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The surface effect on the strain energy release rate of buckling delamination in thin film–substrate systems

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ABSTRACT

Gurtin–Murdoch continuum surface elasticity model is employed to study the buckling delamination of ultra thin film–substrate system. The effects of surface deformation and residual stress on the large deflection of ultra thin film are considered in analysis. A concept of effective bending rigidity (EBR) for ultra thin plate is proposed on the basis of Gurtin–Murdoch continuum theory and the principle of minimum potential energy. The governing equations with EBR are formally consistent with the classical plate theory, including both small deflection and large deflection. A surface effect factor is introduced to decide whether there is need to consider the surface effect or not. Combining the buckling theory and interface fracture mechanics, we obtain analytical solutions of the critical buckling load and the energy release rate of the interface crack in the film–substrate system. It is seen that the surface deformation and residual stress have significant effects on the buckling delamination of ultra thin film–substrate system.

1. Introduction

Film–substrate structures have been widely used in engineering. There are increasing works on the mechanical behavior and reliability of the film–substrate system. Thermal barrier coating (TBC) system is a typical film–substrate structure (Evans & Hutchinson, 1984; Hutchinson, Thouless, & Liniger, 1992; Jensen, 1993). Buckling delamination induced by the thermally grown oxides and the mismatch deformation is one of the most common failure modes in TBC system (Wang & Evans, 1998). The other failure modes of film–substrate system include wrinkling, tunnel cracking and interface debonding (Hutchinson & Suo, 1992; Li & Suo, 2006, 2007).

From the investigation of the buckling delamination of composite plates (Chai, Babcock, & Knauss, 1981), there are many works devoted to this problem. The buckling driven interface delamination and the effect of phase angle were analyzed (Padture, Gell, & Jordan, 2002; Wang & Evans, 1999). The morphology of interface crack propagation such as "telephonecord" was also considered (Abdallah et al., 2006; Evans, Mumm, Hutchinson, Meier, & Pettit, 2000; Jensen & Thouless, 1995), see Hutchinson and Suo (1992) and Gioia and Ortiz (1997) for more details. Using the blister or wedge tests, one can measure the adhesion between thin elastic film/coating and ductile substrate (Begley, Mumm, Evans, & Hutchinson, 2000; Zhou & Hashida, 2003).

When the thickness of film reduces to nanoscale, the effects of surface deformation and residual stress become significant. As to the film–substrate system, the plate theory with surface effect should be invoked. The surface effect on the deflection of ultra-thin films was investigated by incorporating surface elasticity into the von Karman plate theory using the Hamilton's principle (He, Lim, & Wu, 2004; Lim & He, 2004). There are many works focused on the elastic/plastic behavior of nano scale

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structures incorporating surface effects (Lu, He, Lee, & Lu, 2006; Lu, Lim, & Chen, 2009; Ou, Wang, & Wang, 2008, 2009; Zhang & Wang, 2007; Zhang, Wang, & Chen, 2008, 2010).

However, there is no work on the buckling delamination incorporating surface effect. Actually, if the film/coating thickness is small enough, the surface elasticity will have significant effect on the buckling of film and the interface crack propagation. So it is important to consider the surface effect in analyzing the buckling delamination of ultra thin film–substrate system. In the present work, we focus on the investigation of surface effect on the critical buckling load and the energy release rate of interface crack in the film–substrate system and deriving the analytical solutions of the problem. In the solving process, an effective bending rigidity (EBR) for nano-sized plate is proposed, which is convenient to make the nano-plate theory consistent with classical plate theory in form. The paper is organized as follows: the basic theory is formulated in Section 2, including the new concept EBR. Section 3 presents the analytical solutions for the strain energy release rate incorporating surface effect. The results are discussed in Section 4 and conclusion is presented in Section 5.

