The Use of Flat-Ended Projectiles for Determining Dynamic Yield Stress. I. Theoretical Considerations
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Source: *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 194, No. 1038 (Sep. 2, 1948), pp. 289-299
Published by: The Royal Society
Stable URL: http://www.jstor.org/stable/98289
Accessed: 21/06/2009 10:47

The use of flat-ended projectiles for determining dynamic yield stress

I. Theoretical considerations

By Sir Geoffrey Taylor, F.R.S.

(*Received* 24 *July* 1947)

It has long been known that metals may be subjected momentarily to stresses far exceeding their static yield stress without suffering plastic strain. One of the simplest methods for subjecting a metal to a high stress for a short time is to form it into a cylindrical specimen and fire this at a steel target. The front part of this projectile crumples up, but the rear part is left undeformed. If the target is rigid the distance which this portion travels while it is being brought to rest may be taken as the difference between the initial length and the length of the deformed specimen after impact. Knowing the velocity of impact, a minimum possible value can be assigned to the maximum acceleration of the material, and from this a minimum value for the yield stress can be calculated. The actual yield stress is considerably greater than this minimum, and methods are given for calculating a more probable value.

Introduction

When a cylindrical projectile strikes perpendicularly on a flat rigid target, the stress at the impact end immediately rises to the elastic limit, and an elastic compression wave travels towards the rear end. The stress in this wave is equal to the elastic limit. If the material is one in which the stress rises when the strain exceeds that corresponding with the elastic limit, the elastic wave is followed by a plastic one. On reaching the rear end of the projectile, the elastic wave is reflected as a wave of tension which is superposed on the compression wave. At this stage the velocity of the material in the part of the projectile which the reflected wave has not yet reached is $U-S/\rho c$, where S is the yield stress of the material, ρ its density, c the velocity of elastic waves, and U the velocity of impact. The stress in this portion is S. In the reflected wave extending from the wave front to the rear end of the projectile the velocity is $U-2S/\rho c$ and the stress is zero.

The reflected elastic wave runs forward along the projectile until it meets the front of the plastic wave advancing from the target plate. In this plastic wave, the stress will not rise appreciably above the yield stress at any point close to the plastic-elastic boundary, but the velocity may be nearly zero, or, at any rate, will be very different from that in the rear part where plastic flow has not taken place. At the moment when the reflected wave has just reached the plastic boundary, the part of the specimen which lies behind this is stress-free, and is moving as a solid body with velocity $U-2S/\rho c$. It is, therefore, in the same condition as the projectile at the moment of impact, except that its speed is $U-2S/\rho c$ instead of U, and its length is less than the original length L.

The length x of this portion which has not yet suffered plastic strain will depend on the speed of the projectile, the speed of elastic waves in it, and the velocity with which the plastic-elastic boundary moves away from the target plate. Under the conditions of all the experiments which are here considered, c is much greater than U, the velocity of the projectile, or V, the velocity of the plastic-elastic boundary, and S is comparable with ρU^2, but small compared with ρc^2.

If h is the distance of the plastic boundary from the target plate at any time, x the length of the portion which has not yet been plastically compressed, and u the velocity of this rear portion, the above considerations lead to the following equations for the small changes in u, h and x during one passage of an elastic wave from the plastic boundary and back to it. The duration of this double passage is

$$dt = \frac{2x}{c}, \tag{1}$$

so that

$$dh = v\frac{2x}{c}, \tag{2}$$

$$dx = -(u+v)\frac{2x}{c} \tag{3}$$

$$du = -\frac{2S}{\rho c}. \tag{4}$$

Eliminating c, equations (1) to (4) reduce to

$$\frac{dh}{dt} = v, \tag{5}$$

$$\frac{dx}{dt} = -(u+v), \tag{6}$$

$$\frac{du}{dt} = -\frac{2S}{2x\rho} = -\frac{S}{\rho x}. \tag{7}$$

It will be noticed that (5), (6) and (7) are the equations which would be derived if the rear portions of the projectile were regarded as rigid, and all the quantities as continuously varying.

