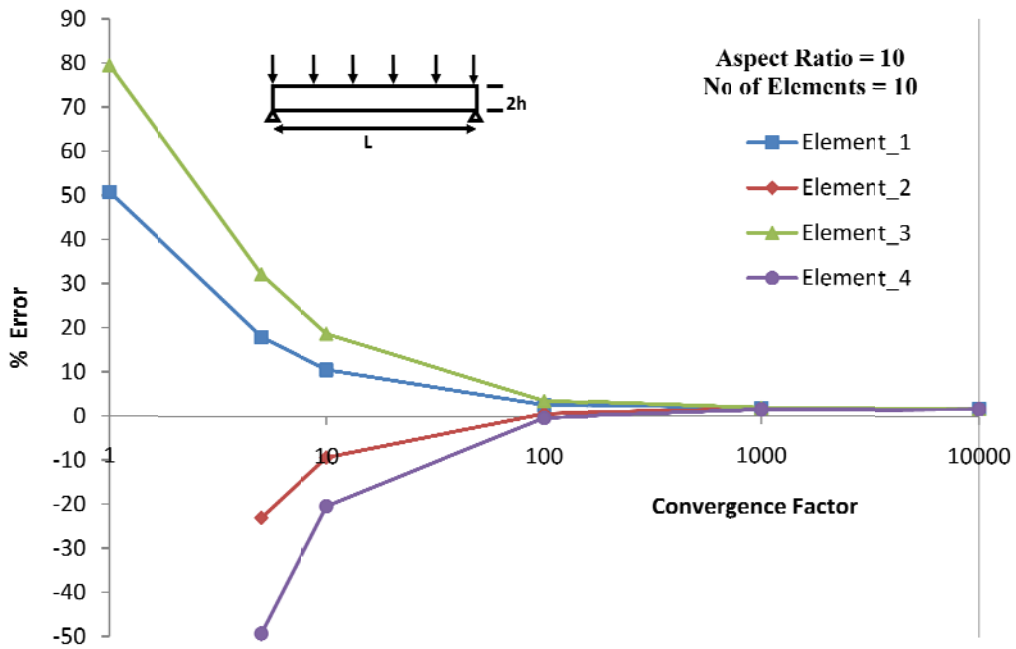


Fig-4_SSB_AR_10_NEL_10

Simply Supported beam with UDL
Error in deflection at the centre of the beam
Aspect ratio = 100

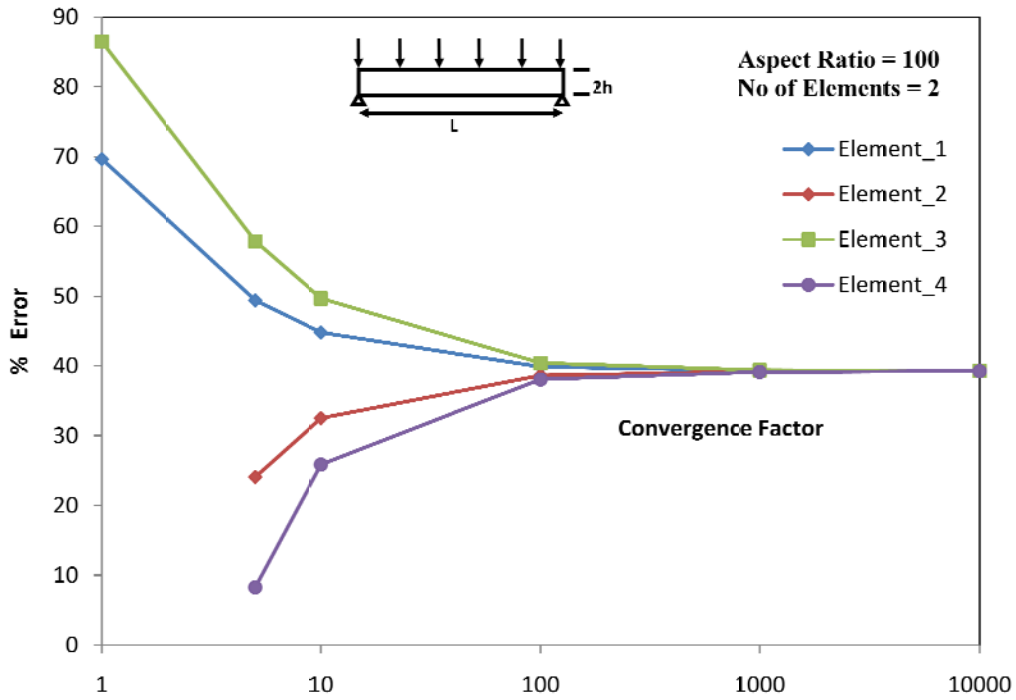


Fig-1_SSB_AR_100_NEL_2

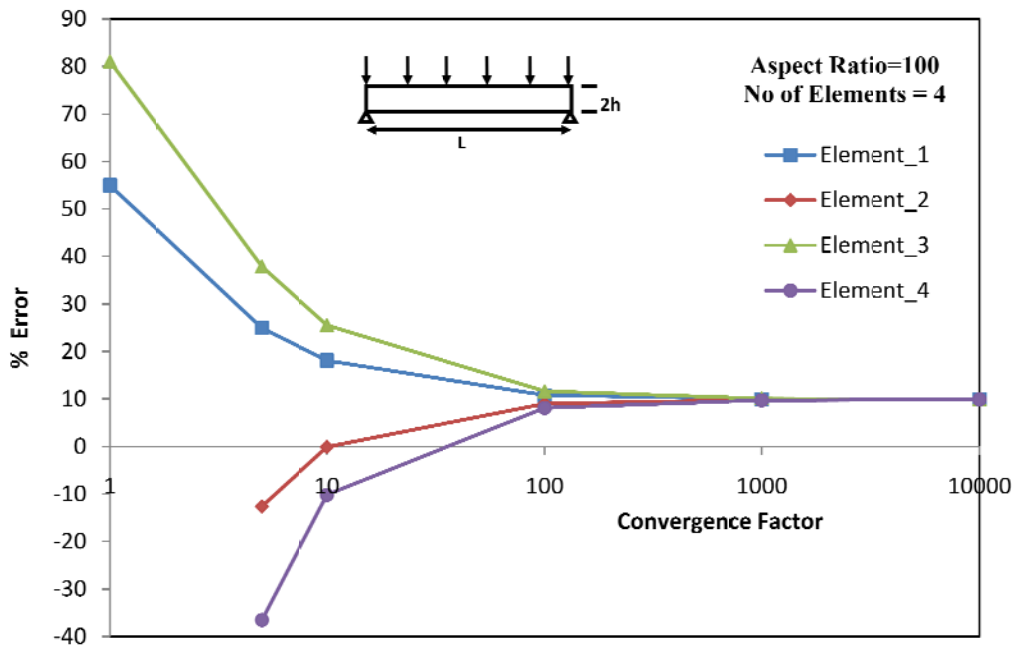


Fig-2_SSB_AR_100_NEL_4

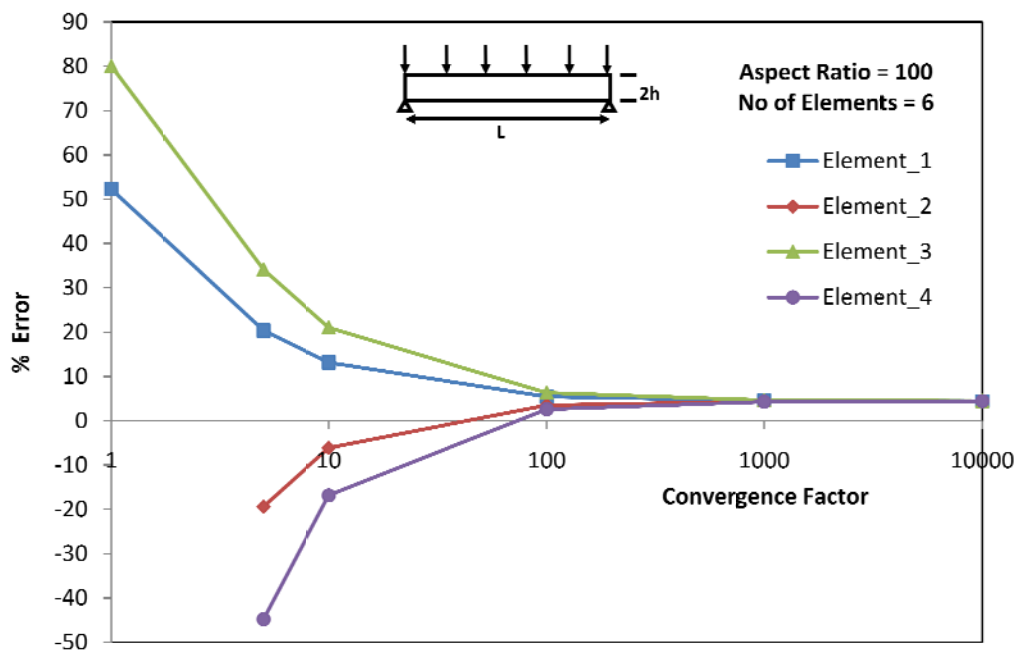


Fig-3_SSB_AR_100_NEL_6

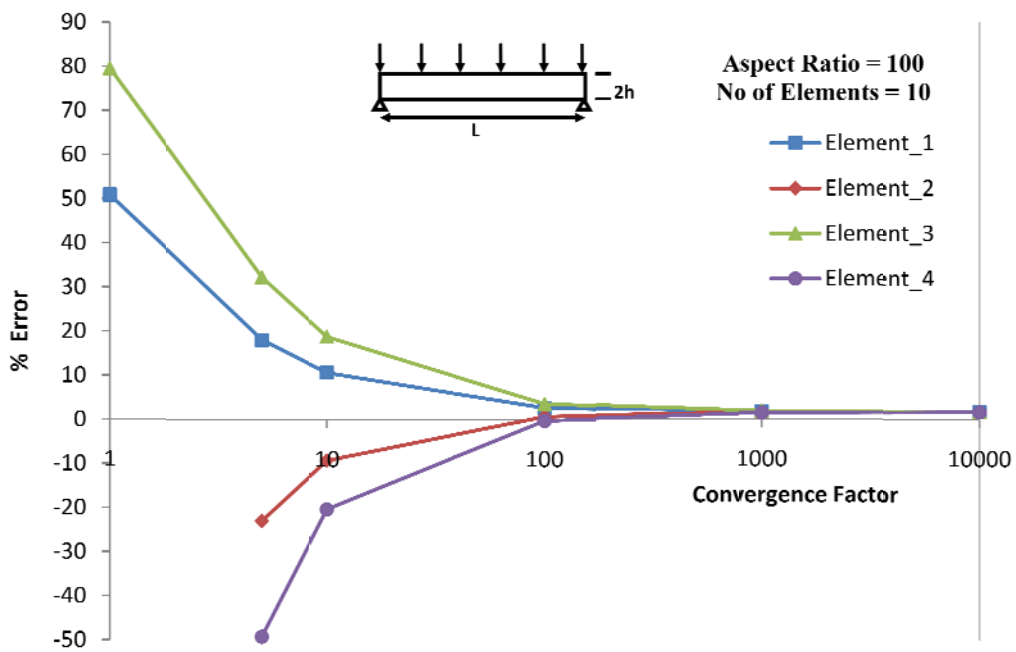


Fig-4_SSB_AR_100_NEL_10

Simply Supported beam with UDL
Error in deflection at the centre of the beam
Aspect ratio = 400

