

Appendix

The incipient viscous flow of the cytosol can be assumed to be in the plane strain deformation state in the r - θ plane (Fig. 1b). The 2-D motion equations in the r - θ plane reduce to

$$\frac{\partial p}{\partial r} = \mu \left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right) \quad (23)$$

$$\frac{1}{r} \frac{\partial p}{\partial \theta} = \mu \left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right) \quad (24)$$

whereas the continuity equation takes the form

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0 \quad (25)$$

By introducing a stream function ψ , that is,

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad v_\theta = -\frac{1}{r} \frac{\partial \psi}{\partial r}, \quad (26)$$

Eqs. (23) and (24) lead to

$$\nabla^4 \psi = 0, \quad (27)$$

or

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left(\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} \right) = 0. \quad (28)$$

Assuming the stream function in the form $\psi = g(r) \sin \theta \sin(kz)$, Eq. (28) reduces to

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \right) \left(\frac{\partial^2 g(r)}{\partial r^2} + \frac{1}{r} \frac{\partial g(r)}{\partial r} - \frac{g(r)}{r^2} \right) = 0, \quad (29)$$

whose solution takes the form

$$g(r) = Ar^3 + Br + C \frac{1}{r} + Dr \ln r, \quad (30)$$

where A, B, C and D are coefficients to be determined.

Substituting Eq. (30) into Eq. (26), we have

$$v_r = -(Ar^2 + B + C \frac{1}{r^2} + D \ln r) \cos \theta \sin(kz), \quad (31)$$

$$v_\theta = -(3Ar^2 + B - C \frac{1}{r^2} + D \ln r + D) \sin \theta \sin(kz). \quad (32)$$

By substituting Eqs. (31) and (32) into the boundary conditions (i.e., Eqs. (6) and (7)), the coefficients A, B, C and D can be solved as

$$A = \frac{-v_0 q^2 \ln q}{(1 - q^2)(1 - q^2 + (1 + q^2) \ln q)}, \quad (33)$$

$$B = \frac{v_0 R_0^2 (2 \ln q + (1 - q^2)(2 \ln R_1 + 1))}{(1 - q^2)(1 - q^2 + (1 + q^2) \ln q)}, \quad (34)$$

$$C = \frac{v_0 R_0^2 R_1^2 (q^2 - 1 - \ln q)}{q^2 (1 - q^2)(1 - q^2 + (1 + q^2) \ln q)}, \quad (35)$$

$$D = \frac{-v_0 q^2}{(1 - q^2)(1 - q^2 + (1 + q^2) \ln q)}, \quad (36)$$

where $q = R_0 / R_1$.

The stress components of the cytosol at the microtubule/cytoplasm interface can be then obtained by substituting Eqs. (31)-(36) into Eq. (4),

$$\sigma_{rr} = -4\mu v_0 \cos \theta \sin(kz) \frac{(4q^4 - 2) \ln q + 3q^2 - q - 2}{R_0((q^4 - 1) \ln q - q^4 + 2q^2 - 1)} \quad (37)$$

$$\sigma_{r\theta} = 2\mu v_0 \sin \theta \sin(kz) \frac{-(3q^4 + 1) \ln q + q^4 - 1}{R_0((q^4 - 1) \ln q - q^4 + 2q^2 - 1)} \quad (38)$$

The surface traction of the cytosol along the interface can be obtained by integrating the stress field,

$$\begin{aligned} F_v &= \int_0^{2\pi} (\sigma_{rr} \cos \theta - \sigma_{r\theta} \sin \theta) R_0 d\theta \\ &= \mu v_0 \pi \sin(kz) \frac{(1 - q^4) \ln q - 12q^2 + 2q^4 + 10}{(q^4 - 1) \ln q + 2q^2 - q^4 - 1} \end{aligned} \quad (39)$$