Numerical Analysis of Strong Buckling Behavior of Square Thin Membranes using ABAQUS

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Outline

- Motivation: design of thermomechanically stable thin films
- Project goal
- Method: pressure application
- Results
  - Code validation: comparison with experiments
  - Stress evolution with geometry change
  - Effect of Poisson’s ratio
  - Stress evolution with temperature change
- Discussion: deviation of FEM results from experimental/analytical results
- Conclusion: effectiveness of this model
- Suggested future work
Motivation

- Released thin film structure for micro solid oxide fuel cell (µSOFC) to increase efficiency
- Residual stress in the film due to fabrication process
- Operation at high temperature required for the electrolyte (Yttria-stabilized Zircons)
- Failure of thin membranes
- Design (geometry, material property, residual stress) to avoid high stress concentration/failure is critical
Project Goal

To achieve thermomechanically stable membrane design

- Analytical prediction is hard due to non-linearity
- Experiments require significant efforts

- Model square thin membranes under in-plane stress with finite element method, and predict their buckling behavior including deflection and stress states

Yttria-stabilized Zirconia (YSZ), ~500nm-thick, buckled after cooling from ~600C after fabrication
Numerical Simulation Method

- Software: ABAQUS v6.6
- 3D deformable shell plate
  sidellength $5-1000\mu$m $>>$ thickness $\sim$300-600nm
- Assumption: isotropy, no plastic deformation, single-layer
- Material Properties of YSZ
  - Young’s modulus: 64 [GPa]
  - Poisson’s ratio: 0.2
  - CTE: $11.4 \times 10^{-6}/K$
- Boundary condition: 4 sides fixed
- Mesh number: $\sim$10x10
Stress Load Application

[Ziebart and Paul, 1999]

- Possible numerical instability at critical buckling points (eigenvalue calculations)
- Thermal simulation of in-plane residual stress
  - Pressure loading to buckle a membrane
  - Thermal loading with temperature field to apply corresponding prestress

\[ \sigma_0 = -(\Delta T)E\alpha/(1-\nu) \]

- Pressure unloading
Results: Deflection Profile after Each Step

- Successfully buckled membranes with center deflection in the right order

3.07 µm (FEM) vs. average 2.46 µm (Experiment)

E = 64 [GPa], ν = 0.2, CTE = 11.39 [10^-6/K], σ₀ = -66 [MPa]
Results: Buckling Pattern Evolution with Sidelength (Experimental)

- First buckling mode: 2-axis and rotation symmetry
- Second buckling mode: only rotation symmetry

First Buckling Mode Evolution with 2-Axis and Rotation Symmetry

Second Buckling Mode Evolution with only Rotation Symmetry

E=24 [GPa], ν=0.2, CTE=9.74 [10^{-6}/K]

σ₀= -38 [MPa]

thickness= 368 [μm]
Results: Buckling Pattern Evolution with Sidelength (FEM)

- Followed first and second buckling modes with increasing sidelength as experiments

\[ E=64 \text{ [GPa]}, \nu=0.2, \text{CTE}=11.39 \times 10^{-6}/\text{K} \ \sigma_0= -66 \text{ [MPa]} \ \text{thickness}=625 \text{ [nm]} \]

Sidelength: [not in scale]

- 20\(\mu\text{m}\)
- 100\(\mu\text{m}\)
- 300\(\mu\text{m}\)
- 500\(\mu\text{m}\)
- 1000\(\mu\text{m}\)
Results: Stress Evolution with Sidelength (EMM)

- Energy Minimization Method (EMM)
  - Model deflection from observed symmetry (only 1\textsuperscript{st} mode)
  - Minimize strain energy to obtain coefficients of deflection function
  - Calculate stress from the deflection

- Three stress regions with varying sidelength

![Graph showing stress evolution with side-length](image-url)
Maximum in-plane stress obtained from FEM showed similar tendency with EMM results, but higher compressive stress.

- EEM Tensile
- EEM Compressive
- FEM Tensile
- FEM Compressive

- $E=64$ [GPa], $\nu=0.2$, $\text{CTE}=11.39 \times 10^{-6}/\text{K}$
- $\sigma_0=-66$ [MPa]
- Thickness=625 [nm]
Results: Effect of Poisson’s Ratio on Center Deflection

- Calculated the center/max deflection of membranes with varying Poisson’s ratio
  - Poisson’s effect has little effect on the center deflection
  - This result coincides with Ziebart and Paul’s analysis (Journal of MEMS, 1999)

<table>
<thead>
<tr>
<th>Poisson’s ratio</th>
<th>Center deflection [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>3.11</td>
</tr>
<tr>
<td>0.2</td>
<td>3.06</td>
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<tr>
<td>0.25</td>
<td>3.01</td>
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</tbody>
</table>

E=64 [GPa], ν=0.2, CTE=11.39 [10^{-6}/K]
σ₀= -66 [MPa] thickness=625 [nm]

[Ziebart and Paul, 1999]
Results: Pattern Evolution with Temperature

- Again, followed the first and second buckling modes with increasing temperature

$E=64\ \text{[GPa]}, \ \nu=0.2, \ \text{CTE}=11.39\ [10^{-6}/\text{K}], \ \text{thickness}=625\ \text{[nm]}, \ \text{sidelength} = 100\ \mu\text{m}$
Discussions

Deviation of FEM results from EEM/experimental results is possibly due to

- Assumptions: no plastic deformation (observed), no expansion of Silicon substrate, uniformity (thickness variation)
- Small number of meshes
- Experimental measurement errors
- EMM analysis which includes only 1st mode buckling
Conclusions

- Buckling behaviors due to residual stresses were successfully modeled with thermal loading.
- Pattern evolutions (1\textsuperscript{st}, 2\textsuperscript{nd} order) with increasing sidelength were predicted.
- Stress evolutions with increasing sidelength, including three regions, were predicted.
- Effect of Poisson’s ratio on center deflections of buckled membranes was confirmed to be weak.
- Stress/deflection evolutions with increasing temperature were predicted.

\textit{This simple FEM analysis can produce preliminary prediction of buckling behavior (stress, deflection) of thin membranes under residual stress or thermal loading.}
Suggested Future Work

- Improvement in modeling
  - Mesh size, number
  - Material property (include plasticity)
  - Modeling of substrates

- Modeling of tri-layers structure (Pt-YSZ/YSZ/Pt-YSZ) of $\mu$SOFC
Thank you!