2. Basic equations

2.1. Continuum surface elasticity theory

In the continuum surface elasticity theory (Gurtin & Murdoch, 1975), the solid surface is regarded as a layer without thickness adhering to the underlying material without slipping, and the elastic constants are different to that of the bulk material. The equilibrium and constitutive equations of the bulk are the same as those in classical elasticity theory, but the boundary conditions must ensure the force balance of the surface layer. As a result, a set of non-classical boundary problems arises in combination with the constitutive relation of surface and classical elasticity theory. Both the bulk and the surface of the film are assumed elastic and isotropic. The stress–strain relation of the bulk material is expressed by

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{1}$$

where σ_{ij} and ε_{ij} are stress and strain tensors, respectively, λ and μ the Lame constants, and δ_{ij} the Kronecker delta. Throughout this paper the Einstein summation convention is adopted and Latin subscripts run from 1 to 3.

For the thin film, normal stress σ_{33} is usually ignored, i.e. $\sigma_{33} = 0$.From Eq. (1) one has $\varepsilon_{33} = -\frac{\lambda}{2\mu}\varepsilon_{kk}$. Using the engineering elastic constants, the simplified constitutive relation of the film is

$$\sigma_{\alpha\beta} = \frac{E}{1+v}\left(\varepsilon_{\alpha\beta} + \frac{v}{1-v}\varepsilon_{vv}\delta_{\alpha\beta}\right) \tag{2}$$

where E is Young's modulus, v is Poisson's ratio and the Greek subscripts take the value of 1 or 2. Such a simplified stress state is enough to analyze the overall response of the film, though the force balance conditions of the surface layers cannot be satisfied (Lim & He, 2004). In the absence of body force, the stress satisfies the static equilibrium equation

$$\sigma_{\alpha\beta,\beta} = 0 \tag{3}$$

The interatomic distance in the surface is different to that in the bulk, which induces the properties of the surface being different to that of the bulk. The interaction between the surface and bulk in freestanding solids results in surface residual stress. The constitutive relations on the surface (Gurtin & Murdoch, 1975) are

$$\tau_{\alpha\beta} = \tau_0\delta_{\alpha\beta} + 2(\mu_0 - \tau_0)\varepsilon_{\alpha\beta} + (\lambda_0 + \tau_0)\varepsilon_{vv}\delta_{\alpha\beta} + \tau_0 u_{\alpha,\beta} \tag{4a}$$

$$\tau_{\alpha 3} = \tau_0 u_{3,\alpha} \tag{4b}$$

where $\tau_{\alpha\beta}$ is surface stress, τ_0 the residual surface tension under unconstrained condition, λ_0 and μ_0 the surface Lame constants, u is the displacement field on the surface. It is seen that the surface effect is composed of two parts, i.e. the surface residual stress τ_0 and the surface elasticity. See Cammarata (1994) for more detail.

2.2. Governing equations of plate incorporating surface effect

The coordinate system of the model is shown in Fig. 1. According to the classical Kirchhoff hypothesis for thin plate, the displacement and strain in the middle surface of the plate are all zero, and the displacement and strain relations with deflection w are

$$u_\alpha = -x_3 w_{,\alpha} \tag{5a}$$

$$u_3 = w \tag{5b}$$

$$\varepsilon_{\alpha\beta} = -x_3 w_{,\alpha\beta} \tag{5c}$$

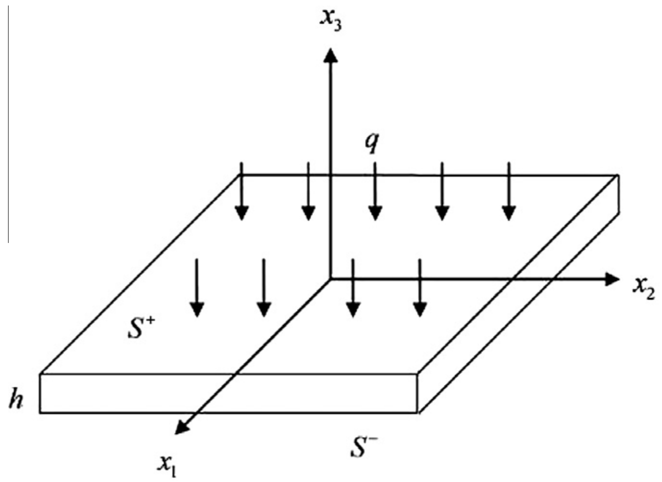

Fig. 1. Sketch of the film.