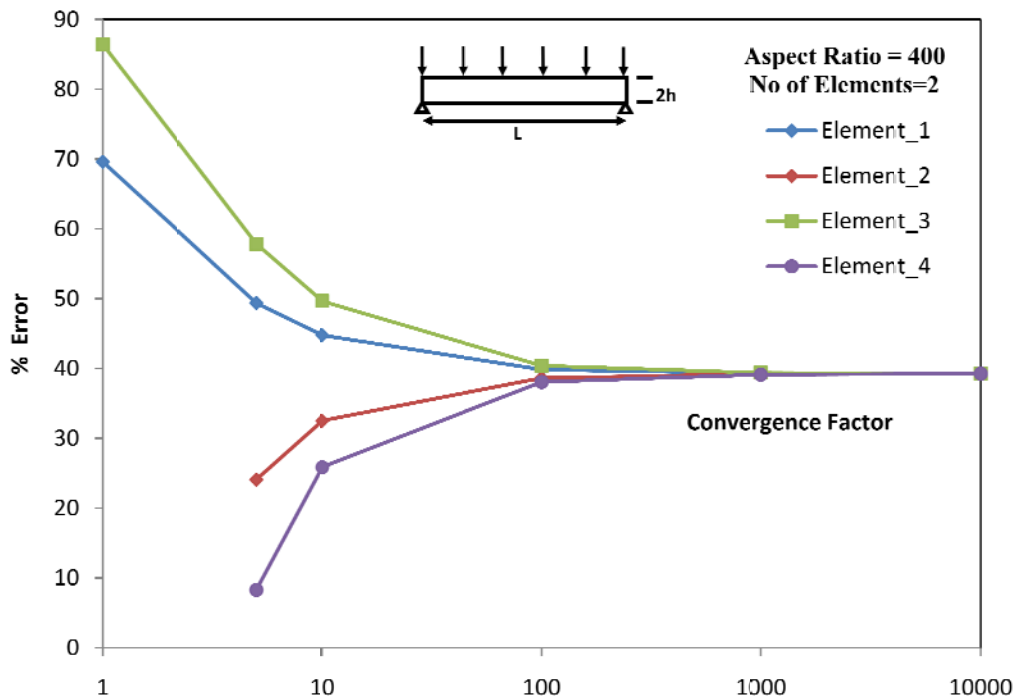


Fig-1_SSB_AR_400_NEL_2

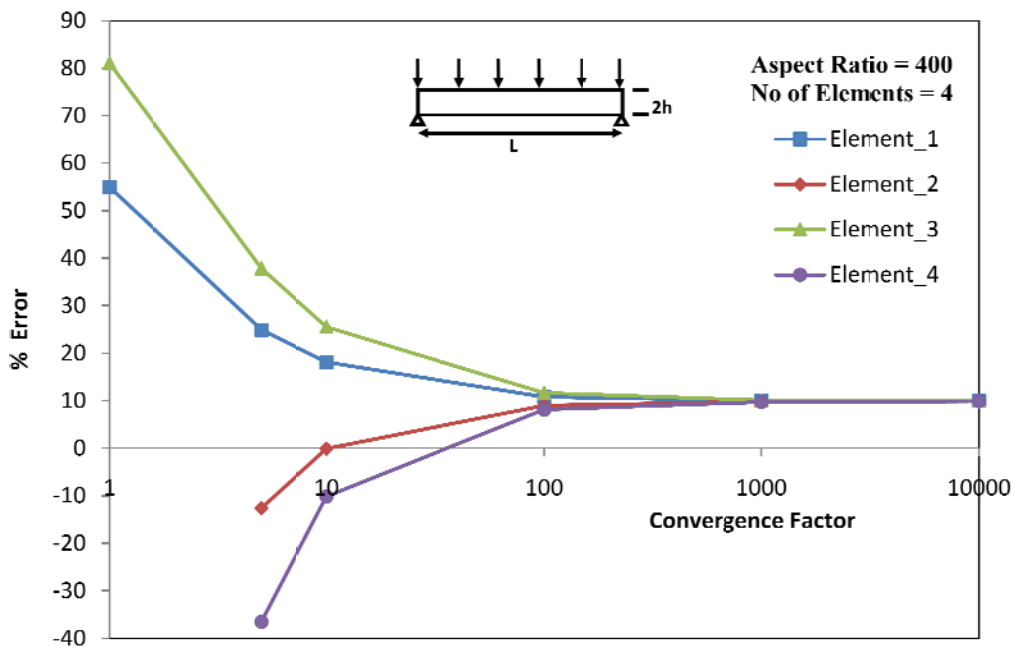


Fig-2_SSB_AR_400_NEL_4

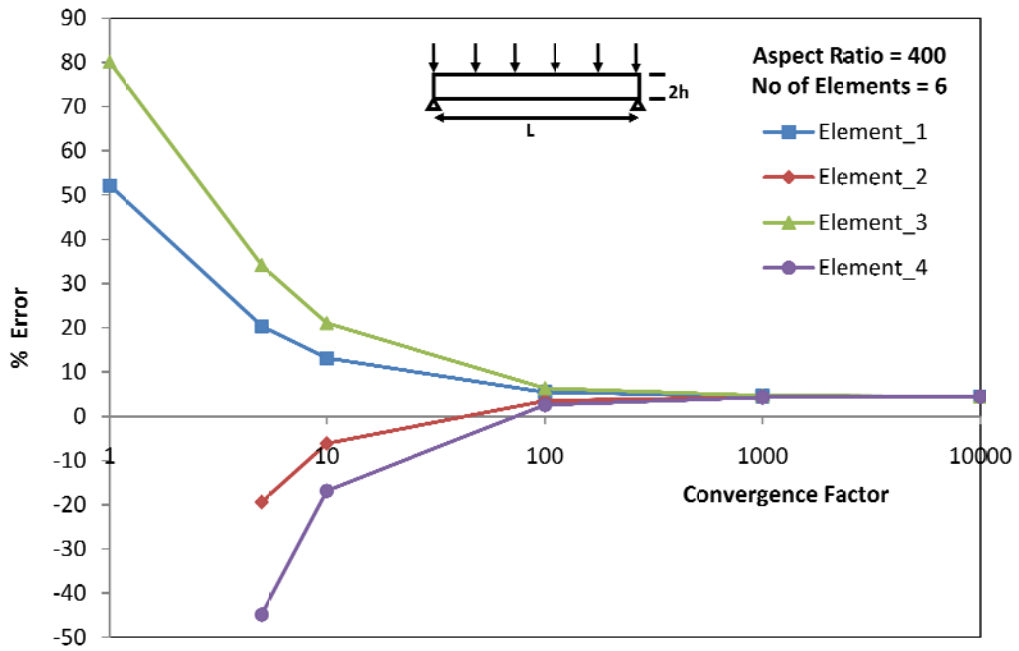


Fig-3_SSB_AR_400_NEL_6

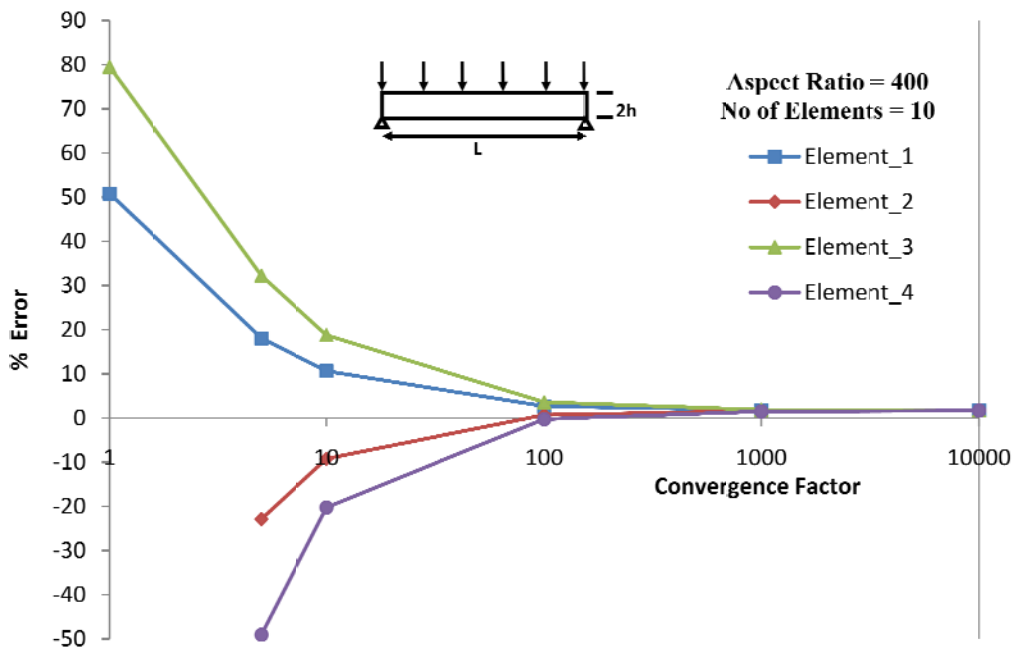
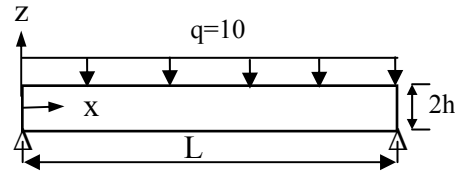


Fig-4_SSB_AR_400_NEL_6

Comparison among the finite elements (1) based on Lagrangian shape functions using Higher order deformation theories developed by me (2) based on new shape functions using Higher order deformation theories and (3) based on new shape functions using Timoshenko beam theory



| | L/2h=160/12 | | | | | | L/2h=80/12 | | | | | |
|-----------------------|--------------------|--------------|---|--|--|---|-------------------|--------------|---|---|---|---|
| N | HFE_1 | HFE_2 | Timo_FE_1 | Timo_FE_3 | FE_NSF_1 | FE_NSF_3 | HFE_1 | HFE_2 | Timo_FE_1 | Timo_FE_3 | FE_NSF_1 | FE_NSF_3 |
| 2 | 0.8254 | 0.8254 | 20.326 (10 elements with 1000 CF) | 20.306 (10 elements with 1000 CF) | 20.374 (10 elements with 1000 CF) | 20.354 (10 elements with 1000 CF) | 0.1849 | 0.1849 | 1.3151 (10 elements with 1000 CF) | 1.3139 (10 elements with 1000 CF) | 1.3271 (10 elements with 1000 CF) | 1.3259 (10 elements with 1000 CF) |
| 4 | 3.6431 | 3.6431 | | | | | 0.6046 | 0.6046 | | | | |
| 8 | 9.8570 | 9.8570 | | | | | 1.0482 | 1.0482 | | | | |
| 12 | 13.9791 | 13.9791 | | | | | 1.2009 | 1.2009 | | | | |
| 20 | 17.7108 | 17.7108 | | | | | 1.2960 | 1.2960 | | | | |
| 24 | 18.5568 | 18.5568 | | | | | 1.3138 | 1.3138 | | | | |
| 30 | 19.3102 | 19.3102 | | | | | 1.3287 | 1.3287 | | | | |
| 36 | 19.7452 | 19.7452 | | | | | 1.3369 | 1.3369 | | | | |
| Reddy * | 20.717 | | | | | | 1.3486 | | | | | |
| Exact Solution | 20.6892 | | | | | | 1.3408 | | | | | |

| | L/2h=40/12 | | | | | | L/2h=12/12 | | | | | |
|-----------------------|-------------------|--------------|--|--|--|--|-------------------|--------------|---|---|---|---|
| N | HFE_1 | HFE_2 | Timo_FE_1 | Timo_FE_3 | FE_NSF_1 | FE_NSF_3 | HFE_1 | HFE_2 | Timo_FE_1 | Timo_FE_3 | FE_NSF_1 | FE_NSF_3 |
| 2 | 0.03475 | 0.03475 | 0.09339 (10 elements with 1000 CF) | 0.09333 (10 elements with 1000 CF) | 0.09634 (10 elements with 1000 CF) | 0.09628 (10 elements with 1000 CF) | 0.0016486 | 0.0016486 | 0.001979 (10 elements with 1000 CF) | 0.001980 (10 elements with 1000 CF) | 0.002230 (10 elements with 1000 CF) | 0.002210 (10 elements with 1000 CF) |
| 4 | 0.07202 | 0.07202 | | | | | 0.0021069 | 0.0021069 | | | | |
| 8 | 0.09064 | 0.09064 | | | | | 0.0022298 | 0.0022298 | | | | |
| 12 | 0.09491 | 0.09491 | | | | | 0.0022529 | 0.0022529 | | | | |
| 20 | 0.09723 | 0.09723 | | | | | 0.0022649 | 0.0022649 | | | | |
| 24 | 0.09764 | 0.09764 | | | | | ----- | ----- | | | | |
| 30 | ----- | ----- | | | | | ----- | ----- | | | | |
| 36 | ----- | ----- | | | | | ----- | ----- | | | | |
| Reddy * | 0.09770 | | | | | | 0.002220 | | | | | |
| Exact Solution | 0.09576 | | | | | | 0.002082 | | | | | |

Stress Results for Simply Supported Beam

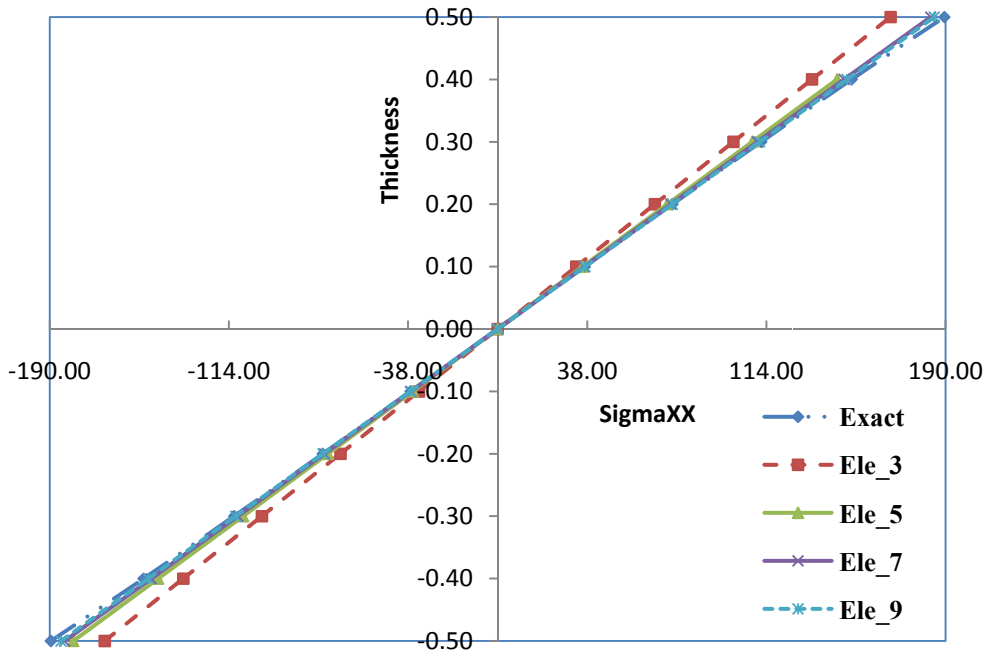
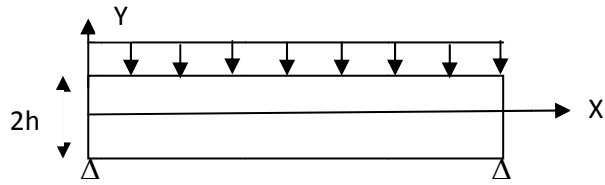
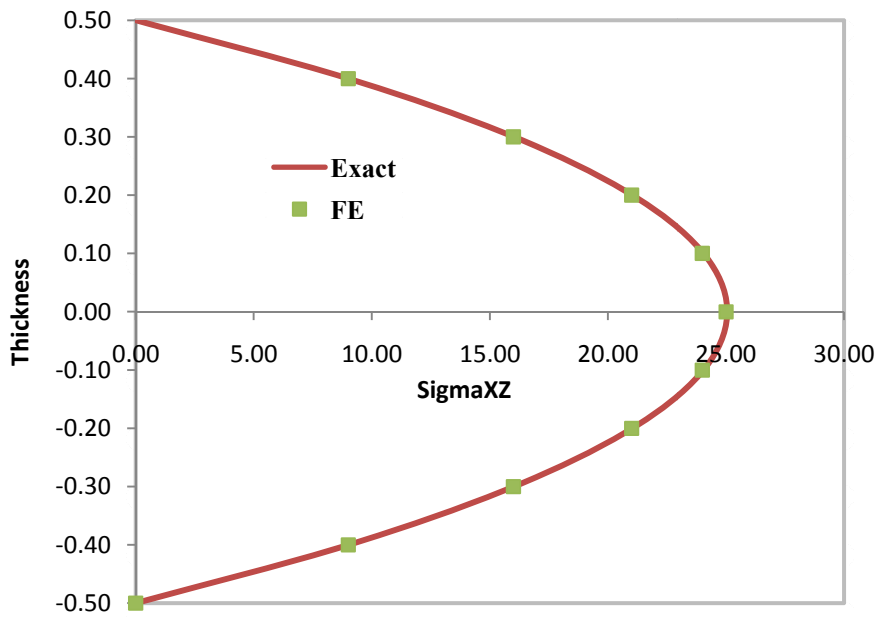


Fig.1 SS_ASR_5_Bending_Stress



Shear stress calculation = 0.83

Fig.2 SS_ASR_5_Shear_Stress

Length =10.0, width=1.0, thickness=1.0
 Maximum Bending Stress at the center of the beam

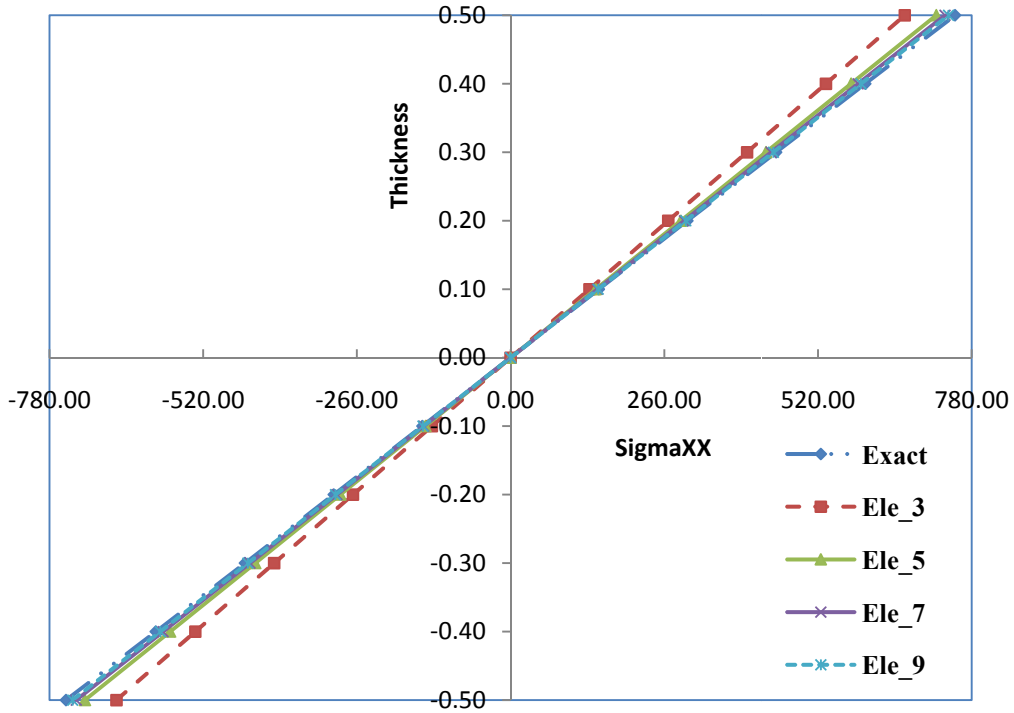
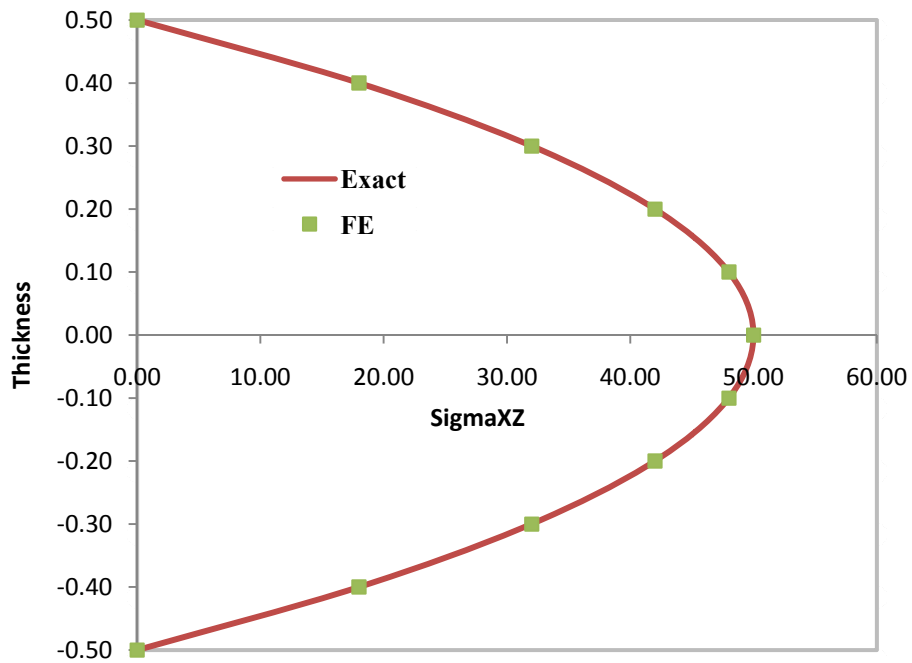


