A very simple estimate of adhesion of hard solids with rough surfaces based on a bearing area model

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Abstract

In the present note, we suggest a single-line equation estimate for adhesion between elastic (hard) rough solids with Gaussian multiple scales of roughness. It starts from the new observation that the entire DMT solution for “hard” spheres (Tabor parameter tending to zero) with the Maugis law of attraction can be obtained using the Hertzian relationship load-indentation and estimating the area of attraction as the increase of the bearing area geometrical intersection when the indentation is increased by the Maugis range of attraction. The bearing area model in fact results in a simpler and even more accurate solution than DMT for intermediate Tabor parameters, although it retains one of the assumptions of DMT, that elastic deformations are not affected by attractive forces. Therefore, a solution is obtained for random rough surfaces combining Persson’s adhesiveless asymptotic simple form solution with the bearing area model, which is trivially computed for a Gaussian. A comparison with recent data from extensive numerical computations involving roughness with wavelength from nano to micrometer scale shows that the approximation is quite good for the pull-off in the simulations, and it remarks the primary importance in this regime of a single parameter, the macroscopic well-defined quantity (rms) amplitude of roughness, and small sensitiveness to rms slopes and curvatures.

Keywords (separated by '-')	Roughness - Adhesion - DMT model - Persson’s theory
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Abstract In the present note, we suggest a single-line equation estimate for adhesion between elastic (hard) rough solids with Gaussian multiple scales of roughness. It starts from the new observation that the entire DMT solution for "hard" spheres (Tabor parameter tending to zero) with the Maugis law of attraction can be obtained using the Hertzian relationship load-indentation and estimating the area of attraction as the increase of the bearing area geometrical intersection when the indentation is increased by AQ1 the Maugis range of attraction. The bearing area model in fact results in a simpler and even more accurate solution than DMT for intermediate Tabor parameters, although it retains one of the assumptions of DMT, that elastic deformations are not affected by attractive forces. Therefore, a solution is obtained for random rough surfaces combining Persson's adhesiveless asymptotic simple form solution with the bearing area model, which is trivially computed for a Gaussian. A comparison with recent data from extensive numerical computations involving roughness with wavelength from nano to micrometer scale shows that the approximation is quite good for the pull-off in the simulations, and it remarks the primary importance in this regime of a single parameter, the macroscopic well-defined quantity (rms) amplitude of roughness, and small sensitiveness to rms slopes and curvatures.

Keywords Roughness · Adhesion · DMT model · Persson's theory

1 Introduction

The classical theories suggest adhesion is usually destroyed very easily by the presence of small *amplitudes* of roughness, even in low modulus materials like smooth rubber lenses against roughened surfaces [1]. Therefore, adhesion to rough surfaces is very difficult to achieve in a macroscopic sense, despite many tribological processes depend on adhesion at asperity scale like in the classical view of Bowden and Tabor [2]. After the introduction of the concepts of multiscale roughness [3, 4], we have recognized that the real area of contact is very loosely defined and it depends, together with some other physical quantities, on the small wavelength truncation of roughness [5], perhaps at atomic scale. Debate between the classical asperity models [6] vs the more accurate Persson model [4], see eg. Putignano et al. [7] have not changed the substance of this: it remains still a problem that no reliable estimates can be made of quantities like real contact area, mean slope or mean curvature of surfaces: for example, advanced multiscale models for example of friction in viscoelastic

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bodies [8] rely on these quantities which are difficult to define.