The equations (6) and (7) are not sufficient to determine the motion. In fact, the velocity of the plastic boundary, v, is determined by the plastic flow between this boundary and the target. To analyze the dynamics completely, it would be necessary to know all the intermediate states of the projectile between the instant of impact and the time when it comes to rest, or leaves the target plate. The object of the present work is to extract as much information as possible from measurements of the projectile recovered after impact. In the absence of measurements made during the impact, it is necessary to make some assumption about how the plastic boundary moves from the surface of the target to the final position in which it is measured after the projectile has come to rest.

Simple theoretical model

In order to obtain a simplified picture of the phenomenon to serve as a framework for thinking about the motion, the simplest possible assumption about the plastic stress-strain relationship was made, namely, that the stress in the part of the projectile where the material is yielding is constant and equal to the yield stress S. Further, the radial inertia is neglected, so that the stress can be considered as constant over any cross-section. It is not possible to discuss the plastic flow between the plastic boundary and the target without analyzing the complete problem of plastic waves. It is possible, however, to imagine a state in which the material which has just passed through the plastic boundary is brought to rest in a very short length. For this to be possible, the material must spread out very rapidly. The appearance of the theoretical model at a time when the rear end is still moving is shown at the right-hand side of figure 1. If A_0 is the cross-section of the projectile before it has been compressed plastically, and A the area at the point where the material is brought to rest, the continuity equation is

$$A_0(u+v) = Av, \tag{8}$$

and if the stress is S on both sides of the thin region where the change in area occurs, the momentum equation is

$$\rho A_0(u+v)\,u = S(A-A_0). \tag{9}$$

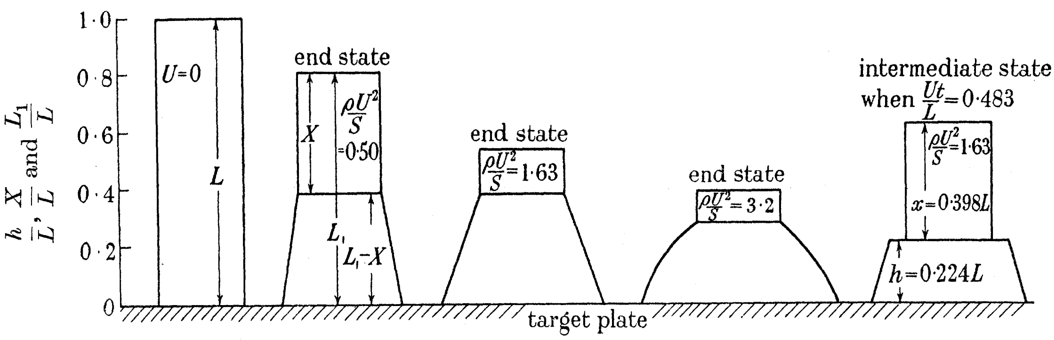

FIGURE 1. Simple theoretical model of flat-ended projectile fired at speed U at flat target.

The longitudinal compressive strain at any point may be defined as

$$e = 1-\frac{A_0}{A}. \tag{10}$$

Eliminating v from (8) and (9), and employing (10) to eliminate A and A_0, we obtain

$$\frac{\rho u^2}{S} = \frac{e^2}{1-e}. \tag{11}$$

From (6), (7), (8) and (10)

$$\frac{dx}{du} = \frac{(u+v)\,\rho x}{S} = \frac{\rho u x}{Se}. \tag{12}$$

Integrating (12)

$$\log_e(x^2) = \int \frac{1}{e}\,d\left(\frac{e^2}{1-e}\right) = \frac{1}{1-e} - \log_e(1-e) + \text{constant}. \tag{13}$$

At the moment of impact $x = L$, and $e = e_1$, say, and from (11)

$$\frac{\rho U^2}{S} = \frac{e_1^2}{1-e_1}. \tag{14}$$

When the projectile is brought to rest, $x = X$ and $e = 0$. X is one of the lengths which can be measured, hence

$$\log_e\left(\frac{x}{L}\right)^2 = \frac{1}{1-e} - \log_e(1-e) - \frac{1}{1-e_1} + \log_e(1-e_1) \tag{15}$$

and

$$\log_e\left(\frac{X}{L}\right)^2 = 1 - \frac{1}{1-e_1} + \log_e(1-e_1). \tag{16}$$

Eliminating e_1 between (14) and (16) gives X/L as a function of $\rho U^2/S$. The results are given in table 1 and shown graphically in figure 2.