Displacements and strains relations of the upper surface:

$$u_{\alpha top} = -\frac{h}{2} w_{,\alpha} \tag{6a}$$

$$u_{3top} = w \tag{6b}$$

$$\varepsilon_{\alpha\beta top} = -\frac{h}{2} w_{,\alpha\beta} \tag{6c}$$

According to the continuum surface elasticity theory (Gurtin & Murdoch, 1975), the strain energy in the whole plate including the bulk and the surface can be calculated as

$$U = \frac{1}{2}\int_V (\sigma_{\alpha\beta}\varepsilon_{\alpha\beta}) dV + \frac{1}{2}\left(\int_{S^+} \tau_{\alpha\beta}\varepsilon_{\alpha\beta}\, dS^+ + \int_{S^-} \tau_{\alpha\beta}\varepsilon_{\alpha\beta}\, dS^-\right) \tag{7}$$

From Eqs. (1)–(7), one has

$$U = \iint \left\{\frac{1}{2} D'(\nabla^2 w)^2 - D''\left[w_{,11} w_{,22} - (w_{,12})^2\right]\right\} dx_1\, dx_2 \tag{8}$$

where

$$D' = \frac{Eh^3}{12(1-\nu^2)} + \frac{(\lambda_0 + 2\mu_0)h^2}{2} \tag{9a}$$

$$D'' = \frac{Eh^3}{12(1+\nu)} + \frac{(2\mu_0 - \tau_0)h^2}{2} \tag{9b}$$

The potential energy V of the plate is equal to the strain energy U plus the work W done by external force, i.e.

$$V = U + W \tag{10}$$

The principle of the minimum potential energy requires the potential energy of the plate should be the minimum at equilibrant state. It equals that the first variation of the potential energy V is zero. If uniform pressure q is loaded on the upper surface of the plate, as shown in Fig. 1, the work is calculated as $W = -\iint q w\, dx_1\, dx_2$. After the variation calculus $\delta V = \delta(U + W)$ we have the governing equations

$$\nabla^4 w = q/D' \tag{11}$$

It is seen from Eq. (11) that the present plate theory has the same form as the classical one except for the expression of bending rigidity D'. Actually, D' is the effective bending rigidity (EBR) including the surface effect. It is obvious from Eq. (9a) that if the surface Lame constants λ_0 and μ_0 are equal to zero then the above derived plate theory incorporating surface effect reduces to the classical thin plate theory. Similarly, one can express the stress component and the bending moment with surface effect using EBR, as shown in Table 1.

Table 1
The forces and bending moments in different plate theories.

Classical plate theory	Plate theory incorporating surface effect
$N_{11} = N_{22} = N_{12} = N_{21} = 0$	$N_{11} = N_{22} = 2\tau_0,\ N_{12} = N_{21} = 0$
$M_{11} = -D(w_{,11} + vw_{,22})$	$M_{11} = -D'(w_{,11} + v'w_{,22})$
$M_{22} = -D(w_{,22} + vw_{,11})$	$M_{22} = -D'(w_{,22} + v'w_{,11})$
$M_{12} = -D(1-v)w_{,12}$	$M_{12} = -D'(1 - v')w_{,12}$
$Q_1 = -D(\nabla^2 w)_{,1}$	$Q_1 = -D'(\nabla^2 w)_{,1}$
$Q_2 = -D(\nabla^2 w)_{,2}$	$Q_2 = -D'(\nabla^2 w)_{,2}$
where $v' = \begin{cases} v & \text{in the bulk} \\ \frac{\lambda_0+\tau_0}{\lambda_0+2\mu_0} & \text{on the surface} \end{cases}$	