This problem of sensitiveness to "small scale" truncation was not perceived 40 years ago, at the time of Fuller-Tabor [1], although it is clear that the FT adhesion stickiness parameter contains the radius of asperities. The emphasis was however on the macroscopically well defined rms amplitude of asperities heights h_{rms}, which indeed was found to critically affect pull-off in the experiments, and no attempt was made to fit the experiments with various possible choices of "truncation" or different resolution measurement of the asperity features. But it wasn't until perhaps 40 years later (Pastewka and Robbins [9], PR in the following) that an attempt was made of a fully numerical investigation of pull-off for a reasonably "multiscale" self-affine rough surfaces (still limited by computational capabilities to less than 3 orders of magnitude in wavelengths from nano/atomic scale wavelengths to microscale one). PR concluded that pull-off data decay (Fig. S3) did *not* correlate well with classical Fuller-Tabor [1] asperity model predictions by various orders of magnitude. PR own attempt to define a stickiness parameter (based on the slope of the area-load curve) involves only slopes and curvature, and this was suggested in *strong contrast* with asperity models whose emphasis is on rms amplitude.[1] However, when studying and re-elaborating in different form the results of PR investigation, is become clear to the present author that the pull-off data in PR indeed *mainly depend* on rms amplitude of roughness, like in a rigid model [10] where the finite realization of a real Gaussian surface necessarily involve a highest point, being it at 2 or 3 or 4 standard deviations, as it is common experience.

Persson [11] and Persson and Tosatti [12] develop a theory of adhesion of rough surfaces which is aimed at the JKR regime, which surprisingly seems perfectly reversible, and is not simple nor corroborated so far by numerical or experimental results. Persson and Scaraggi [13], attempts to solve the problem using the DMT approximations originally developed for the spherical problem [14], see also Maugis [15]). In DMT, the contact is assumed to be split into "repulsive" contact areas and "attractive" contact areas, and no effect of tensile tractions occurs within the repulsive contact area. Tensile tractions can therefore be estimated by convolution with the attractive forces if one knows the repulsive contact solution force vs mean separation which was estimated by Persson [16], and finally the entire distribution of separations [17]. This state of not trivial calculation results in an apparent dependence of the results on the entire power spectrum of the surface, and Persson and Scaraggi do not show the dependence of pull-off on the simple parameters like rms amplitude, slopes and curvatures. Further, significant corrective factors are needed for Persson [16], see below, and therefore it is not clear the accuracy of Persson and Scaraggi [13].

1.1 Outline of the present paper

The present paper suggests a very simple approach, which is original:

- first estimating the *attractive* area with a purely geometrical approach, as the difference between two bearing area estimates, irrespective of any elastic deformation (and using the Maugis attractive force function which is constant to the theoretical strength σ_{th} up to a distance of the order of atomic spacing Δr)
- then, using an adhesiveless theory for the relationship force with separation for the adhesiveless contact.

This elementary derivation, in the case of the sphere, results very simply in the *entire* DMT solution (in the most meaningful form, given by Maugis [15]) and wasn't noticed before. We sheare with a DMT method that the contact area is identified as the part in contact with purely repulsive (compressive) tractions. However, while we find the contact area after adhesive tractions have been estimated, or independently anyway, a DMT model as attempted by Persson and Scaraggi [13] find attraction forces by integrating them *outside* the repulsive contact area: this requires a detailed knowledge of separation between the deformed bodies, for which Persson and Scaraggi use some elaborate approximate solutions [17] which instead we don't attempt.

After discussing this elementary result, and showing that the bearing area model is in fact both simpler and possibly similarly accurate than DMT (see also Ciavarella [18]), we then apply to the case of Gaussian random rough surfaces, using Persson's solution [16]

[1] Further aspects of the PR criterion are discussed in Ciavarella (2016a, b, c, d, e and Ciavarella and Papangelo, 2016).

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for the force vs mean separation, but only in the simplest, closed form, case—which we correct in the multiplier. Finally, we compare this estimate with PR numerical extensive set of results.

2 The basic case of the sphere

To show that the idea works in a simple case, we first consider the case of the sphere of radius R, against a flat rigid surface, as represented in Fig. 1a. We assume the Maugis simplified attractive forces (Fig. 1b) in which the tension σ is constant to theoretical strength σ_{th} up the gap equal to Δr, and $w = \sigma_{th} \Delta r$ is the work of adhesion.