TABLE 1. RESULTS OF CALCULATION BASED ON SIMPLE THEORETICAL MODEL

e_1	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
X/L	1·0	0·897	0·789	0·675	0·555	0·430	0·299	0·171	0·061	0·003	0
$\rho U^2/S$	0	0·011	0·050	0·128	0·267	0·500	0·900	1·633	3·200	8·10	—
h/L	—	—	—	—	—	0·382	—	0·376	0·288	—	—
L_1/L	—	—	—	—	—	0·812	—	0·547	0·349	—	—

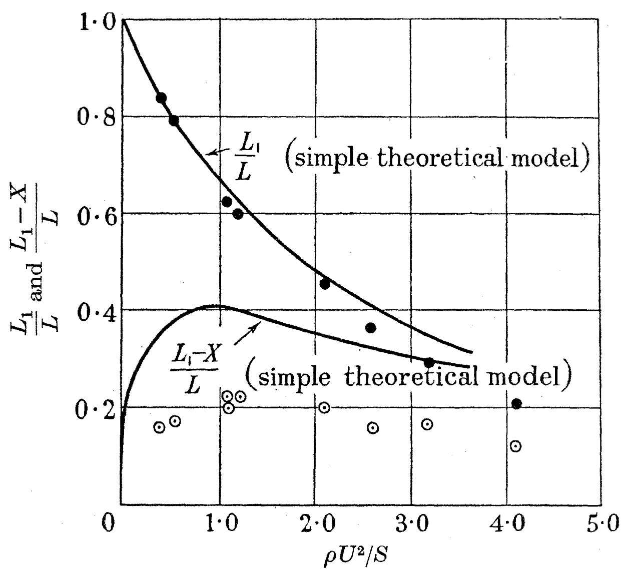

FIGURE 2. Results of calculation based on a simple theoretical model. ● measured values L_1/L; ⊙ measured values $(L_1-X)/L$.

To find the shape of the projectile after impact, we obtain from (5) and (6)

$$\frac{dh}{dx} = \frac{-v}{u+v} = -1+e,$$

so that

$$h = -\int_L^x (1-e)\,dx. \tag{17}$$

This integration was performed numerically in three cases, $e_1 = 0·5$, $0·7$ and $0·8$. Taking a given value of e_1, $\rho U^2/S$ was taken from table 1, and (x/L) calculated from (15) for the range of values of e from 0 to e_1. The resulting values of h for $e_1 = 0·5$ and

$e_1 = 0{\cdot}7$ are given in table 2. The corresponding cross-section is $A = A_0/(1-e)$, so that the diameter of the projectile at this point is $d = d_0\sqrt{\{1/(1-e)\}}$, where d_0 is the diameter of the projectile before firing. The values of d/d_0 and h/L are given in table 2. Similar calculations were made for $e_1 = 0{\cdot}8$.

TABLE 2. CALCULATIONS FOR SIMPLE THEORETICAL MODEL OF PROJECTILE FOR TWO IMPACT VELOCITIES

$e_1 = 0{\cdot}70, \quad \rho U^2/S = 1{\cdot}633$

e	0	0·05	0·10	0·15	0·20	0·25	0·30	0·35
x/L	0·171	0·180	0·190	0·202	0·216	0·233	0·253	0·277
h/L	0·376	0·368	0·358	0·348	0·336	0·323	0·309	0·293
Ut/L	0·720	0·708	0·695	0·681	0·665	0·646	0·625	0·599
d/d_0	1·00	1·026	1·054	1·087	1·118	1·155	1·196	1·241

e	0·40	0·45	0·50	0·55	0·60	0·65	0·70
x/L	0·307	0·346	0·398	0·468	0·571	0·730	1·000
h/L	0·274	0·251	0·224	0·191	0·147	0·088	0
Ut/L	0·568	0·531	0·483	0·420	0·333	0·207	0
d/d_0	1·292	1·349	1·414	1·492	1·583	1·690	1·827