Fig.3 SS_ASR_10_Bending_Stress



Shear stress calculation = 1.666

Fig.4 SS_ASR_10_Shear_Stress

Length =200.0, width=1.0, thickness=1.0
 Maximum Bending Stress at the center of the beam

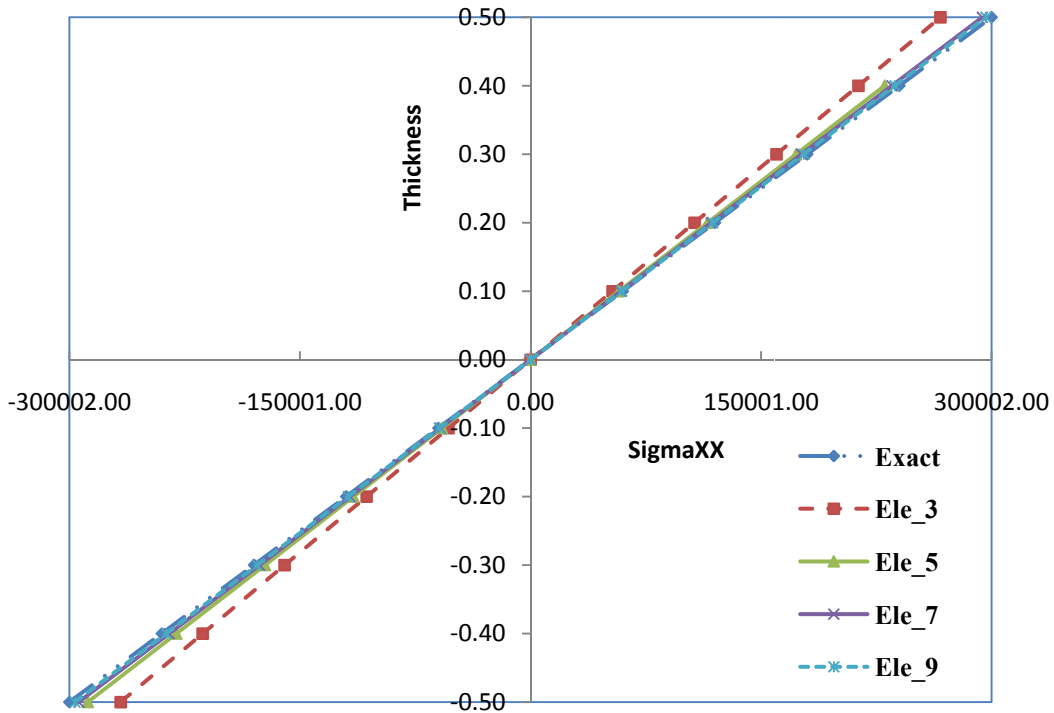
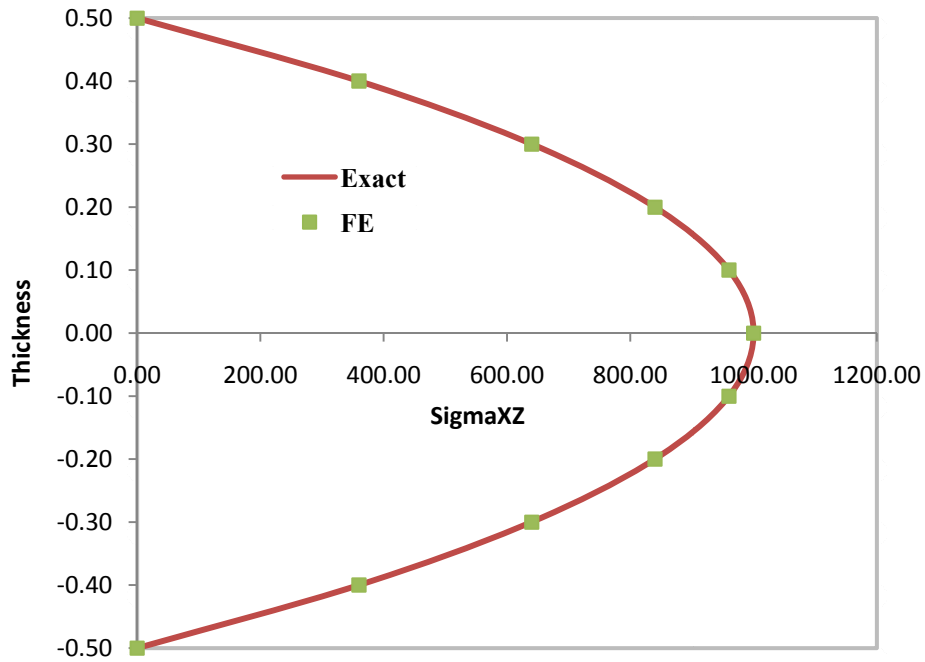


Fig.5 SS_ASR_200_Bending_Stress



Shear stress calculation = 33.33

Fig.6 SS_ASR_200_Shear_Stress

Length =400.0, width=1.0, thickness=1.0
 Maximum Bending Stress at the center of the beam

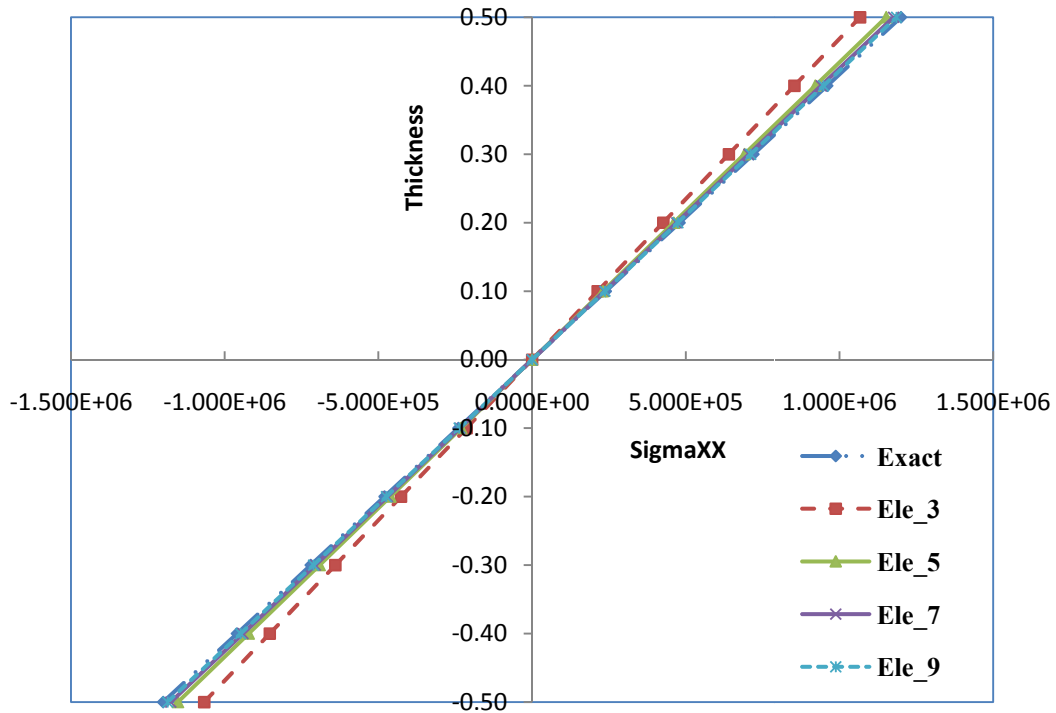
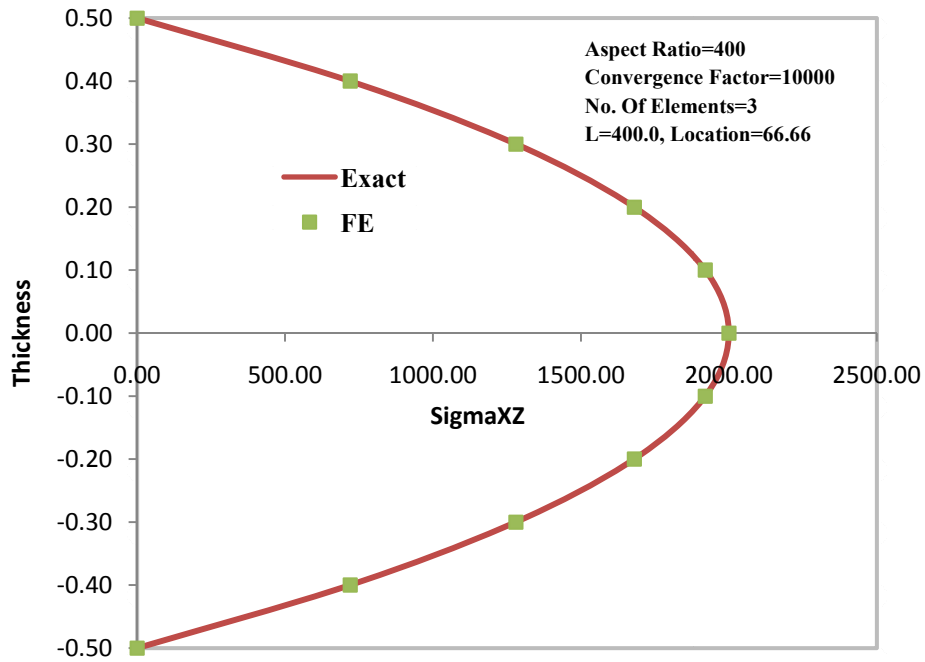