It is seen from Eq. (9a) that there are two parts in the expression of D'. The surface effect is reflected by the latter. Herein, we define a parameter ξ

$$\xi = \frac{\frac{1}{2}(\lambda_0 + 2\mu_0)h^2}{\frac{Eh^3}{12(1-v^2)}} = \frac{6(\lambda_0 + 2\mu_0)(1 - v^2)}{Eh} \tag{12}$$

which depends on the plate thickness h, Young's modulus and Poisson's ratio of the bulk and surface Lame constants λ_0 and μ_0. It seen from Eq. (12) that the surface effect becomes significant with decreasing the plate thickness h. For the case A listed in Table 2 as an example, $\xi \sim O(1)$ and $O(100)$ for h = 100 nm and 1 nm, respectively, which indicates that the surface has significant effect on the deformation of the plate. If h = 10 μm, then $\xi \sim O(0.01)$ for case A, which means the surface effect is negligible. The parameter ξh is a material parameter and only depends on the Lame constants of the surface and the elastic constants of the bulk, which can be used as a characteristic length for given material.

It is well known that bending rigidity is the essential property of plate itself. It is independent of the plate theory. So, the EBR derived from the plate theory of small deflection should be suitable for the case of large deflection. Substitution of EBR into the classical von Karman plate theory for large deflection yields,

$$N_{\alpha\beta,\beta} = 0 \tag{13a}$$

$$Q_{\alpha,\alpha} + N_{\alpha\beta}w_{,\alpha\beta} + q = 0 \tag{13b}$$

which are the same as that in Lim and He (2004) derived from Hamilton principle, but the membrane forces take the following forms:

$$N_{11} = 2\tau_0 + [h(\lambda + 2\mu) + 2\lambda_0 + 4\mu_0]\left[u_{1,1} + \frac{1}{2}(w_{,1})^2\right] + (h\lambda + 2\lambda_0 + 2\tau_0)\left[u_{2,2} + \frac{1}{2}(w_{,2})^2\right] + h\lambda(u_{1,2} + u_{2,1} + w_{,1}w_{,2}) \tag{14a}$$

$$N_{22} = 2\tau_0 + [h(\lambda + 2\mu) + 2\lambda_0 + 4\mu_0]\left[u_{2,2} + \frac{1}{2}(w_{,2})^2\right] + (h\lambda + 2\lambda_0 + 2\tau_0)\left[u_{1,1} + \frac{1}{2}(w_{,1})^2\right] + h\lambda(u_{1,2} + u_{2,1} + w_{,1}w_{,2}) \tag{14b}$$

$$N_{12} = [h\mu + 2(\mu_0 - \tau_0)](u_{1,2} + u_{2,1} + w_{,1}w_{,2}) + 2\tau_0 u_{1,2} \tag{14c}$$

$$N_{21} = [h\mu + 2(\mu_0 - \tau_0)](u_{1,2} + u_{2,1} + w_{,1}w_{,2}) + 2\tau_0 u_{2,1} \tag{14d}$$

The expressions of the moments and transverse load are the same as those in Table 1.

Using a potential function F, one has

$$N_{11} = hF_{,22}, \quad N_{22} = hF_{,11}, \quad N_{12} = -hF_{,12}, \quad N_{21} = -hF_{,21} \tag{15}$$

and then the governing equations become

$$\frac{D'}{h}\nabla^2\nabla^2 w = \frac{q}{h} + F_{,22}w_{,11} + F_{,11}w_{,22} - F_{,12}w_{,12} - F_{,21}w_{,21} \tag{16}$$

Eq. (16) includes the effect of surface and has the same form as the classical one.

Table 2
Surface parameters (Gurtin & Murdoch, 1978) used in analysis.

Cases	E (N/m^2)	v	ρ (kg/m^3)	λ_0 (N/m)	μ_0 (N/m)	τ_0 (N/m)
A	5.625×10^{10}	0.25	3×10^3	7×10^3	8×10^3	110
B	17.73×10^{10}	0.27	7×10^3	−8	2.5	1.7

3. Strain energy release rate for the thin film–substrate system with surface effect

As mentioned above, when the thickness of the film goes down to nanoscale the surface effect on the film–substrate system cannot be neglected. In this section we apply the plate theory derived in Section 2 to analyze the surface effect on the strain energy release rate in thin film–substrate system.