$e_1 = 0{\cdot}50, \quad \rho U^2/S = 0{\cdot}50$

e	0	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
x/L	0·430	0·452	0·478	0·508	0·543	0·585	0·635	0·696	0·773	0·871	1·000
h/L	0·382	0·361	0·337	0·311	0·282	0·250	0·213	0·172	0·124	0·067	0
Ut/L	0·332	0·315	0·298	0·277	0·255	0·229	0·199	0·163	0·119	0·067	0
d/d_0	1·000	1·026	1·054	1·087	1·118	1·155	1·196	1·241	1·292	1·349	1·414

The shapes of the projectile in the three cases are shown in figure 1 for the case where the diameter was initially 0·3 of the height. Comparing these with the profiles of the steel slugs shown in figure 1 of Mr Whiffin's paper (see part II), it will be seen that the calculated shape for $\rho U^2/S = 0{\cdot}5$ is very similar to that of the slug fired at 810 ft./sec. The shapes of the slugs fired at greater speeds do not resemble at all closely those calculated for $\rho U^2/S = 1{\cdot}63$ or 3·2. The plastically strained parts of the slugs fired at speeds greater than 810 ft./sec. have a concave profile, due, apparently, to the high radial velocity imparted to the material near the target which was neglected in the analysis.

As previously stated, the object of these calculations was to form the basis for an appropriate rough assumption for the rate at which the plastic boundary reaches its final position. To determine how h varies with t, the time since the beginning of the impact, equation (12) was integrated numerically. Using equations (7) and (11)

$$t = -\int \frac{\rho x}{S}\,du = -\int \frac{\rho x}{S}\sqrt{\frac{S}{\rho}}\,d\left(\frac{e}{\sqrt{(1-e)}}\right) = L\sqrt{\frac{\rho}{S}}\int \frac{x}{L}\frac{(1-\frac{1}{2}e)}{(1-e)^{\frac{3}{2}}}\,de,$$

and, using (14), this may be written

$$\frac{Ut}{L} = \frac{e_1}{\sqrt{(1-e_1)}}\int_e^{e_1} \frac{x}{L}\frac{(1-\frac{1}{2}e)}{(1-e)^{\frac{3}{2}}}\,de. \tag{18}$$

The integration of (18) was performed numerically in the cases when $e_1 = 0{\cdot}5$ and $e_1 = 0{\cdot}7$. The results are given in table 2, together with the values of h. The relationship between Ut/L and h/L for these two values of e_1 are shown in figure 3. It will be seen that h increases nearly uniformly with Ut/L, so that the velocity of the plastic boundary, as it moves away from the target, is nearly uniform.

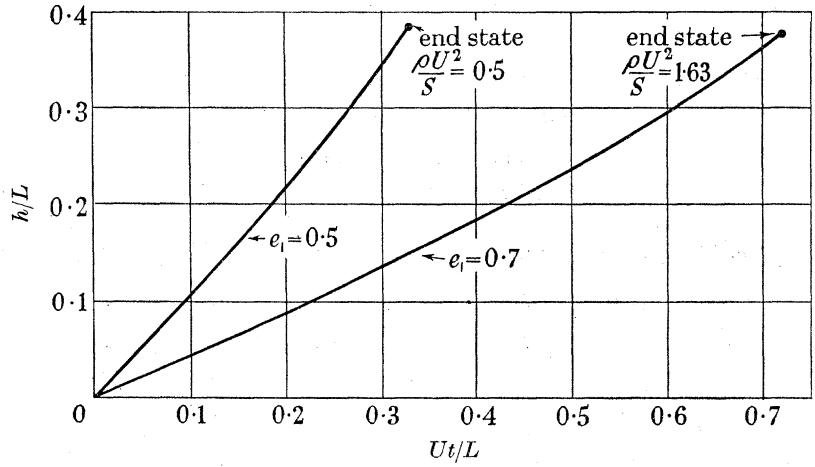

FIGURE 3. Propagation of plastic boundary in a simple theoretical model.