Fig.7 SS_ASR_400_Bending_Stress



Shear stress calculation = 66.66

Fig.8 SS_ASR_400_Shear_Stress

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 5

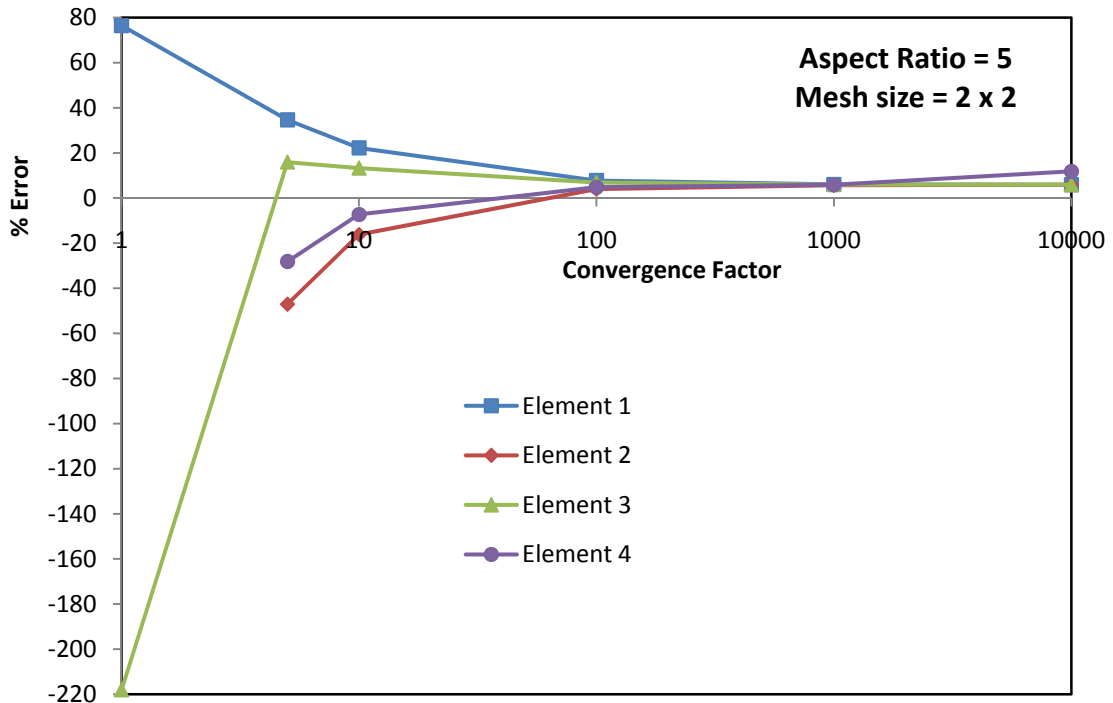


Fig-1_SSP_AR_5_NEL_2x2

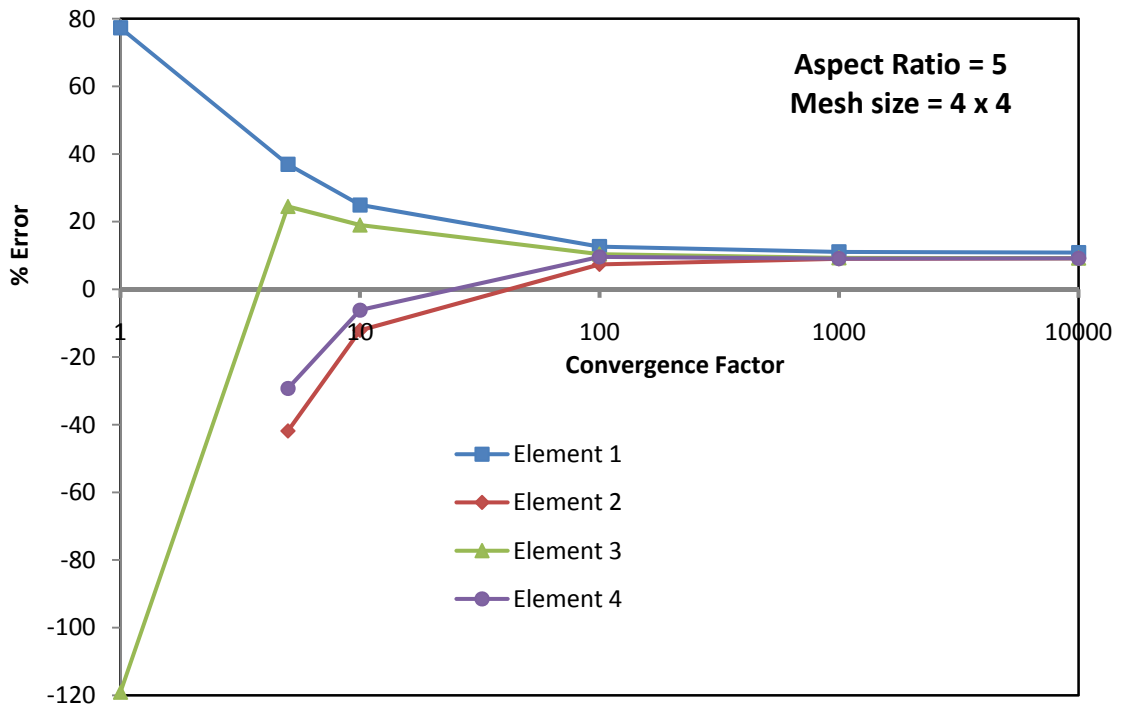


Fig-2_SSP_AR_5_NEL_4x4

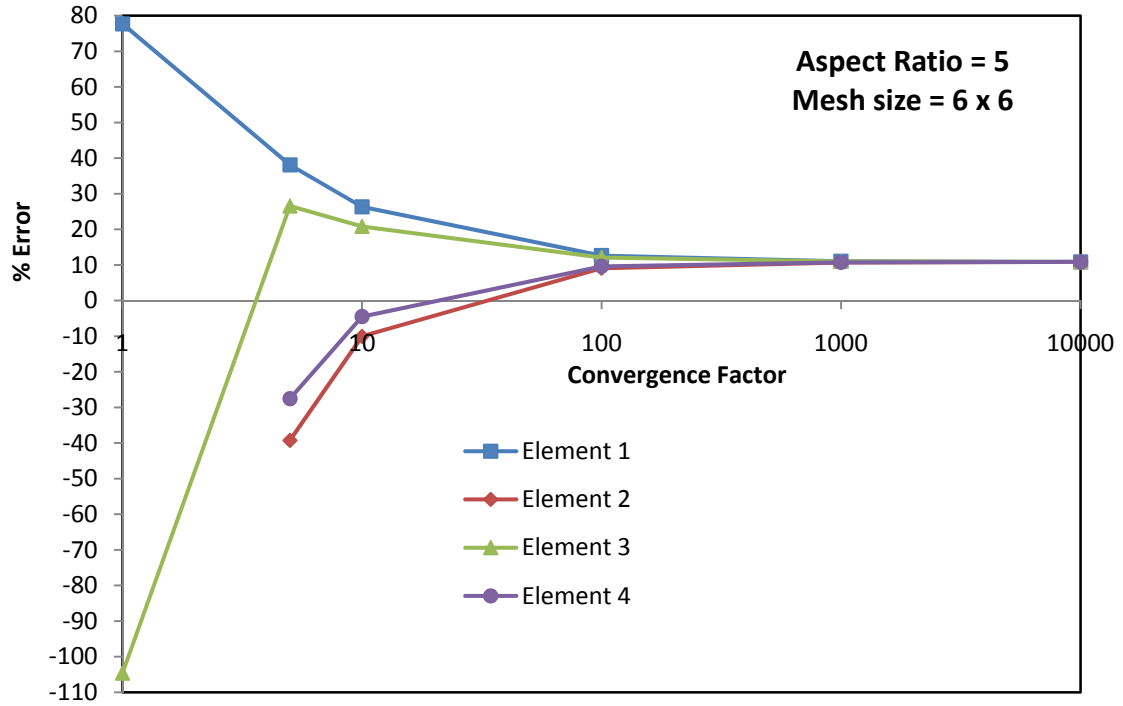


Fig-3_SSP_AR_5_NEL_6x6

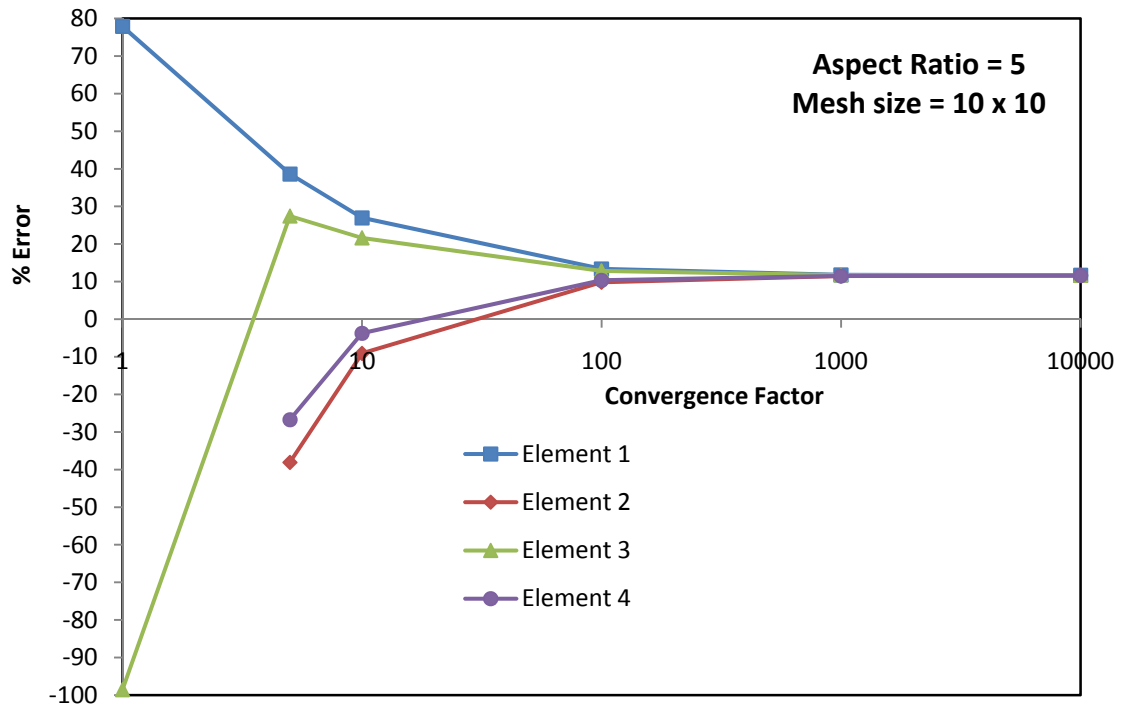


Fig-4_SSP_AR_5_NEL_10x10

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 7.14

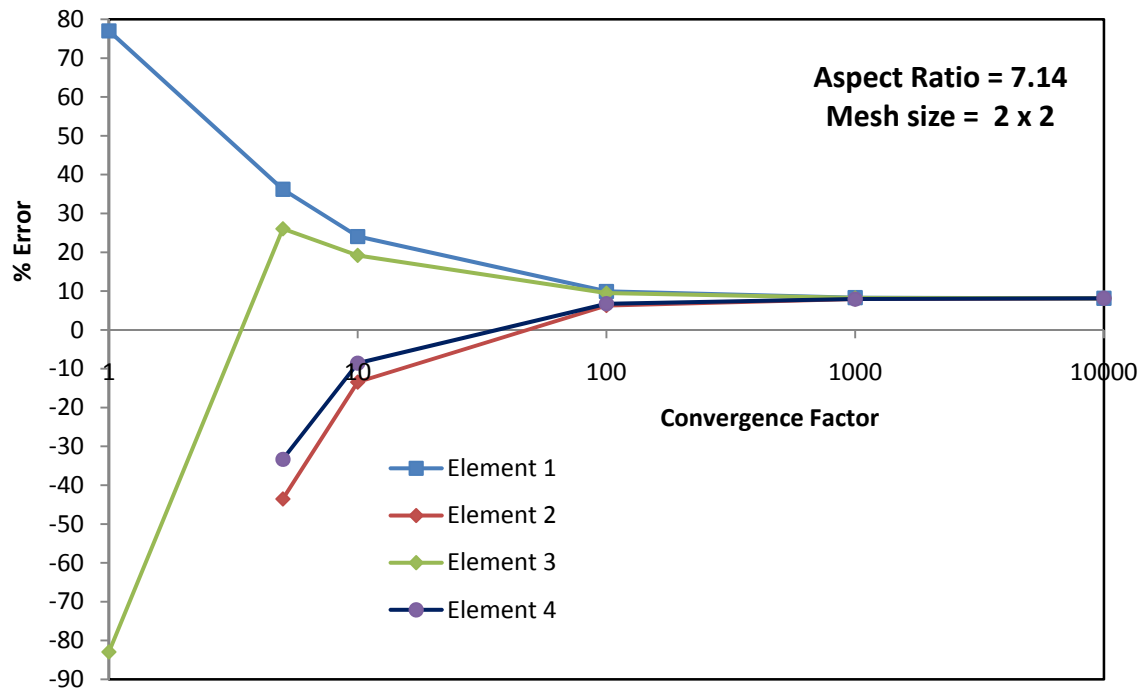


Fig-1_SSP_AR_7.14_NEL_2x2

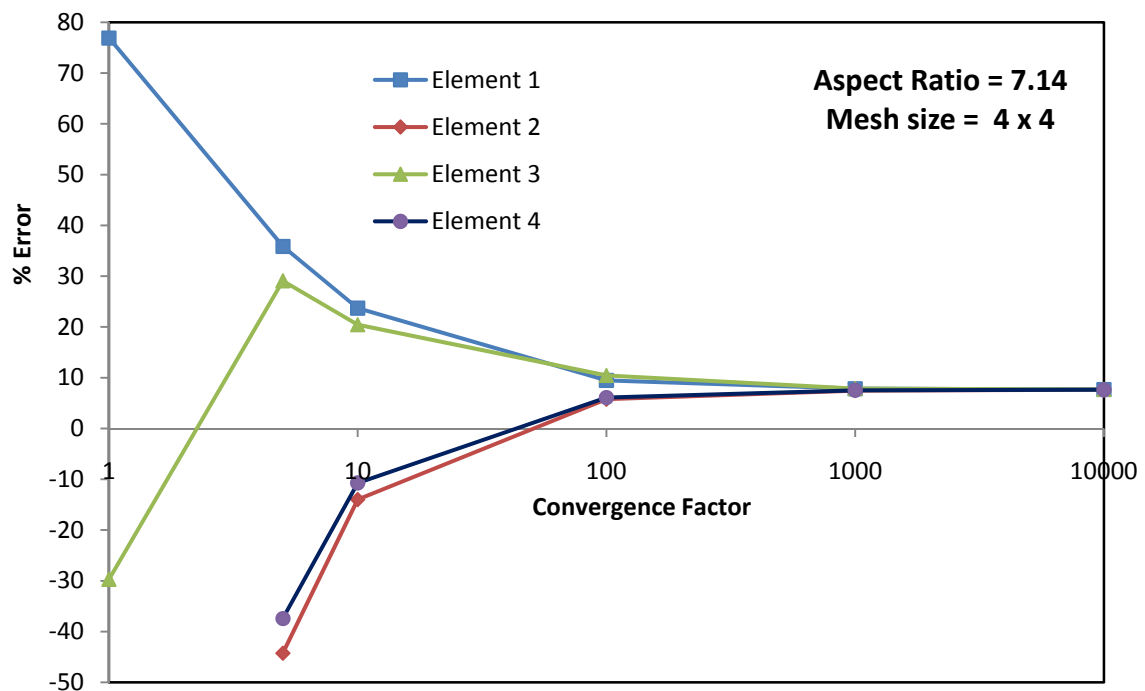