3.1. Thin film–substrate system

The through-width delamination in the film–substrate interface is considered herein, as shown in Fig. 2. Similar to Hutchinson and Suo (1992), we separate the film that detaches the substrate from the system. Then, the effect of substrate on the film is attributed to the boundary conditions of the separated film. It is assumed that stiffness of the substrate is much higher than that of the film. Then, we can use the fully clamped boundary conditions for the separated film, which is precise enough for engineering application. If the substrate is very compliant, then the boundary condition should be modified (Cotterell & Chen, 2000; Hong & Hutchinson, 2002), which will not be considered in this work.

We consider an isotropic and linear elastic film with thickness h, Young's modulus E and Poisson's ratio ν. The length of delamination is l and the aspect ratio is $s = l/h$. Usually, thermal and/or lattice mismatch in the film–substrate system will induce axial load P, which may result in the buckling of film as P attends its critical value. In this case Eq. (13) reduce to

$$N_{11,1} = 0 \tag{17a}$$

$$D' w_{,1111} + N_{11} w_{,11} = 0 \tag{17b}$$

and the fully clamped boundary conditions are

$$N_{11} = -P, \quad w = 0, \quad w_{,1} = 0 \text{ at } x = 0, l \tag{18}$$

where P is the axial force at the edge.

3.2. Solution for the strain energy release rate

Interfacial crack propagation is one of the main mechanisms for the failure of film–substrate system. It is well known that strain energy release rate G is a measure of the driving force for crack propagation. When G attains its critical value G_c, the crack propagates. For a crack in homogeneous solid body G_c is a material constant, while for interface crack it is dependent on some interface parameters such as phase angle, see Hutchinson and Suo (1992) for more detail. For the interface crack problem, as shown in Fig. 2, Suo and Hutchinson (1990) derived a formula to calculate strain energy release rate

$$G = \frac{6(1-\nu^2)}{Eh^3}(M^2 + h^2 \Delta N^2/12) \tag{19}$$

where M is the bending moment and ΔN is the change of axial force.

It is seen from Eq. (17a) that the axial force N_{11} is a constant and equals to P. Substitution of N_{11} into Eq. (17b) yields

$$D' w_{,1111} + P w_{,11} = 0 \tag{20}$$

A nontrivial solution of Eq. (20) with boundary conditions (18) is

$$w(x_1) = w_0 \left(1 - \cos\frac{2\pi x_1}{l}\right) \tag{21}$$

where w_0 is the central deflection of the plate. Substituting Eq. (21) into Eq. (20), we have

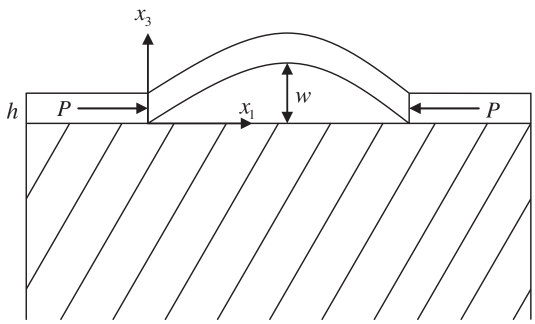

Fig. 2. Sketch of the film–substrate system.

$$\left[D'\left(\frac{2\pi}{l}\right)^2 - P\right]w_0 = 0 \tag{22}$$

and then the critical buckling load N_{cr} is

$$N_{cr} = D'\left(\frac{2\pi}{l}\right)^2 \tag{23}$$

The non-dimensional form of N_{cr} is

$$N_{nor} = \frac{D'}{D}\left(\frac{2\pi}{s}\right)^2 = (1+\xi)\left(\frac{2\pi}{s}\right)^2 \tag{24}$$

The solution Eq. (24) is the same as that derived by Lim and He (2004) on the basis of Hamilton principle. However, it is seen that the present method is simpler.