The complete system of plastic waves occurring during an impact has been analyzed by Messrs E. H. Lee and S. J. Tupper in a case where the plastic stress-strain relationship was assumed known. Good agreement was found with the strains calculated using the simple theoretical model here described. The main difference is that the strains calculated by the complete plastic-wave theory change in discontinuous jumps along the length of the projectile. This, as the authors point out in a paper which is not yet published, is due to the neglect in their analysis of the radial inertia of the plastic material as it spreads out near the target plate. The present calculations also neglect this radial inertia, but the discontinuities of strain disappear from the calculations when the finite difference equations (1), (2) and (3) are replaced by the differential equations (5), (6) and (7).

It is worth pointing out that though the plastic wave system can be calculated with considerable difficulty, when the plastic stress-strain relationship is known, this relationship can only be measured at low rates of straining. It is not possible to use these calculations directly to determine the stress-strain relationships from the shape of a projectile measured after an impact. It appears that the most which can be done at present is to determine approximately the yield stress using simplifying assumptions as to the general nature of the plastic stress waves. This is the line of attack developed in the present work.

Approximate formula for estimating yield point from measurements of slugs after impact

In developing a simple formula for estimating the yield point from measurements of the position of the yield boundary after the impact, it will be assumed that the plastic-elastic boundary moves outwards at a uniform velocity from the impact end to its final position. This, together with the assumption that the yield boundary represents the position where a definite compressive stress S is reached, is sufficient to fix the whole history of the deceleration of the rear part of the projectile.

If C is written for the constant velocity of the plastic boundary, equations (6) and (7) give

$$\frac{du}{dx} = \frac{S}{\rho x(u+C)}, \tag{19}$$

which may be integrated to give

$$\frac{S}{\rho}\log_e\left(\frac{x}{L}\right) = \tfrac{1}{2}u^2 + Cu - \tfrac{1}{2}U^2 - CU. \tag{20}$$

When $u = 0$, $x = X$, so that

$$\frac{S}{\rho}\log_e\left(\frac{X}{L}\right) = -\tfrac{1}{2}U^2 - CU. \tag{21}$$

If the deceleration of the rear of the projectile were exactly uniform, C/U could be determined simply from the fact that the time of deceleration T is equal to $(L_1 - X)/C$ (where L_1 = overall length of the projectile after test), and is also equal to $2(L-L_1)/U$, so that $C/U = \frac{1}{2}(L_1 - X)/(L-L_1)$, and equation (21) would then assume the form

$$\frac{S_1}{\rho U^2} = \frac{(L-X)}{2(L-L_1)}\frac{1}{\log_e(L/X)}, \tag{22}$$

where S_1 has been used instead of S to distinguish it from the value calculated by more exact methods. In fact, the deceleration is not uniform, so that (22) is only approximate. It is, however, the formula used with success by Mr Whiffin to interpret his experiments (see part II). In Mr Whiffin's experiments, slugs fired at very different speeds gave different values of X/L and L_1/L, but, when the measured values were inserted in (22), the values of S so deduced were found to be nearly independent of U. This fact affords a strong confirmation that (22) is an approximate equation which can be relied on to give the values of S from measurements of U, X/L and L_1/L. In practice, it is sometimes found that the slug makes a depression in the target. In this case, equation (22) can still be applied, but if L_1 is the total measured final length, and d the depth of the hole into which it fitted at the end of the impact, L_1 in (22) must be replaced by $(L_1 - d)$.