Fig-2_SSP_AR_7.14_NEL_4x4

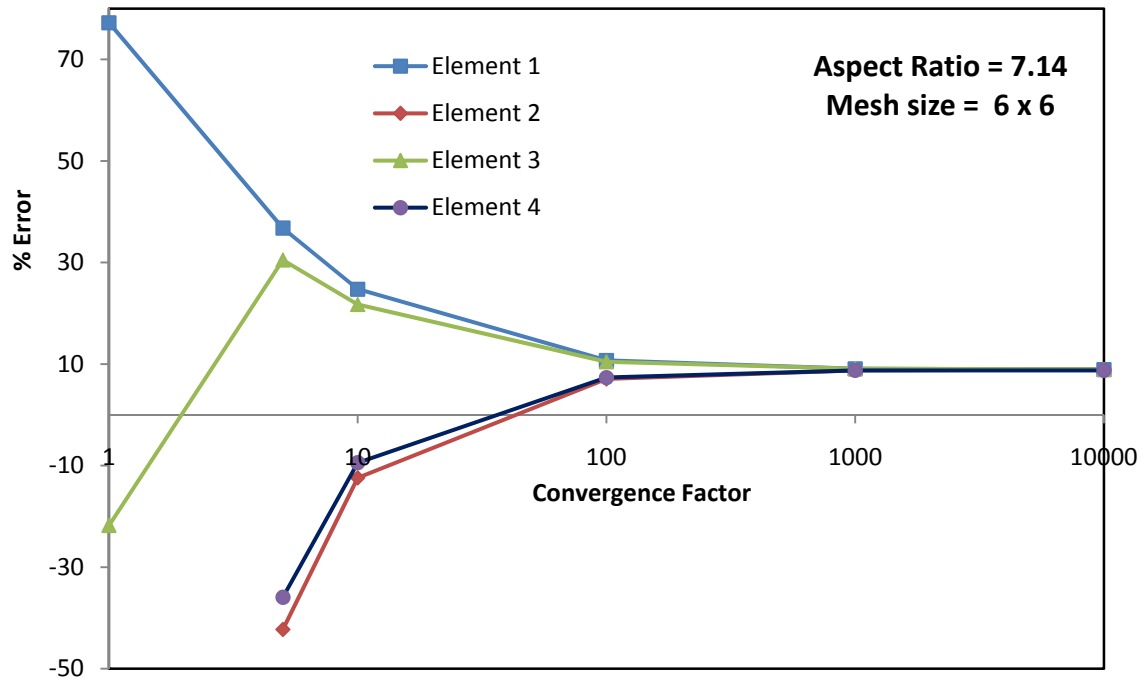


Fig-3_SSP_AR_7.14_NEL_6x6

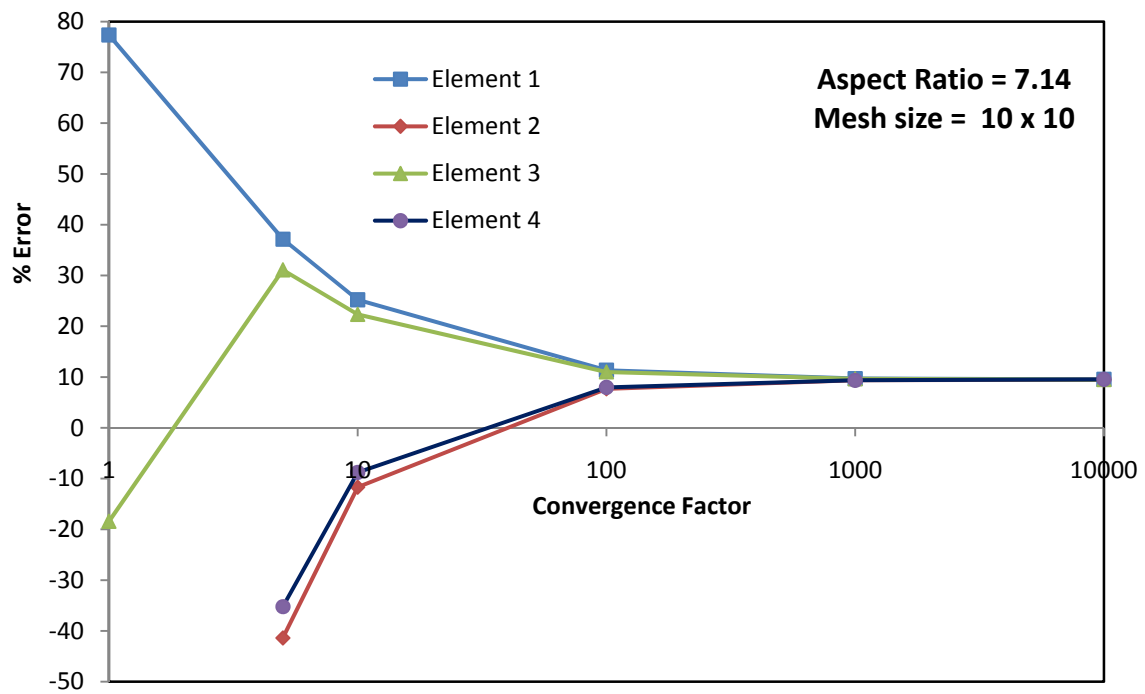


Fig-4_SSP_AR_7.14_NEL_10x10

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 10

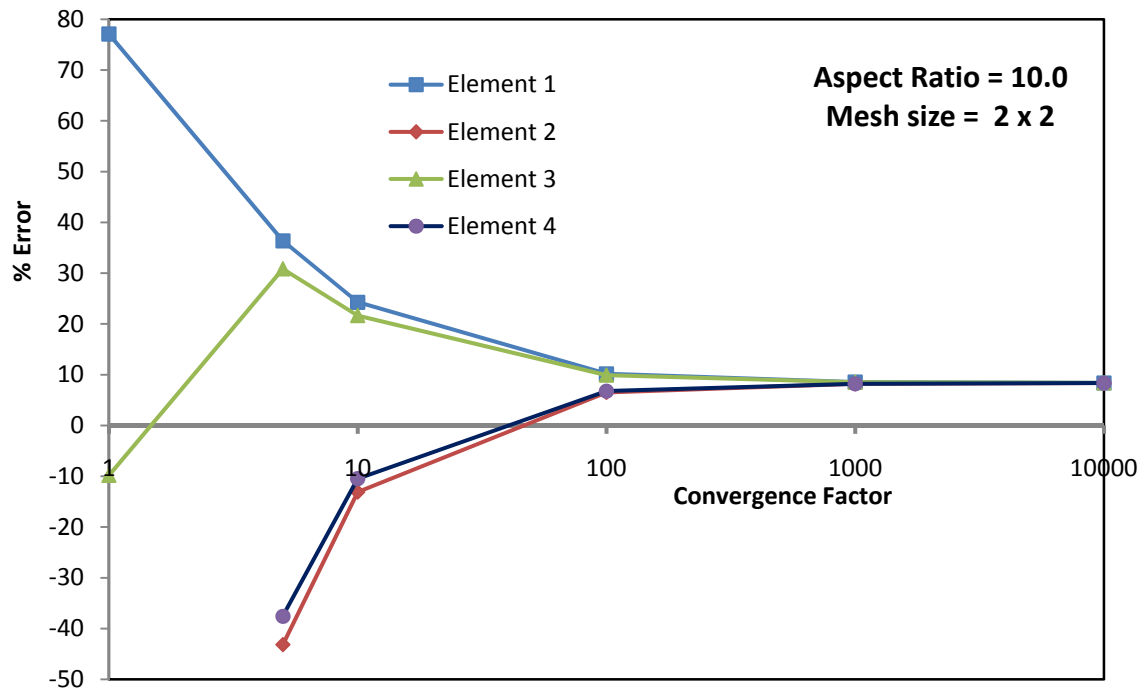


Fig-1_SSP_AR_10_NEL_2x2

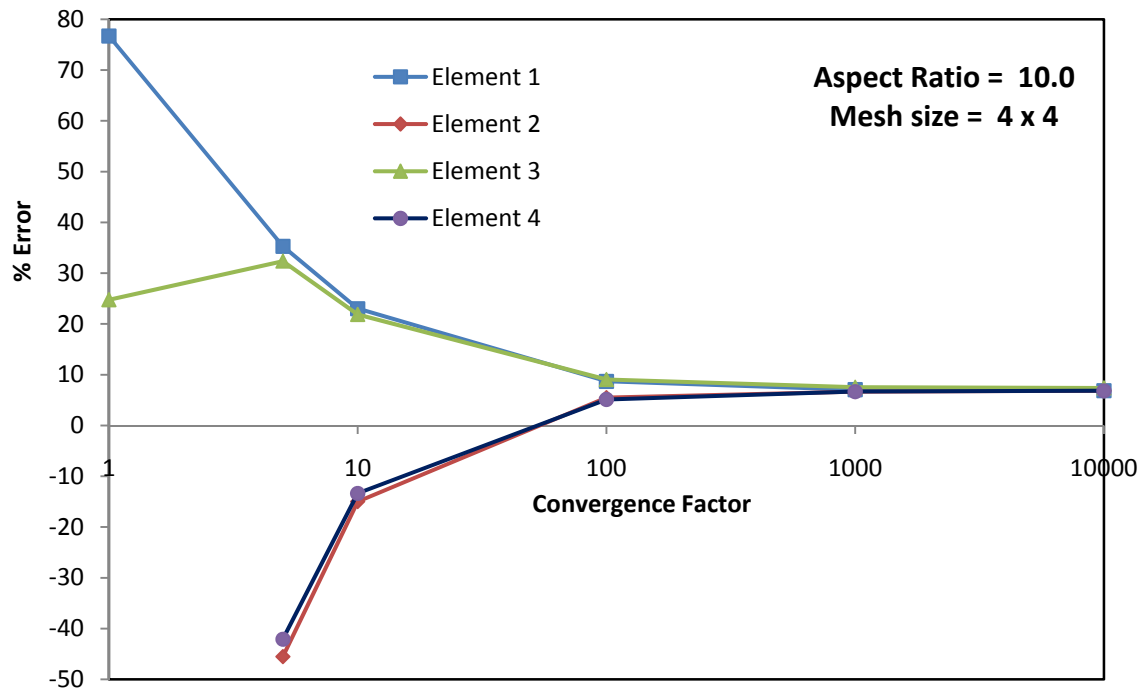


Fig-2_SSP_AR_10_NEL_4x4

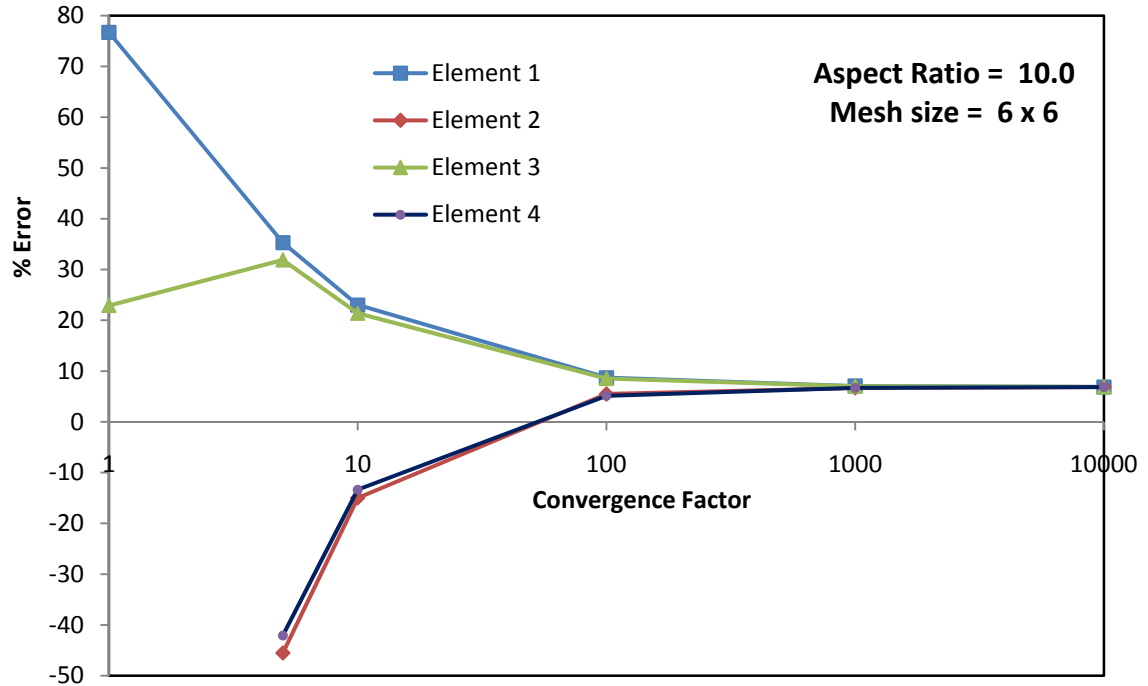


Fig-3_SSP_AR_10_NEL_6x6

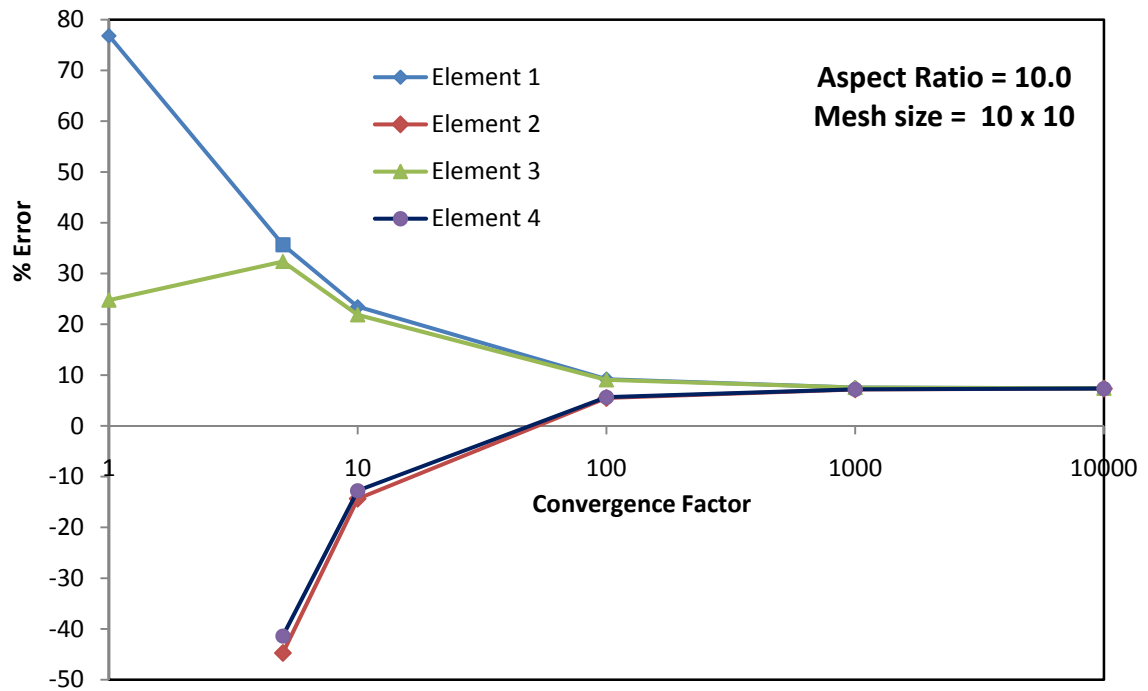


Fig-4_SSP_AR_10_NEL_10x10

Simply Supported plate with UDL
Error in deflection at the centre of the plate
Aspect ratio = 20