It is well known that buckling will induce the variation of internal force in the film, i.e. $\Delta N > 0$. Define σ and σ_m are the current postbuckling stress and critical buckling stress, respectively. Moreover, we assume the deflection shape keeps the same form as function (21) if the postbuckling deformation is relatively small. Then we obtain the change of the internal force as

$$\Delta N = |N_{cr}|\left(1 - \frac{\sigma_m^2}{\sigma^2}\right) \tag{25}$$

Eq. (25) is accurate when the postbuckling stress is within several times of critical buckling stress (Hutchinson & Suo, 1992). The precise solution for ΔN can be numerically obtained.

Integrating Eq. (14a) with the condition $u_1 = 0$ at the edge and neglecting the effect of deformation along x_2 direction, one has

$$w_0^2 = \frac{(1-\nu^2)l^2\Delta N}{Eh + 2(\lambda_0 + 2\mu_0 - \tau_0)(1-\nu^2)} \tag{26}$$

Substituting Eq. (26) into Eq. (21) yields the solution for deflection, i.e.

$$w(x_1) = \sqrt{\frac{(1-\nu^2)l^2\Delta N}{Eh + 2(\lambda_0 + 2\mu_0 - \tau_0)(1-\nu^2)}}\left(1 - \cos\frac{2\pi x_1}{l}\right) \tag{27}$$

Using Eq. (26) and the expressions in Table 1, we obtain the bending moment at the edge

$$M_{11} = \frac{|N_{cr}|h}{\sqrt{3}}\frac{\sigma_m^2}{\sigma^2}\sqrt{\frac{D'}{D''}\left(\frac{\sigma^2}{\sigma_m^2} - 1\right)} \tag{28}$$

Substituting Eqs. (25) and (28) into Eq. (19), one has

$$G = \frac{(1-\nu^2)|N_{cr}|^2}{2Eh}\left[\frac{4[Eh + 6(\lambda_0 + 2\mu_0)(1-\nu^2)]}{Eh + 2(\lambda_0 + 2\mu_0 - \tau_0)(1-\nu^2)}\left(\frac{\sigma_m^2}{\sigma^2} - \frac{\sigma_m^4}{\sigma^4}\right) + \left(1 - \frac{\sigma_m^2}{\sigma^2}\right)^2\right] \tag{29}$$

Obviously, Eq. (29) involves the effect of surface stress and deformation on the strain energy release rate in the thin film–substrate system. It is clear that if surface effect is neglected, i.e. $\lambda_0 = 0$, $\mu_0 = 0$ and $\tau_0 = 0$, then Eq. (29) reduces to the solution derived by Hutchinson and Suo (1992)

$$G = \frac{(1-\nu^2)|N_{cr}|^2}{2Eh}\left(1 - \frac{\sigma_m^2}{\sigma^2}\right)\left(1 + 3\frac{\sigma_m^2}{\sigma^2}\right) \tag{30}$$

4. Results and discussion

Two kinds of surface data (Gurtin & Murdoch, 1978) listed in Table 2 are used as examples to discuss the surface effect on the buckling delamination in thin film–substrate system. Without losing the generality, we fix the aspect ratio $s = 10$. Fig. 3 shows the effect of surface Lame constants on the critical buckling load, in which the parameter ξh is a characteristic length. The effect of surface Lame constants $\lambda_0 + 2\mu_0$ are the same as that of ξh for given E and ν. It is seen that the critical buckling load may be negative, which depends on the surface property. This means that surface property may result in the film buckling itself without external load (Kornev & Srolovitz, 2004). In what follows, we focus on the surface effect on the strain energy release rate of the interface crack.