We shall see that, very surprisingly[2], the *entire* DMT solution (Derjaguin et al. [14]), but in the most commonly accepted form, that given by Maugis [15], is obtained *exactly* when removing one approximation with another one, which turns out also (as very rarely happens) much simpler. Indeed, we do assume that for a given separation, the repulsive component of the load does not change from when there is no adhesion (Hertz theory in this case), but we change the estimation of the adhesive force second with a purely “geometrical” one, based on the “bearing area” or “Abbott-Firestone curve” concept (Johnson [19], par.13.3), having two advantages, which not so often combine:

- 1) simplicity: we do not compute the contact area and do not need to work out the complicate expressions for gap outside a Hertzian contact. This a fortiori is extremely more complex for random rough surfaces.
- 2) better approximation, at least in the Hertzian case. In fact, we obtain the DMT solution given by Maugis [15], and not the much worse forms obtained by “thermodynamic” or “force” methods, in the original DMT solutions (see Ciavarella [18])

Consider the parabolic elastic solid in Fig. 1a of equation $y = r^2/(2R)$. The bearing area at a given indentation δ is simply

$$A = \pi r^2 = 2\pi R\delta \quad (1)$$

Let’s assume the attractive area A_{att} at *any value of*

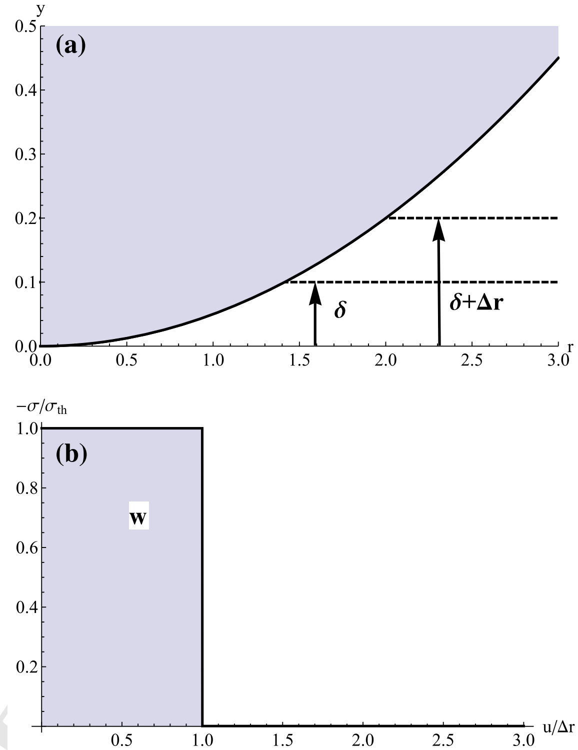

Fig. 1 **a** A parabolic elastic body in adhesive contact with a rigid plane; **b** Maugis forces of attraction *(“Maugis–Dugdale potential”)*

indentation to be given by the difference between the “bearing area” at indentation $\delta + \Delta r$, and the “bearing area” at indentation δ, which are obviously

$$A_{att} = 2\pi R(\delta + \Delta r - \delta) = 2\pi R\Delta r \quad (2)$$

and multiplying by the theoretical strength σ_{th}, results in the constant force of adhesion $P_{att} = 2\pi R\sigma_{th}\Delta r = 2\pi Rw$ which holds also at pull-off. This is indeed the DMT solution as presented in (Maugis [15]), called also DMT-M, the sum of the adhesiveless force due to a Hertzian contribution at given indentation δ, and a constant adhesion force $P_{att} = 2\pi Rw$.

$$P = P_H - P_{att} = \frac{4}{3}E^* \delta^{3/2} R^{1/2} - 2\pi Rw \quad (3)$$

3 Corrective factors

The DMT-M theory is the correct limit for extremely low Tabor parameter

[2] It is not known to the present authors that this fact has been noticed before in the Literature.