More exact calculation

The fact that the deceleration of the rear of the projectile is not uniform under the conditions assumed introduces an error which can be calculated. Since

$$\frac{dx}{dt} = -(u+C),$$

equation (20) may be written

$$\left(\frac{dx}{dt}\right)^2 = \frac{2S}{\rho}\log_e\left(\frac{x}{L}\right) + (U+C)^2, \tag{23}$$

and when $u = 0$, $x = X$, so that

$$\frac{2S}{\rho}\log_e\frac{X}{L} = C^2 - (U+C)^2. \tag{24}$$

Writing $2S/\rho = a^2$, $K = (U+C)/a$, $x_1 = x/L$, $t_1 = at/L$, and $T_1 = aT/L$, where T is the duration of the impact, the non-dimensional form of (23) is obtained:

$$\frac{dx_1}{dt_1} = \sqrt{(K^2 + \log_e x_1)},$$

so that

$$T_1 = \int_1^{e^{-K^2}} \frac{dx_1}{\sqrt{(K^2+\log_e x_1)}}, \tag{25}$$

and, putting $K^2 + \log_e x_1 = z^2$,

$$T_1 = 2e^{-K^2} \int_{C/a}^{K} e^{z^2}\, dz. \tag{26}$$

Values of $F(K) = e^{-K^2}\int_0^K e^{z^2}\,dz$ have been tabulated (Miller & Gordon 1931) in the range $K = 0$ to $K = 12$. To use these tables, (26) can be expressed as

$$T_1 = 2[F(K) - \exp[-K^2 + (C/a)^2]\, F(C/a)]. \tag{27}$$

Since the plastic-elastic boundary moves with velocity C, $CT = L_1 - X$, which, in non-dimensional form, becomes

$$\frac{C}{a} T_1 = \frac{L_1}{L} - \frac{X}{L}, \tag{28}$$

while (24) becomes

$$\log_e (L/X) = K^2 - (C/a)^2. \tag{29}$$

Equations (27), (28) and (29), together with

$$K = U/a + C/a, \tag{30}$$

are sufficient to determine C/a, U/a, K and T, when X/L and L_1/L are known.

To estimate the error arising from the use of equation (22) instead of the more accurate equations (27) to (30) in any particular case where L_1 and X are measured, it is necessary to solve transcendental equations. This laborious calculation was carried out by Mr Whiffin in the case of several of the experimentally determined values of L_1 and X. It was found that, in all cases, equation (22) underestimated the yield stress as compared with the equations (27) to (30). Subsequently, the corrections were calculated systematically in the form of a correcting factor S/S_1, which, when applied to S_1, as calculated from (22), gives the value which would have been obtained for S if (27) to (30) had been solved.

Combining equations (27), (28) and (29)

$$\frac{L_1}{L} = 2\frac{C}{a}F(K) - \left[2\frac{C}{a}F(C/a) - 1\right]\frac{X}{L}. \tag{31}$$

Since

$$\frac{2S}{\rho U^2} = \frac{a^2}{U^2} = \frac{1}{(K - C/a)^2}$$

the correcting factor is

$$\frac{S}{S_1} = \frac{L - L_1}{L - X}\frac{\log_e (L/X)}{(K - C/a)^2}. \tag{32}$$

In performing the calculation, a value of X/L was taken, and a series of values of C/a covering the range from 0 to infinity were used to calculate the corresponding values of L_1/L and S/S_1. These were plotted on a diagram with ordinates S/S_1 and abscissae L_1/L. A series of curves were thus obtained, each corresponding with a given value of X/L. The values of L_1/L corresponding with definite values of S/S_1, namely, 1·0, 1·05, 1·10, ..., 2·0, were taken from these curves. In this way, and with the help of scattered values calculated by Mr Whiffin, the diagram of figure 4 was constructed. Here the ordinates are X/L and the abscissae L_1/L. The curves on the diagram are contours of equal correction factors. It will be seen that the curvature of these contours is very slight except near the axis $X/L = 0$.