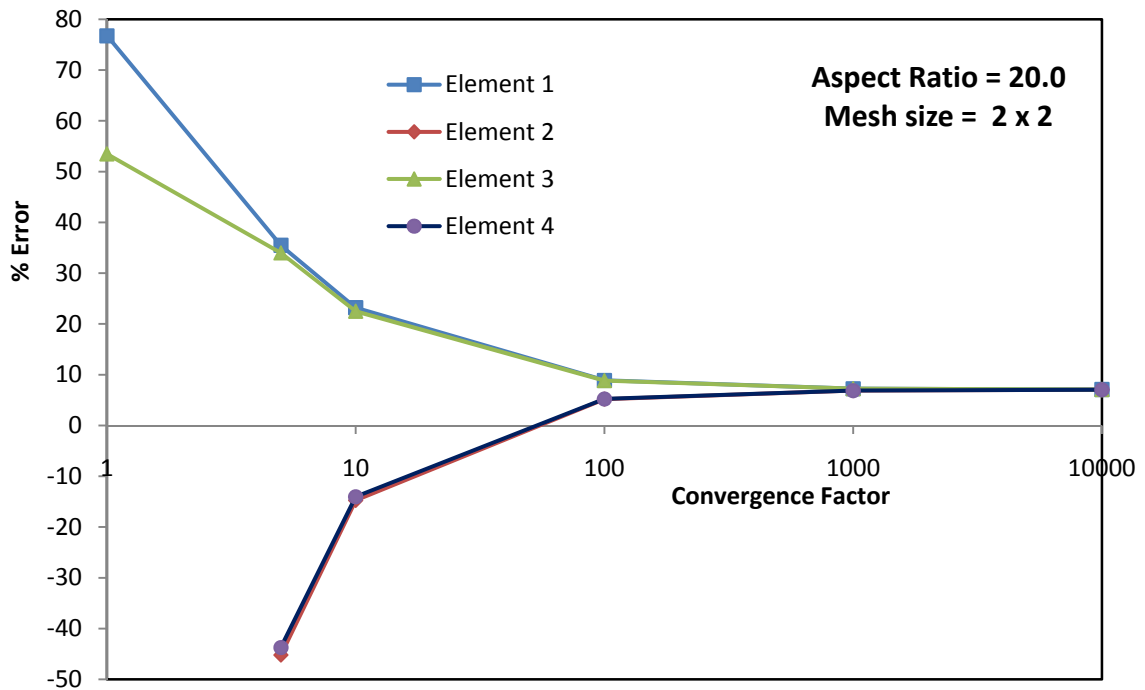


Fig-1_SSP_AR_20_NEL_2x2

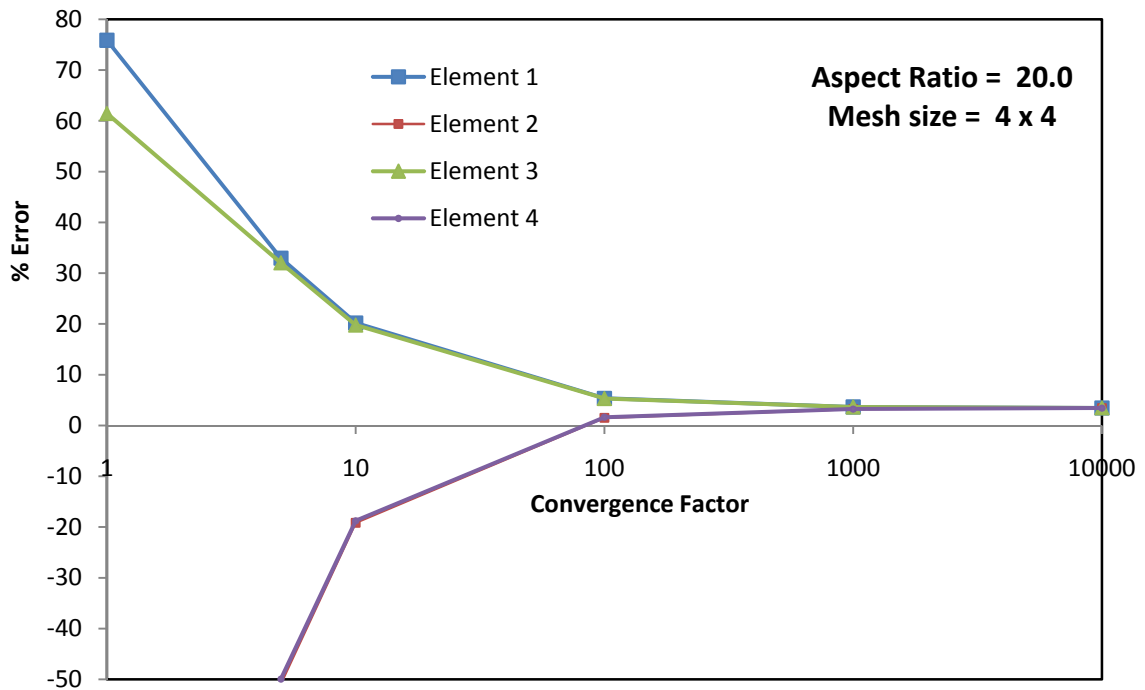


Fig-2_SSP_AR_20_NEL_4x4

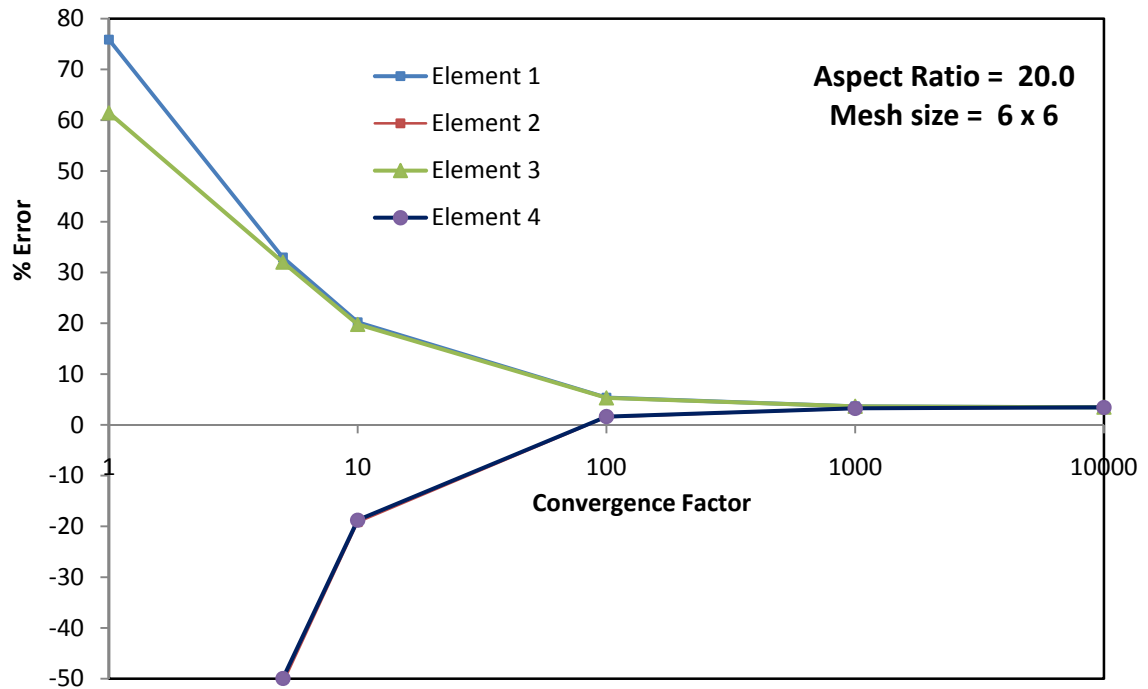


Fig-3_SSP_AR_20_NEL_6x6

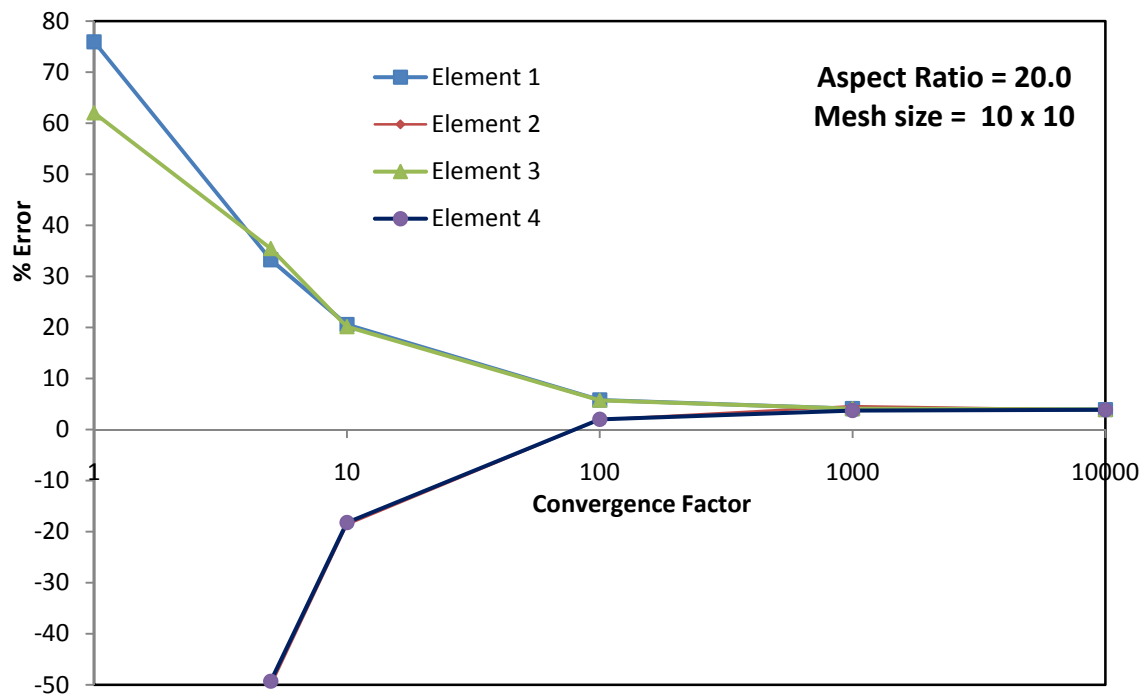
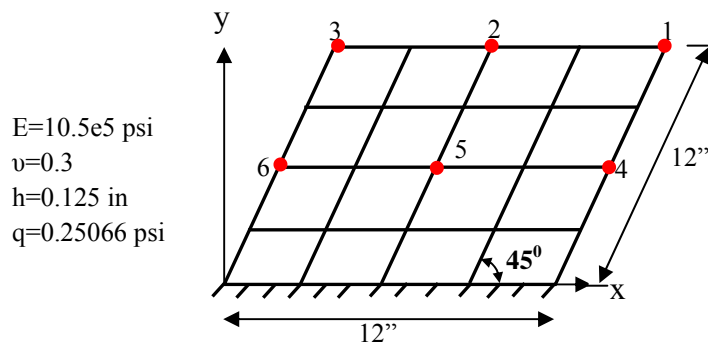


Fig-4_SSP_AR_20_NEL_10x10

Rhombic Plate under UDL

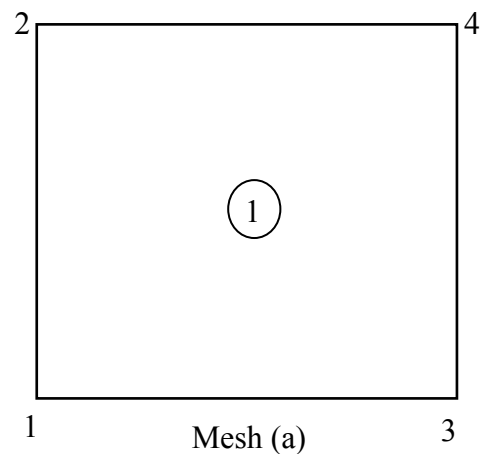
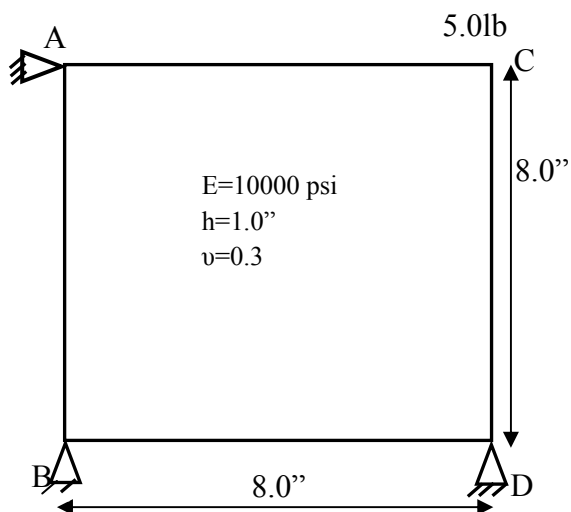


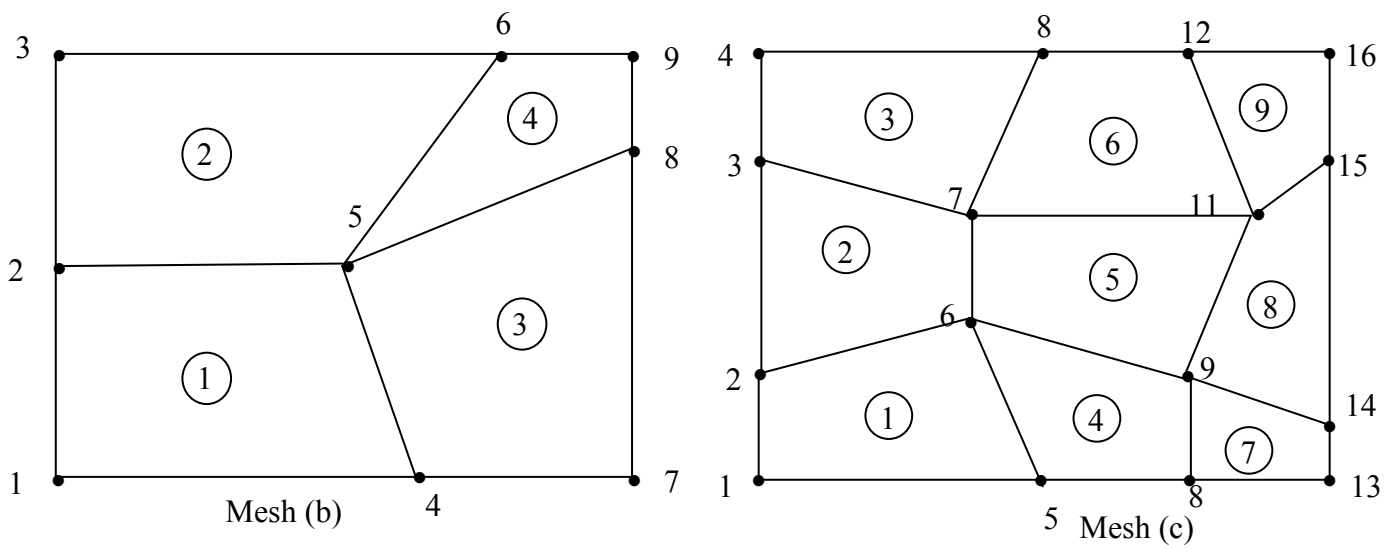
| Element | Mesh | DOF | 1 | 2 | 3 | 4 | 5 | 6 |
|----------------------------|-------|-----|---------------|---------------|---------------|---------------|----------------|----------------|
| MR_FE_1¹ | 4 x 4 | 75 | 0.3132 | 0.2059 | 0.1144 | 0.1255 | 0.05239 | 0.02053 |
| MR_FE_2¹ | 4 x 4 | 75 | 0.3133 | 0.2010 | 0.1144 | 0.1256 | 0.05241 | 0.02054 |
| MR_FE_3¹ | 4 x 4 | 75 | 0.3132 | 0.2059 | 0.1144 | 0.1255 | 0.05239 | 0.02053 |
| MR_FE_4¹ | 4 x 4 | 75 | 0.3133 | 0.2060 | 0.1144 | 0.1256 | 0.05241 | 0.02054 |
| DKT² | 4 x 4 | 75 | 0.304 | 0.198 | 0.113 | 0.121 | 0.055 | 0.028 |
| HSM | 4 x 4 | 75 | 0.264 | 0.173 | 0.100 | 0.095 | 0.043 | 0.021 |
| ACM | 8 x 6 | 189 | 0.296 | 0.198 | 0.114 | 0.114 | 0.052 | 0.020 |
| HCT | 8 x 6 | 189 | 0.281 | 0.188 | 0.111 | 0.111 | 0.049 | 0.018 |
| Experimental Value | | | 0.297 | 0.204 | 0.121 | 0.129 | 0.056 | 0.022 |

¹ **one point integration;** ² Three point integration;

DKT – Discrete Krichhoff Theory element , Jean-Louis Batoz et al, A study of three-node triangular plate bending elements, international journal for numerical methods in engineering, Vol. 15, 1771-1812 (1980).

Twisting of a square plate





Mesh (b)

| Node No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------|---|---|---|---|---|-----|---|-----|---|
| X-co | 0 | 0 | 0 | 5 | 5 | 6.2 | 8 | 8 | 8 |
| Y-co | 0 | 4 | 8 | 0 | 4 | 8 | 0 | 6.2 | 8 |

Mesh (c)

| Node No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| X-co | 0 | 0 | 0 | 0 | 4 | 3 | 3 | 4 | 6 | 6 | 7 | 6 | 8 | 8 | 8 | 8 |
| Y-co | 0 | 2 | 6 | 8 | 0 | 3 | 5 | 8 | 0 | 2 | 5 | 8 | 0 | 1 | 5 | 8 |

| Element Type | DKT | HSM | ACM | HCT | Mesh | MR_FE_1 | MR_FE_2 | MR_FE_3 | MR_FE_4 |
|-------------------------------------|---------|---------|--------|---------|------|---------|---------|---------|---------|
| Point C* | 0.24960 | 0.24960 | .24972 | 0.25002 | a | 0.25740 | 0.25740 | 0.25742 | 0.25739 |
| | | | | | b | 0.18963 | 0.18963 | 0.18945 | 0.19218 |
| | | | | | c | 0.19229 | 0.19229 | 0.19218 | 0.19239 |
| 0.24960 (Exact thin plate solution) | | | | | | | | | |

* Three point integration

New Numerical algorithm

Currently I am developing an efficient numerical method (based on Newton-Raphson method) to find out the roots of real-valued functions in one variable. The preliminary results for some functions are given below and the refinement of this work is going on. The time achievement is also being examined.

Polynomial functions

* -Two roots can be obtained at the same time. In this case both are the same.