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$$\mu = \left(\frac{Rw^2}{E^{*2}\Delta r^3}\right)^{1/3} = \left(\frac{Rl_a^2}{\Delta r^3}\right)^{1/3} \to 0 \tag{4}$$

where E^* is plane strain modulus of the material ($l_a = w/E^*$ is an alternative way to measure adhesion as a length scale).

Instead, we want in general to cover cases of intermediate Tabor parameters. Indeed, we may further improve the bearing area model and show the transition between the DMT to the JKR solution, by making a small correction to consider the dependence on Tabor parameter. In Fig. 2 the load vs indentation curves are plotted using the dimensionless notation ($\widehat{\delta} = \delta/(\mu\Delta r)$ for indentation and $\widehat{P} = P/(\pi R w)$ for total load) for different values of the Tabor parameter μ. For $\mu > 1$, the JKR solution (Johnson et al. [20]) is more appropriate—and the curve is nearly parallel to the DMT one. The "bearing area" model gives always $\widehat{P} = -2$ at zero indentation, hence a simple correction is to reduce the constant adhesive force using the value of $\widehat{P}$ at $\widehat{\delta} = 0$ taken from the Maugis–Dugdale solution. For example, when $\mu > 1$ the JKR curve at zero indentation as and attractive load of $1.35\pi R w$, thus we divide the adhesive contribution in the bearing area model by a factor

$$\beta_1 = 2/1.35 = 1.5 \tag{5}$$

Moreover, the geometry of contacts in rough contact is *not* spherical. Indeed, despite asperities per se are not very elliptical, the description by asperities fails even at modest indentations (Greenwood [21]), and the contacts form rather elongated shapes (PR [9]), having actually a nearly constant average characteristic diameter fixed only by the geometry (whereas in asperity models it is the average diameter of contacts which is constant).

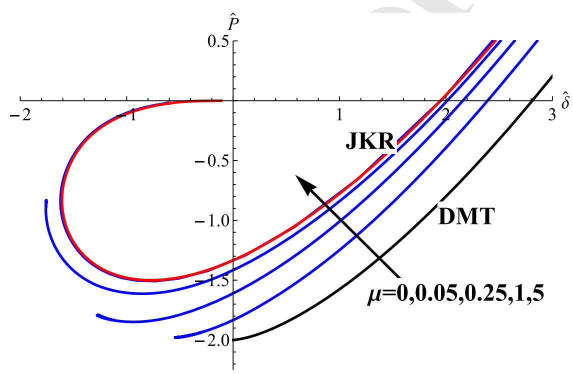

Fig. 2 Solutions of JKR, DMT-M and Maugis intermediate Tabor parameter range $\mu = 0, 0.05, 0.25, 1, 5$. JKR is obtained very closely at positive indentations for $\mu \simeq 1$. Notice that the proposed bearing area model correspons exactly with DMT-M with no corrective factors

We should warn the reader that we *shall not assume* the contact to be consistent of spherical asperities: the "bearing area" model is not a good approximation only for the spherical geometry. Let us consider the JKR solution for elliptical contacts (Johnson and Greenwood, 2005): from geometrical considerations, it is immediate to show that our bearing area model leads to

$$P_{att} = 2\pi R_e w \tag{6}$$

where $R_e = \sqrt{R_1 R_2}$ is the geometric mean of the principal radii of the surfaces. We don't know of a DMT solution for elliptical contacts, but Johnson and Greenwood [22] show in the JKR regime an approximate solution for which $P_{att} = \frac{3}{2}\pi R_e w$ is a good approximation, and an even better approximation is

$$P_{att} = \frac{3}{2}\pi R_m w \quad \text{where} \quad \frac{1}{\mathrm{R_m}} = \frac{1}{\mathrm{R_e^{3/4}}}\left[\frac{1}{2}\left(\frac{1}{\mathrm{R_1}} + \frac{1}{\mathrm{R_2}}\right)\right]^{1/4} \tag{7}$$