4.1. Effect of surface Lame constants on strain energy release rate

To discuss the effect of surface Lame constants on strain energy release rate in the film–substrate system, the following normalized strain energy release rate is defined,

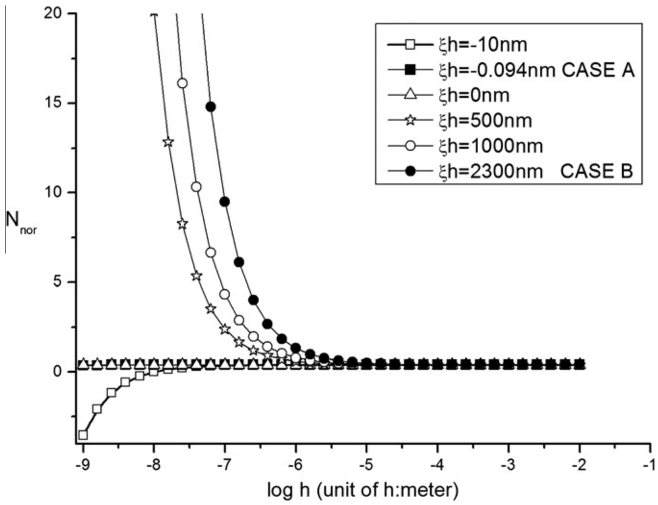

Fig. 3. The effect of surface Lame constants on the critical buckling load. The parameter $\xi h = 6(\lambda_0 + 2\mu_0)(1-v^2)/E$ is the characteristic length. For given E and v, the effect of surface Lame constants $\lambda_0 + 2\mu_0$ are the same as ξh.

$$G_{nor} = \frac{2G}{D}h^2 = \left(M_{nor}^2 + h^2\frac{\Delta N_{nor}^2}{12}\right)$$

where

$$N_{nor} = \frac{N}{D}h^2 \quad \text{and} \quad M_{nor} = \frac{M}{D}h.$$

Two kinds of surface data (Gurtin & Murdoch, 1978) listed in Table 2 are used as examples to show the surface effect on G_{nor} for the case of $\sigma/\sigma_m = 2$, as shown in Figs 4 and 5, respectively. For case A shown in Fig. 4, G_{nor} increases rapidly as the thickness of film approaches to nanoscale. When the thickness of the film is of 10^{-9} m order, G_{nor} attains 10^4 order. Usually, the material has already yielded under such a load. Nevertheless, the probable case is that the film buckles before yielding if the thickness of the film has the order of 10^2 nm. In this case, the normalized critical buckling load is about 10^1 order and G_{nor} is of 10^2 order.

For case B shown in Fig. 5, it is on the contrary to case A. As the thickness of the film approaches to nanoscale, G_{nor} decreases and the value of G_{nor} is rather small. Obviously, different surface constants result in different tendency and extent of size-dependence of strain energy release rate. G_{nor} increases as the absolute value of $\lambda_0 + 2\mu_0$ increases and equals to the value of macroscopic film problem as $\lambda_0 + 2\mu_0$ approaches to zero. It is seen from Eq. (24) that if the absolute value of the negative $\lambda_0 + 2\mu_0$ is large, there will be large negative critical buckling load. The film will buckle itself, so Eqs. (25) and (28) are not suitable. The calculation of the strain energy release rate should be changed.

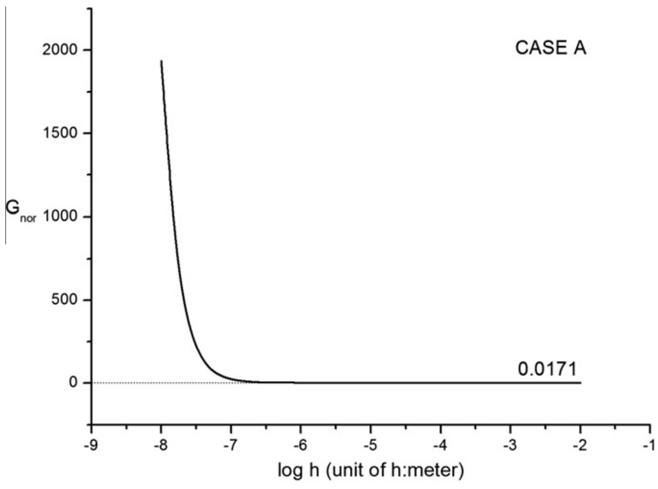

Fig. 4. The strain energy release rate involving surface effect (case A).