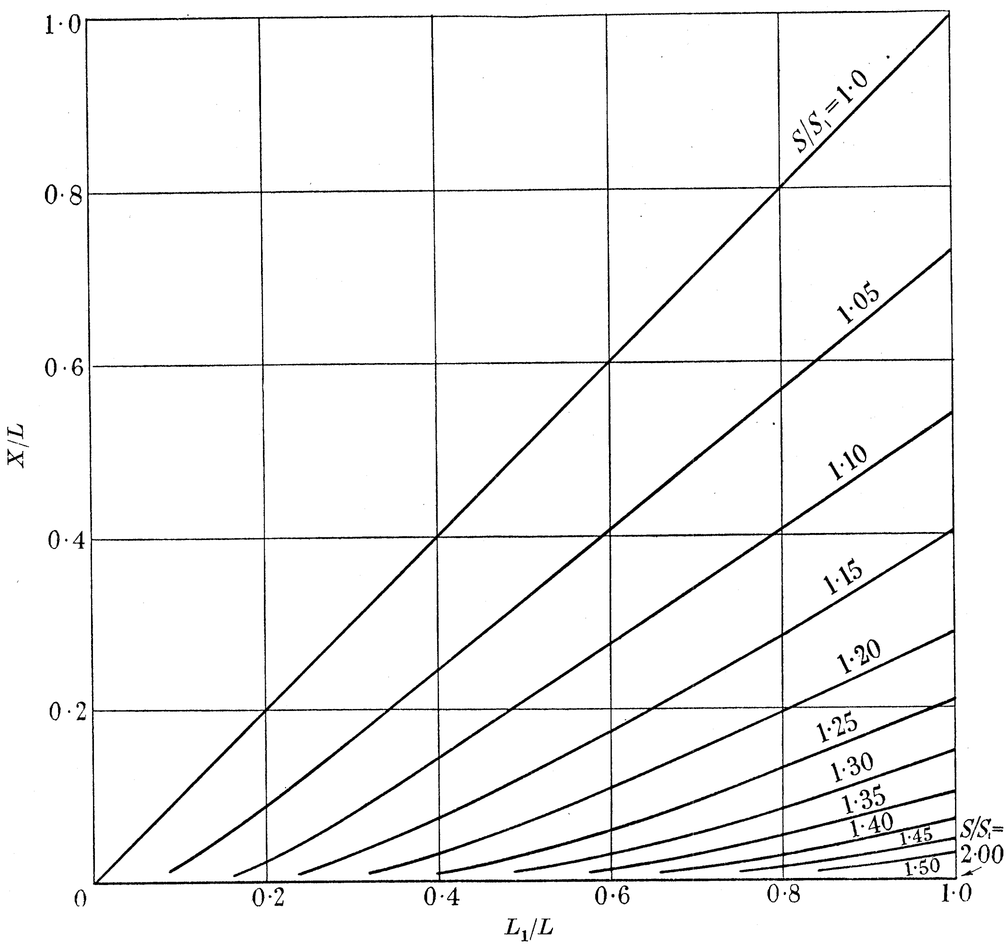

FIGURE 4. Contours of ratio of yield value S computed by exact formula to value S_1 obtained by approximate formula.

It will be noticed that the contours cut the line $L_1/L = 1·0$ without any apparent singularity. The limit points on $L_1/L = 1·0$ were calculated using the asymptotic form of $F(x)$ as $x \to \infty$, namely

$$\lim_{x\to\infty} F(x) = \frac{1}{2x} + \frac{1}{4x^3} + \dots. \tag{33}$$

In this way it is found that, when C/a and K are infinite,

$$L_1/L = 1{\cdot}0, \tag{34}$$

and

$$\frac{S}{S_1} = 2\left[\frac{1}{1-X/L} - \frac{1}{\log_e(L/X)}\right]. \tag{35}$$

The limiting points on the line $L_1/L = 1{\cdot}0$ in figure 4 were calculated from equation (35).