1. **Function:** $x^3 - 3x^2 - 9x + 27 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 25000 | 3.000000000 | 296 | -3.000000000 | 7 | -3.000000000 | 7 |
| 15000 | 2.999999999 | 623 | -3.000000000 | 7 | -3.000000000 | 7 |
| 10000 | 2.999999999 | 458 | -3.000000000 | 7 | -3.000000000 | 7 |
| 7000 | 3.000000000 | 322 | -3.000000000 | 7 | -3.000000000 | 7 |
| 500 | 2.999999999 | 292 | -3.000000000 | 7 | -3.000000000 | 7 |
| 10 | 3.000000000 | 122 | -3.000000000 | 7 | -3.000000000 | 7 |
| 1 | 3.000000000 | 680 | ----- | ----- | 3.000000000 | 126,7 |
| -1 | ----- | ----- | -3.000000000 | 6,11 | ----- | ----- |
| -5 | -3.000000000 | 6 | -3.000000000 | 7 | -3.000000000 | 7,9 |
| -600 | -3.000000000 | 18 | -3.000000000 | 7 | -3.000000000 | 19,19 |
| -9000 | -3.000000000 | 25 | -3.000000000 | 7 | -3.000000000 | 26,26 |
| -12500 | -3.000000000 | 26 | -3.000000000 | 7 | -3.000000000 | 27,27 |
| -26000 | -3.000000000 | 27 | -3.000000000 | 7 | -3.000000000 | 28,28 |

2. Function: $x^5 - 11x^4 + 46x^3 - 90x^2 + 81x - 27 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 98765 | 3.000009907 | 1475 | 1.000000000 | 2,2 | 1.000000000 | 635 |
| 3995 | 2.999989503 | 1251 | 1.000000000 | 2,2 | 1.000000000 | 635 |
| 987 | ----- | ----- | 1.000000000 | 2,2 | 1.000000000 | 635 |
| 195 | 2.999982054 | 1233 | 1.000000000 | 2,2 | 1.000000000 | 635 |
| -95 | 0.999999999 | 547 | 1.000000000 | 2,2 | 1.000000000 | 198 |
| -987 | 1.000000009 | 1522 | 1.000000000 | 2,2 | 0.999999999 | 70 |
| -9865 | 0.999999999 | 368 | 1.000000000 | 2,2 | 1.000000013 | 326 |
| -98765 | 1.000000012 | 95 | 1.000000000 | 2,2 | 1.000000020 | 94 |

3. Function: $x^3 - x - 1.0 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 126779 | 1.324717957 | 33 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 87765 | 1.324717957 | 33 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 8725 | 1.324717957 | 28 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 675 | 1.324717957 | 26 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 65 | 1.324717957 | 15 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 1 | 1.324717957 | 6 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| -1 | 1.324717957 | 22 | 1.324717957 | 5,24 | 1.324717957 | 15,19 |
| -107 | 1.324717957 | 21 | 1.324717957 | 5,24 | 1.324717957 | 37,42 |
| -9008 | 1.324717957 | 48 | 1.324717957 | 5,24 | 1.324717957 | 42,50 |
| -77864 | 1.324717957 | 77 | 1.324717957 | 5,24 | 1.324717957 | 62,69 |

4. Function: $x^3 - 2x^2 - 11x - 12 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------------------|--------------------|-----------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 987432 | 4.000000000 | 37 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 87546 | 4.000000000 | 31 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 576 | 4.000000000 | 19 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 61 | 4.000000000 | 13 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 1 | 1.000000000 | 1 | 4.000000000 1.000000000 | 1 10 | ----- | ---- |
| -1 | 4.000000000 | 2 | -3.000000000 1.000000000 | 6 6 | -3.000000000 1.000000000 | 19 5 |
| -68 | -3.000000000 | 13 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| -743 | -3.000000000 | 19 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| -56732 | -3.000000000 | 30 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| -487472 | -3.000000000 | 35 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |

5. Function $x^3 - 2x + 2 = 0$ (Special case)

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 874531 | -1.769292354 | 67 | -1.769292354 | 6,17 | ---- | ---- |
| 34561 | ----- | ----- | -1.769292354 | 6,17 | | |
| 562 | -1.769292354 | 107 | -1.769292354 | 6,17 | | |
| 43 | ----- | --- | -1.769292354 | 6,17 | | |
| 2 | -1.769292354 | 10 | -1.769292354 | 6,17 | | |
| -3 | -1.769292354 | 7 | -1.769292354 | 6,17 | | |
| -71 | -1.769292354 | 14 | -1.769292354 | 6,17 | | |
| -8934 | -1.769292354 | 27 | -1.769292354 | 6,17 | | |
| -76987 | -1.769292354 | 31 | -1.769292354 | 6,17 | | |

6. Function: $x^2 - 612.0 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 345698 | 24.7386337537 | 19 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| 14567 | 24.7386337537 | 14 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| 456 | 24.7386337537 | 9 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| 5 | 24.7386337537 | 8 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| -7 | -24.7386337537 | 7 | 24.7386337537 | 2 | 24.7386337537 | 9 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 9 |
| -654 | -24.7386337537 | 10 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| -19654 | -24.7386337537 | 15 | 24.7386337537 | 2 | 24.7386337537 | 8 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 8 |
| -786542 | -24.7386337537 | 20 | 24.7386337537 | 2 | 24.7386337537 | 11 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 11 |

7. Function: $(x - 1)^3 + 0.512 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 99998.5 | 1.800000012 | 34 | 1.800000012 | 9,13 | 1.800000012 | 17,10 |
| 12675.5 | 1.800000012 | 29 | 1.800000012 | 9,13 | 1.800000012 | 17,10 |
| 175.5 | 1.800000012 | 19 | 1.800000012 | 9,13 | 1.800000012 | 17,10 |
| 1.5 | 1.800000012 | 7 | 1.800000012 | 9,13 | 1.800000012 | 17,7 |
| -0.5 | 1.800000012 | 8 | 1.800000012 | 5,9 | 1.800000012 | 12,20 |
| -4.5 | 1.800000012 | 14 | 1.800000012 | 5,9 | 1.800000012 | 19,19 |
| -453.5 | 1.800000012 | 21 | 1.800000012 | 5,9 | 1.800000012 | 23,25 |
| -15673.5 | 1.800000012 | 34 | 1.800000012 | 5,9 | 1.800000012 | 35,35 |
| -78432.5 | 1.800000012 | 38 | 1.800000012 | 5,9 | 1.800000012 | 39,39 |

8. Function: $x^3 - x^2 - 10x - 8 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 876234 | 4.000000000 | 36 | -1.000000000 | 6 | -1.000000000 | 1 |
| | | | -2.000000000 | 11 | -2.000000000 | 8 |
| 34672 | 4.000000000 | 28 | -1.000000000 | 6 | -1.000000000 | 1 |
| | | | -2.000000000 | 11 | -2.000000000 | 8 |
| 764 | 4.000000000 | 19 | -1.000000000 | 6 | -1.000000000 | 1 |
| | | | -2.000000000 | 11 | -2.000000000 | 8 |
| 1 | -1.000000000 | 1 | -1.000000000 | 7 | -1.000000000 | 1 |
| | | | 4.000000000 | 8 | -2.000000000 | 8 |
| -1 | -1.000000000 | 1 | -1.000000000 | 1 | -1.000000000 | 1 |
| | | | -2.000000000 | 7 | -2.000000000 | 8 |
| -467 | -2.000000000 | 20 | -1.000000000 | 6 | -2.000000000 | 21,21 |
| | | | -2.000000000 | 11 | | |
| -6542 | -2.000000000 | 26 | -1.000000000 | 6 | -2.000000000 | 27,27 |
| | | | -2.000000000 | 11 | | |
| -87321 | -2.000000000 | 33 | -1.000000000 | 6 | -2.000000000 | 34,34 |
| | | | -2.000000000 | 11 | | |

9. Function: $x^3 - 3x^2 + 4 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500000 | 2.000000000 | 90 | -1.000000000 | 6 | -1.000000000 | 6 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| 73256 | 2.000000000 | 55 | -1.000000000 | 6 | -1.000000000 | 7 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| 235 | 2.000000000 | 41 | -1.000000000 | 6 | -1.000000000 | 7 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| 5 | 2.000000000 | 76 | -1.000000000 | 6 | -1.000000000 | 7 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| -10 | -1.000000000 | 10 | -1.000000000 | 6 | -1.000000000 | 11 |
| | | | 2.000000000 | 28 | -1.000000000 | 12 |
| -563 | -1.000000000 | 20 | -1.000000000 | 6 | -1.000000000 | 11 |
| | | | 2.000000000 | 28 | -1.000000000 | 12 |
| -96342 | -1.000000000 | 32 | -1.000000000 | 6 | -1.000000000 | 33 |
| | | | 2.000000000 | 28 | -1.000000000 | 33 |

10. Function: $x^4 + 3x - 4 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 107890 | 1.000000000 | 45 | 1.000000000 | 2,8 | 1.000000000 -1.742959202 | 2 12 |
| 76543 | 1.000000000 | 45 | 1.000000000 | 2,7 | 1.000000000 -1.742959202 | 2 12 |
| 546 | 1.000000000 | 27 | 1.000000000 | 2,8 | 1.000000000 -1.742959202 | 2 12 |
| 3 | 1.000000000 | 9 | 1.000000000 | 2,8 | 1.000000000 -1.742959202 | 2 12 |
| -2 | -1.742959202 | 6 | -1.742959202 | 5,32 | -1.742959202 | 10,25 |
| -4321 | -1.742959202 | 32 | -1.742959202 | 5,8 | -1.742959202 | 34, >1500 |
| -432678 | -1.742959202 | 49 | 1.000000000 | 2,8 | -1.742959202 | 50, >1500 |

11. Function: $x^3 - 0.165x^2 + 3.993e - 04 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|------------------|--------------------|------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 100000 | 0.146359512 | 40 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| 35678 | 0.146359512 | 38 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| 453 | 0.146359512 | 27 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| 1.0 | 0.146359512 | 27 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| -15 | -4.373708626E-02 | 19 | 6.237758018E-02 | 5 | -4.373708626E-02 | 23,23 |
| | | | -4.373708626E-02 | 6 | | |
| -6743 | -4.373708626E-02 | 34 | 6.237758018E-02 | 5 | -4.373708626E-02 | 23,23 |
| | | | -4.373708626E-02 | 6 | | |
| -98456 | -4.373708626E-02 | 40 | 6.237758018E-02 | 5 | -4.373708626E-02 | 35,35 |
| | | | -4.373708626E-02 | 6 | | |

12. Function: $x^3 - 0.03x^2 + 2.4e - 04 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|------------------|--------------------|-------------------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 100000 | 2.661081797E-02 | 45 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| 67543 | 2.661081797E-02 | 44 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| 723 | 2.661081797E-02 | 32 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| 2 | 2.661081797E-02 | 18 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| -20 | -7.952197683E-03 | 18 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 36 20 |
| -67845 | -7.952197683E-03 | 43 | -7.952197642E-03 | 6,13 | -7.952197683E-03 | 44 |
| -120007 | -7.952197683E-03 | 43 | -7.952197642E-03 | 6,13 | -7.952197683E-03 | 44 |

13. Function: $2x^3 + 3.5x^2 - 4.21x + 0.546 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|----------------------------|--------------------|-----------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 564327 | 0.700000007 | 38 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| 5637 | 0.700000007 | 28 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| 74 | 0.700000007 | 17 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| 0 | 0.149999999 | 5 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| -13 | -2.600000005 | 10 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| -2451 | -2.600000005 | 23 | 0.700000006 0.149999999 | 8 5 | -2.600000005 | 24,25 |
| -123867 | -2.600000005 | 33 | 0.700000006 0.149999999 | 8 5 | -2.600000005 | 24,25 |

14. Function: $0.004x^3 - 0.2x^2 + 2.45x - 5.4 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|------------------------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 34562 | 32.363031067 | 20 | 32.363031067 2.814202526 | 12 5 | 2.814202526 | 6,8 |
| 6534 | 32.363031067 | 21 | 32.363031067 2.814202526 | 12 5 | 2.814202526 | 6,8 |
| 31 | 32.363031067 | 5 | 32.363031067 14.822764775 | 4 6 | 32.363031067 | 5,8 |
| 0 | 2.814202526 | 6 | 2.814202526 32.363031067 | 5 12 | 32.363031067 | 5,8 |
| -10 | 2.814202526 | 8 | 2.814202526 32.363031067 | 5 12 | 2.814202526 | 8,9 |
| -5678 | 2.814202526 | 21 | 2.814202526 32.363031067 | 5 12 | 2.814202526 | 8,9 |
| -26524 | 2.814202526 | 21 | 2.814202526 32.363031067 | 5 12 | 2.814202526 | 8,9 |

Transcendental functions

1. Function: $e^{-x} - x = 0$

| | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|--------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 100000 | 0.567143290 | 6 | 0.567143290 | 6,7 | 0.567143290 | 6,7 |
| 8525 | 0.567143290 | 6 | 0.567143290 | 5,7 | 0.567143290 | 6,9 |
| 525 | 0.567143290 | 6 | 0.567143290 | 6,7 | 0.567143290 | 6,7 |
| 25 | 0.567143290 | 6 | 0.567143290 | 6,7 | 0.567143290 | 6,9 |
| 1 | 0.567143290 | 5 | 0.567143290 | 5,7 | 0.567143290 | 6,7 |
| -1 | 0.567143290 | 6 | 0.567143290 | 5,7 | 0.567143290 | 7,10 |
| -20 | 0.567143290 | 25 | 0.567143290 | 5,7 | 0.567143290 | 26,28 |
| -105 | 0.567143290 | 110 | 0.567143290 | 5,7 | 0.567143290 | 111,113 |
| -515 | 0.567143290 | 520 | 0.567143290 | 5,7 | 0.567143290 | 521,523 |

2. Function: $xe^x - 3.0 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500 | 1.049908899 | 500 | 1.049908899 | 4,16 | | |
| 150 | 1.049908899 | 159 | 1.049908899 | 4,16 | | |
| 11 | 1.049908899 | 18 | 1.049908899 | 4,16 | | |
| 0 | 1.049908899 | 9 | ----- | ---- | | |
| -1 | ---- | ----- | ---- | ----- | | |
| -10 | | | 1.049908899 | 4,16 | | |
| -135 | | | 1.049908899 | 4,16 | | |
| -550 | | | 1.049908899 | 4,16 | | |

3. Function: $e^x - 3x = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500 | 1.512134551 | 506 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| 200 | 1.512134551 | 206 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| 53 | 1.512134551 | 59 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| 0 | 1.512134551 | 6 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| -24 | 1.512134551 | 7 | ----- | ----- | 0.619061286 | 8 |
| | | | | | 1.512134551 | 31 |
| -2234 | 1.512134551 | 7 | 0.619061286 | 5 | ----- | ----- |
| | | | 1.512134551 | 6 | | |
| -67456 | 1.512134551 | 7 | 0.619061286 | 5 | ----- | ----- |
| | | | 1.512134551 | 6 | | |

4. Function: $e^x + x - 2 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500 | 0.442854401 | 505 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| 150 | 0.442854401 | 155 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| 2 | 0.442854401 | 7 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| 0 | 0.442854401 | 5 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| -75 | 0.442854401 | 8 | 0.442854401 | 6,7 | 0.442854401 | 5,8 |
| -2350 | 0.442854401 | 8 | 0.442854401 | 5,9 | 0.442854401 | 9,9 |
| -98754 | 0.442854401 | 8 | 0.442854401 | 5,9 | 0.442854401 | 9,9 |

Conclusion

1. The first algorithm is independent of initial guess.
2. It converges very faster than the other two, that is, the Newton-Raphson method and New Method-2.
3. The modifications carried out in the present two algorithms are very simple and the mathematical expressions used are very short statements involving only arithmetic operations.
4. The performance of the first algorithm is far better than that of the other two and in some cases the performance of the second algorithm is similar to that of the Newton-Raphson method.
5. In some cases, two different roots are simultaneously obtained by these two algorithms. In the case of the NR method, it is not possible.

Future Work

The above work is extended to the analysis of a system of non linear equations in 'n' variables to examine the performance of the algorithm.

New Numerical algorithm

Currently I am developing an efficient numerical method (based on Newton-Raphson method) to find out the roots of real-valued functions in one variable. The preliminary results for some functions are given below and the refinement of this work is going on. The time achievement is also being examined.

Polynomial functions

* -Two roots can be obtained at the same time. In this case both are the same.