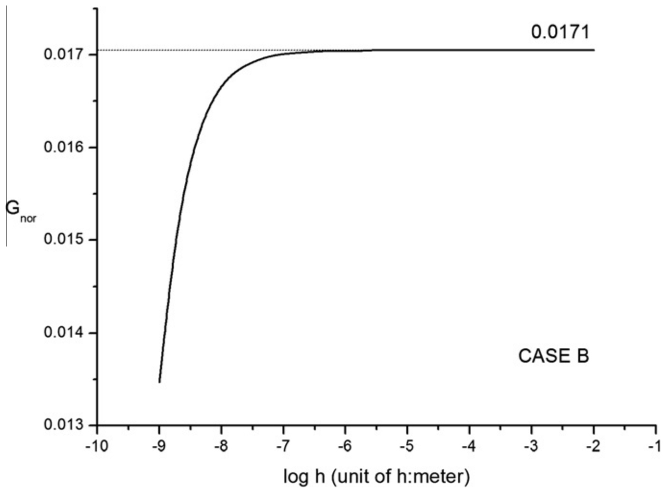

Fig. 5. The strain energy release rate involving surface effect (case B).

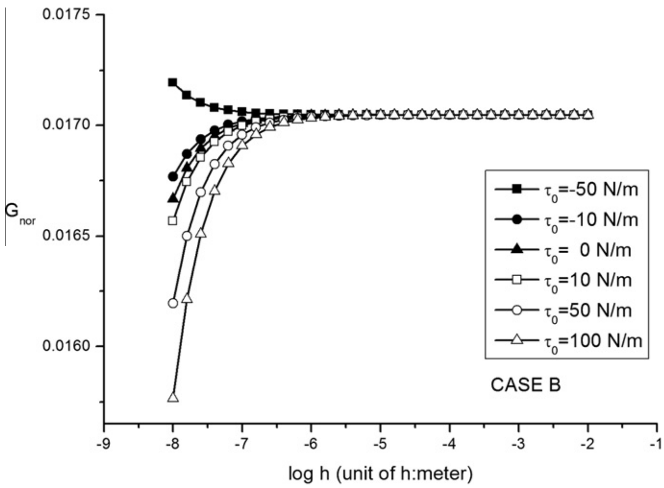

Fig. 6. The effect of surface residual stress on the strain energy release rate.

4.2. Effect of surface residual stress on strain energy release rate

Fig. 6 shows the effect of surface residual stress τ_0 on the normalized strain energy release rate G_{nor} for constant values of λ_0 and μ_0. It is seen that the effect of surface residual stress is remarkable as the Lame constants have the same order as τ_0. For case A mentioned above, the effect of τ_0 is negligible. For case B, the effect of τ_0 is significant, as shown in Fig. 6. So, the effect of surface residual stress on strain energy release rate depends on the surface parameters of materials.

5. Conclusion

The continuum surface elasticity theory is employed to investigate the buckling delamination of ultra thin film–substrate system. An effective bending rigidity (EBR) parameter is proposed to incorporate surface effect in analysis. Substituting EBR into the classical von Karman plate theory, one can easily obtain the governing equations for thin plate incorporating surface effect, which are the same as that derived from Hamilton principle. Moreover, the parameter ξ is introduced to reflect the effect of surface Lame constants. Finally, analytical solutions for the critical buckling load and the strain energy release rate of the problem are obtained. Two kinds of surface data are used as examples to discuss the surface effect on the strain energy release rate.

It can be concluded that if the thickness of film is of 10^2 nm order the normalized critical buckling load attains several times of that neglecting surface effect, and the normalized strain energy release rate of the interface crack can be of 10^2

order. The critical buckling load and strain energy release rate increase quickly as the absolute value of $\lambda_0 + 2\mu_0$ increases, and then attain to the value without surface effect as $\lambda_0 + 2\mu_0$ approaches to zero. The critical buckling load can be negative, which means that the film can buckle itself due to surface effect. The surface residual stress τ_0 has no effect on the critical buckling load, but has significant effect on strain energy release rate.

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