Experiments with paraffin wax

The method described above was first applied to find the dynamic yield stress of paraffin wax. Transparent cylinders of this material were cut from cast blocks, and these were projected by means of a catapult, capable of giving them a speed of 125 ft./sec., at a heavy anvil hung as a ballistic pendulum. The cylinders were 1·75 cm. long. After the impact they were found to be shorter, but they remained coherent. Paraffin wax has the property of remaining transparent under compressive stressing until a sudden collapse occurs. The material which has yielded is full of small cracks, which give it an opaque white appearance. The cylinders which had been projected were found to be opaque at the impact end, but they remained transparent at the rear end. The yield point was taken to correspond with the boundary between the transparent and opaque portions. The length of the transparent portion was taken as X. Static tests were also made by compressing paraffin wax cylinders between polished plates in a parallel-motion compression machine, and it was found that sudden and catastrophic breakdown occurred at a certain load, the wax remaining transparent up to the instant of breakdown.

Some results are given in table 3. It will be seen that the dynamic yield stress varied from 840 to 930 lb./sq.in., while the static yield stress was only 485 lb./sq.in. The ratio (dynamic yield stress)/(static yield stress) was therefore about 1·8.

Table 3. Experiments with cylinders of paraffin wax

L	U	L_1	X	S dynamic yield stress from eqn. (22)	S static yield stress (mean of 6 observations)
(cm.)	(ft./sec.)	(cm.)	(cm.)	(lb./sq.in.)	(lb./sq.in.)
1·774	126	1·635	0·95	854	—
1·757	128	1·625	0·95	930	485
1·779	132	1·625	0·95	840	—

Interpretation of results

The measurements made by Mr Whiffin show that his specimens maintained the stresses, which are here called dynamic yield stresses, instantaneously without suffering strain greater than 0·2 %. When considering the results of mechanical tests at comparatively low speeds, in which the elastic limit is passed and plastic strain occurs, the rate of strain is definable in terms of the experimentally measured quantities. In fact, if l is the length of the specimen at time t, and L its original length, one definition of rate of strain is simply $1/L.dl/dt$. In experiments of the type here considered, it is possible to define the mean rate of strain of the plastically distorted portion of the projectile. If the approximation represented by (22) is adopted, the mean rate of strain is $U/2(L-X)$, because the reduction in length is $(L-L_1)$, and this is entirely confined to the material whose initial length was $(L-X)$. The total strain of the portion which has yielded is therefore $(L-L_1)/(L-X)$. Since the deceleration is assumed uniform, its duration is $2(L-L_1)/U$, so that the mean rate of strain is

$$\frac{(L-L_1)}{(L-X)}\frac{\frac{1}{2}U}{(L-L_1)} = \frac{\frac{1}{2}U}{(L-X)}. \tag{36}$$

Values of this mean rate of strain are given in Mr Whiffin's paper because they are the only rates of strain which can be deduced from his measurements. It must, however, be remembered that, at all stages of the impact, the analysis refers only to the part of the projectile which has not yet suffered plastic compression, so that the connexion between the rate of strain just defined and the yield stress can only be an indirect one.

It seems impossible to derive further information about the physical factors which determine the dynamic yield stress without making more complete measurements of the successive states of the projectile during impact. It is, however, worth noticing that, if the deceleration of the rear portion of the projectile is continuous, as is contemplated in equations (5), (6) and (7), the maximum stress S at a distance X from the rear end is only attained instantaneously at the end of the impact. At the beginning of the impact, the stress in this part of the projectile is XS/L. If the stress at distance X rises uniformly from the value XS/L at the beginning to S at the end, the stress at time t would be

$$S\left[\frac{X}{L}+\frac{t}{T}\frac{(L-X)}{L}\right].$$

The duration of stress greater than, say, $S(1-y)$, where y is small, is $L/(L-X).Ty$. To a rough approximation, therefore, one may expect that, at the place where the yield point is found, the stress has exceeded, say, 99 % of S for a duration of the order of $(0{\cdot}01)\ L(L-X).T$, and, according to the simple theory of equation (22), $T = 2(L-L_1)/U$. In one group of Mr Whiffin's experiments, recorded in his table 2, the estimated value of T lies between $5{\cdot}2$ and $7{\cdot}0\times10^{-5}$ sec., so that there is very little variation in the duration of the impact, although there is great variation in U and $(L-L_1)$. It seems possible that the constancy of S, which is found for varying velocities of the projectile, is due to the constancy of the duration of impact.

Reference

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