1. **Function:** $x^3 - 3x^2 - 9x + 27 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 25000 | 3.000000000 | 296 | -3.000000000 | 7 | -3.000000000 | 7 |
| 15000 | 2.999999999 | 623 | -3.000000000 | 7 | -3.000000000 | 7 |
| 10000 | 2.999999999 | 458 | -3.000000000 | 7 | -3.000000000 | 7 |
| 7000 | 3.000000000 | 322 | -3.000000000 | 7 | -3.000000000 | 7 |
| 500 | 2.999999999 | 292 | -3.000000000 | 7 | -3.000000000 | 7 |
| 10 | 3.000000000 | 122 | -3.000000000 | 7 | -3.000000000 | 7 |
| 1 | 3.000000000 | 680 | ----- | ----- | 3.000000000 | 126,7 |
| -1 | ----- | ----- | -3.000000000 | 6,11 | ----- | ----- |
| -5 | -3.000000000 | 6 | -3.000000000 | 7 | -3.000000000 | 7,9 |
| -600 | -3.000000000 | 18 | -3.000000000 | 7 | -3.000000000 | 19,19 |
| -9000 | -3.000000000 | 25 | -3.000000000 | 7 | -3.000000000 | 26,26 |
| -12500 | -3.000000000 | 26 | -3.000000000 | 7 | -3.000000000 | 27,27 |
| -26000 | -3.000000000 | 27 | -3.000000000 | 7 | -3.000000000 | 28,28 |

2. Function: $x^5 - 11x^4 + 46x^3 - 90x^2 + 81x - 27 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 98765 | 3.000009907 | 1475 | 1.000000000 | 2,2 | 1.000000000 | 635 |
| 3995 | 2.999989503 | 1251 | 1.000000000 | 2,2 | 1.000000000 | 635 |
| 987 | ----- | ----- | 1.000000000 | 2,2 | 1.000000000 | 635 |
| 195 | 2.999982054 | 1233 | 1.000000000 | 2,2 | 1.000000000 | 635 |
| -95 | 0.999999999 | 547 | 1.000000000 | 2,2 | 1.000000000 | 198 |
| -987 | 1.000000009 | 1522 | 1.000000000 | 2,2 | 0.999999999 | 70 |
| -9865 | 0.999999999 | 368 | 1.000000000 | 2,2 | 1.000000013 | 326 |
| -98765 | 1.000000012 | 95 | 1.000000000 | 2,2 | 1.000000020 | 94 |

3. Function: $x^3 - x - 1.0 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 126779 | 1.324717957 | 33 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 87765 | 1.324717957 | 33 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 8725 | 1.324717957 | 28 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 675 | 1.324717957 | 26 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 65 | 1.324717957 | 15 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| 1 | 1.324717957 | 6 | 1.324717957 | 5,24 | 1.324717957 | 6,17 |
| -1 | 1.324717957 | 22 | 1.324717957 | 5,24 | 1.324717957 | 15,19 |
| -107 | 1.324717957 | 21 | 1.324717957 | 5,24 | 1.324717957 | 37,42 |
| -9008 | 1.324717957 | 48 | 1.324717957 | 5,24 | 1.324717957 | 42,50 |
| -77864 | 1.324717957 | 77 | 1.324717957 | 5,24 | 1.324717957 | 62,69 |

4. Function: $x^3 - 2x^2 - 11x - 12 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------------------|--------------------|-----------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 987432 | 4.000000000 | 37 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 87546 | 4.000000000 | 31 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 576 | 4.000000000 | 19 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 61 | 4.000000000 | 13 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| 1 | 1.000000000 | 1 | 4.000000000 1.000000000 | 1 10 | ----- | ---- |
| -1 | 4.000000000 | 2 | -3.000000000 1.000000000 | 6 6 | -3.000000000 1.000000000 | 19 5 |
| -68 | -3.000000000 | 13 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| -743 | -3.000000000 | 19 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| -56732 | -3.000000000 | 30 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |
| -487472 | -3.000000000 | 35 | -3.000000000 1.000000000 | 8 5 | -3.000000000 1.000000000 | 19 5 |

5. Function $x^3 - 2x + 2 = 0$ (Special case)

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 874531 | -1.769292354 | 67 | -1.769292354 | 6,17 | ---- | ---- |
| 34561 | ----- | ----- | -1.769292354 | 6,17 | | |
| 562 | -1.769292354 | 107 | -1.769292354 | 6,17 | | |
| 43 | ----- | --- | -1.769292354 | 6,17 | | |
| 2 | -1.769292354 | 10 | -1.769292354 | 6,17 | | |
| -3 | -1.769292354 | 7 | -1.769292354 | 6,17 | | |
| -71 | -1.769292354 | 14 | -1.769292354 | 6,17 | | |
| -8934 | -1.769292354 | 27 | -1.769292354 | 6,17 | | |
| -76987 | -1.769292354 | 31 | -1.769292354 | 6,17 | | |

6. Function: $x^2 - 612.0 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 345698 | 24.7386337537 | 19 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| 14567 | 24.7386337537 | 14 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| 456 | 24.7386337537 | 9 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| 5 | 24.7386337537 | 8 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| -7 | -24.7386337537 | 7 | 24.7386337537 | 2 | 24.7386337537 | 9 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 9 |
| -654 | -24.7386337537 | 10 | 24.7386337537 | 2 | 24.7386337537 | 6 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 6 |
| -19654 | -24.7386337537 | 15 | 24.7386337537 | 2 | 24.7386337537 | 8 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 8 |
| -786542 | -24.7386337537 | 20 | 24.7386337537 | 2 | 24.7386337537 | 11 |
| | | | -24.7386337537 | 2 | -24.7386337537 | 11 |

7. Function: $(x - 1)^3 + 0.512 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 99998.5 | 1.800000012 | 34 | 1.800000012 | 9,13 | 1.800000012 | 17,10 |
| 12675.5 | 1.800000012 | 29 | 1.800000012 | 9,13 | 1.800000012 | 17,10 |
| 175.5 | 1.800000012 | 19 | 1.800000012 | 9,13 | 1.800000012 | 17,10 |
| 1.5 | 1.800000012 | 7 | 1.800000012 | 9,13 | 1.800000012 | 17,7 |
| -0.5 | 1.800000012 | 8 | 1.800000012 | 5,9 | 1.800000012 | 12,20 |
| -4.5 | 1.800000012 | 14 | 1.800000012 | 5,9 | 1.800000012 | 19,19 |
| -453.5 | 1.800000012 | 21 | 1.800000012 | 5,9 | 1.800000012 | 23,25 |
| -15673.5 | 1.800000012 | 34 | 1.800000012 | 5,9 | 1.800000012 | 35,35 |
| -78432.5 | 1.800000012 | 38 | 1.800000012 | 5,9 | 1.800000012 | 39,39 |

8. Function: $x^3 - x^2 - 10x - 8 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 876234 | 4.000000000 | 36 | -1.000000000 | 6 | -1.000000000 | 1 |
| | | | -2.000000000 | 11 | -2.000000000 | 8 |
| 34672 | 4.000000000 | 28 | -1.000000000 | 6 | -1.000000000 | 1 |
| | | | -2.000000000 | 11 | -2.000000000 | 8 |
| 764 | 4.000000000 | 19 | -1.000000000 | 6 | -1.000000000 | 1 |
| | | | -2.000000000 | 11 | -2.000000000 | 8 |
| 1 | -1.000000000 | 1 | -1.000000000 | 7 | -1.000000000 | 1 |
| | | | 4.000000000 | 8 | -2.000000000 | 8 |
| -1 | -1.000000000 | 1 | -1.000000000 | 1 | -1.000000000 | 1 |
| | | | -2.000000000 | 7 | -2.000000000 | 8 |
| -467 | -2.000000000 | 20 | -1.000000000 | 6 | -2.000000000 | 21,21 |
| | | | -2.000000000 | 11 | | |
| -6542 | -2.000000000 | 26 | -1.000000000 | 6 | -2.000000000 | 27,27 |
| | | | -2.000000000 | 11 | | |
| -87321 | -2.000000000 | 33 | -1.000000000 | 6 | -2.000000000 | 34,34 |
| | | | -2.000000000 | 11 | | |

9. Function: $x^3 - 3x^2 + 4 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500000 | 2.000000000 | 90 | -1.000000000 | 6 | -1.000000000 | 6 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| 73256 | 2.000000000 | 55 | -1.000000000 | 6 | -1.000000000 | 7 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| 235 | 2.000000000 | 41 | -1.000000000 | 6 | -1.000000000 | 7 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| 5 | 2.000000000 | 76 | -1.000000000 | 6 | -1.000000000 | 7 |
| | | | 2.000000000 | 28 | 2.000000000 | 28 |
| -10 | -1.000000000 | 10 | -1.000000000 | 6 | -1.000000000 | 11 |
| | | | 2.000000000 | 28 | -1.000000000 | 12 |
| -563 | -1.000000000 | 20 | -1.000000000 | 6 | -1.000000000 | 11 |
| | | | 2.000000000 | 28 | -1.000000000 | 12 |
| -96342 | -1.000000000 | 32 | -1.000000000 | 6 | -1.000000000 | 33 |
| | | | 2.000000000 | 28 | -1.000000000 | 33 |

10. Function: $x^4 + 3x - 4 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 107890 | 1.000000000 | 45 | 1.000000000 | 2,8 | 1.000000000 -1.742959202 | 2 12 |
| 76543 | 1.000000000 | 45 | 1.000000000 | 2,7 | 1.000000000 -1.742959202 | 2 12 |
| 546 | 1.000000000 | 27 | 1.000000000 | 2,8 | 1.000000000 -1.742959202 | 2 12 |
| 3 | 1.000000000 | 9 | 1.000000000 | 2,8 | 1.000000000 -1.742959202 | 2 12 |
| -2 | -1.742959202 | 6 | -1.742959202 | 5,32 | -1.742959202 | 10,25 |
| -4321 | -1.742959202 | 32 | -1.742959202 | 5,8 | -1.742959202 | 34, >1500 |
| -432678 | -1.742959202 | 49 | 1.000000000 | 2,8 | -1.742959202 | 50, >1500 |

11. Function: $x^3 - 0.165x^2 + 3.993e - 04 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|------------------|--------------------|------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 100000 | 0.146359512 | 40 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| 35678 | 0.146359512 | 38 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| 453 | 0.146359512 | 27 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| 1.0 | 0.146359512 | 27 | 6.237758018E-02 | 5 | 0.146359512 | 11 |
| | | | -4.373708626E-02 | 6 | -4.373708626E-02 | 12 |
| -15 | -4.373708626E-02 | 19 | 6.237758018E-02 | 5 | -4.373708626E-02 | 23,23 |
| | | | -4.373708626E-02 | 6 | | |
| -6743 | -4.373708626E-02 | 34 | 6.237758018E-02 | 5 | -4.373708626E-02 | 23,23 |
| | | | -4.373708626E-02 | 6 | | |
| -98456 | -4.373708626E-02 | 40 | 6.237758018E-02 | 5 | -4.373708626E-02 | 35,35 |
| | | | -4.373708626E-02 | 6 | | |

12. Function: $x^3 - 0.03x^2 + 2.4e - 04 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|------------------|--------------------|-------------------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 100000 | 2.661081797E-02 | 45 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| 67543 | 2.661081797E-02 | 44 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| 723 | 2.661081797E-02 | 32 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| 2 | 2.661081797E-02 | 18 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 16 36 |
| -20 | -7.952197683E-03 | 18 | -7.952197642E-03 | 6,13 | 2.661081797E-02 -7.952197683E-03 | 36 20 |
| -67845 | -7.952197683E-03 | 43 | -7.952197642E-03 | 6,13 | -7.952197683E-03 | 44 |
| -120007 | -7.952197683E-03 | 43 | -7.952197642E-03 | 6,13 | -7.952197683E-03 | 44 |

13. Function: $2x^3 + 3.5x^2 - 4.21x + 0.546 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|----------------------------|--------------------|-----------------------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 564327 | 0.700000007 | 38 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| 5637 | 0.700000007 | 28 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| 74 | 0.700000007 | 17 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| 0 | 0.149999999 | 5 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| -13 | -2.600000005 | 10 | 0.700000006 0.149999999 | 8 5 | -2.600000005 0.149999999 | 8 6 |
| -2451 | -2.600000005 | 23 | 0.700000006 0.149999999 | 8 5 | -2.600000005 | 24,25 |
| -123867 | -2.600000005 | 33 | 0.700000006 0.149999999 | 8 5 | -2.600000005 | 24,25 |

14. Function: $0.004x^3 - 0.2x^2 + 2.45x - 5.4 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|------------------------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 34562 | 32.363031067 | 20 | 32.363031067 2.814202526 | 12 5 | 2.814202526 | 6,8 |
| 6534 | 32.363031067 | 21 | 32.363031067 2.814202526 | 12 5 | 2.814202526 | 6,8 |
| 31 | 32.363031067 | 5 | 32.363031067 14.822764775 | 4 6 | 32.363031067 | 5,8 |
| 0 | 2.814202526 | 6 | 2.814202526 32.363031067 | 5 12 | 32.363031067 | 5,8 |
| -10 | 2.814202526 | 8 | 2.814202526 32.363031067 | 5 12 | 2.814202526 | 8,9 |
| -5678 | 2.814202526 | 21 | 2.814202526 32.363031067 | 5 12 | 2.814202526 | 8,9 |
| -26524 | 2.814202526 | 21 | 2.814202526 32.363031067 | 5 12 | 2.814202526 | 8,9 |

Transcendental functions

1. Function: $e^{-x} - x = 0$

| | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|--------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 100000 | 0.567143290 | 6 | 0.567143290 | 6,7 | 0.567143290 | 6,7 |
| 8525 | 0.567143290 | 6 | 0.567143290 | 5,7 | 0.567143290 | 6,9 |
| 525 | 0.567143290 | 6 | 0.567143290 | 6,7 | 0.567143290 | 6,7 |
| 25 | 0.567143290 | 6 | 0.567143290 | 6,7 | 0.567143290 | 6,9 |
| 1 | 0.567143290 | 5 | 0.567143290 | 5,7 | 0.567143290 | 6,7 |
| -1 | 0.567143290 | 6 | 0.567143290 | 5,7 | 0.567143290 | 7,10 |
| -20 | 0.567143290 | 25 | 0.567143290 | 5,7 | 0.567143290 | 26,28 |
| -105 | 0.567143290 | 110 | 0.567143290 | 5,7 | 0.567143290 | 111,113 |
| -515 | 0.567143290 | 520 | 0.567143290 | 5,7 | 0.567143290 | 521,523 |

2. Function: $xe^x - 3.0 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500 | 1.049908899 | 500 | 1.049908899 | 4,16 | | |
| 150 | 1.049908899 | 159 | 1.049908899 | 4,16 | | |
| 11 | 1.049908899 | 18 | 1.049908899 | 4,16 | | |
| 0 | 1.049908899 | 9 | ----- | ---- | | |
| -1 | ---- | ----- | ---- | ----- | | |
| -10 | | | 1.049908899 | 4,16 | | |
| -135 | | | 1.049908899 | 4,16 | | |
| -550 | | | 1.049908899 | 4,16 | | |

3. Function: $e^x - 3x = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500 | 1.512134551 | 506 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| 200 | 1.512134551 | 206 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| 53 | 1.512134551 | 59 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| 0 | 1.512134551 | 6 | 0.619061286 | 5 | 0.619061286 | 7 |
| | | | 1.512134551 | 10 | 1.512134551 | 7 |
| -24 | 1.512134551 | 7 | ----- | ----- | 0.619061286 | 8 |
| | | | | | 1.512134551 | 31 |
| -2234 | 1.512134551 | 7 | 0.619061286 | 5 | ----- | ----- |
| | | | 1.512134551 | 6 | | |
| -67456 | 1.512134551 | 7 | 0.619061286 | 5 | ----- | ----- |
| | | | 1.512134551 | 6 | | |

4. Function: $e^x + x - 2 = 0$

| Initial Value | Newton-Raphson Method | | New Method-1 | | New Method-2 | |
|---------------|-----------------------|-------------------|-----------------|--------------------|-----------------|--------------------|
| | Converged Value | No. of Iterations | Converged Value | No. of Iterations* | Converged Value | No. of Iterations* |
| 500 | 0.442854401 | 505 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| 150 | 0.442854401 | 155 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| 2 | 0.442854401 | 7 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| 0 | 0.442854401 | 5 | 0.442854401 | 5,9 | 0.442854401 | 5,8 |
| -75 | 0.442854401 | 8 | 0.442854401 | 6,7 | 0.442854401 | 5,8 |
| -2350 | 0.442854401 | 8 | 0.442854401 | 5,9 | 0.442854401 | 9,9 |
| -98754 | 0.442854401 | 8 | 0.442854401 | 5,9 | 0.442854401 | 9,9 |

Conclusion

1. The first algorithm is independent of initial guess.
2. It converges very faster than the other two, that is, the Newton-Raphson method and New Method-2.
3. The modifications carried out in the present two algorithms are very simple and the mathematical expressions used are very short statements involving only arithmetic operations.
4. The performance of the first algorithm is far better than that of the other two and in some cases the performance of the second algorithm is similar to that of the Newton-Raphson method.
5. In some cases, two different roots are simultaneously obtained by these two algorithms. In the case of the NR method, it is not possible.

Future Work

The above work is extended to the analysis of a system of non linear equations in 'n' variables to examine the performance of the algorithm.