As a consequence of the fact that contact patches in rough contacts are very elongated (PR [9]), we may assume $\frac{R_1}{R_2} = 10 - 100$ which leads to a reduction of the order of $\frac{R_m}{R_e} = 0.87 - 0.6$, and hence consider as mean value 0.75. This results in a total corrective factor for the adhesive force

$$k = \frac{\beta_1}{R_m/R_e} = \frac{1.5}{0.75} \simeq 2 \tag{8}$$

4 Application to random rough surfaces

In applying the bearing area to a random rough surface, it will become evident the advantage of not having to estimate during the process the elastic contact area, nor the separations outside the contact area due to elastic deformations to integrate them as in the force method of DMT theory. Indeed, we estimate the attractive forces by a simple geometrical estimate which doesn't consider elastic deformations at all,

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although now this geometrical estimate will depend on the actual level of mean separation between surfaces.

Persson [16] derives from elastic energy concepts and with various approximations and corrections, a mean repulsive pressure σ_{rep} vs mean separation u law which, for the most practical case of self-affine surfaces of low fractal dimension (Persson et al. [5]), assumes the form valid for not too large σ_{rep} ([16] Eq. 20)

$$\frac{\sigma_{rep}}{E^*} \simeq \frac{3}{8\gamma} q_0 h_{rms} \exp\left(\frac{-u}{\gamma h_{rms}}\right) \tag{9}$$

where $\gamma \simeq 0.4$, q_0 is the smallest wavevector in the self-affine process, and h_{rms} is the rms amplitude of roughness. The original theory fits the multiplier inside the exponential with some FEM data, but doesn't provide a fit for the multiplier outside the exponential, for which we find a much better agreement in the present form, with the factor γ outside the exponential, with detailed calculations reported elsewhere [23]. The factor changes also weakly with the fine scale content of roughness, and particularly for high fractal dimensions, but as a first approximation we keep it constant.

Therefore, we compute the difference of the bearing area at separation $u_{att} = u - \Delta r$, and u, and the attractive pressure can be estimated in a single line as

$$\frac{\sigma_{att}}{\sigma_{th}} = -\frac{1}{2k}\left[Erfc\left(\frac{u_{att}}{\sqrt{2}h_{rms}}\right) - Erfc\left(\frac{u}{\sqrt{2}h_{rms}}\right)\right] \tag{10}$$

where Erfc is the error complementary function, and the factor $k \simeq 2$ was described in (8).

Therefore, summing up repulsive (9) and attractive (10) contributions,

$$\frac{\sigma(u)}{\sigma_{th}} \simeq \frac{3}{8\gamma} q_0 h_{rms} \frac{E^*}{\sigma_{th}} \exp\left(\frac{-u}{\gamma h_{rms}}\right) - \frac{1}{2k} \times \left[Erfc\left(\frac{u-\Delta r}{\sqrt{2}h_{rms}}\right) - Erfc\left(\frac{u}{\sqrt{2}h_{rms}}\right)\right] \tag{11}$$

This is the single closed form approximate result for the entire curve of pressure vs mean separation, which obviously results in a pull off finding the minimum as a function of u. The equation depends only on h_{rms}, q_0 and no other aspect of Power Spectrum, which may be some relief for those hoping that results of physical quantities do not depend too much on small scale details. In particular, notice that using the constant $k \simeq 2$ comes at the expense of modelling the very low h_{rms} as obviously the limit becomes $\frac{\sigma(u)}{\sigma_{th}} \simeq -\frac{1}{k}$ and not -1. However, as it will appear in the PR data below, we are talking in that case of a range where the amplitude of roughness is subatomic (!), and even if this made any sense with atomic roughness, the asymptotic version of Persson's theory starts to be problematic (the pressure deviates from (9) at $\frac{u}{h_{rms}} < 1$), and therefore the limit, as postulated, hardly makes any sense: either the surface is perfectly flat, or it has some atomic roughness.

4.1 Area–load

Persson's original contact theory [4] has a prediction for the proportion of actual contact at a given nominal pressure which, after the corrective factor of Putignano et al. [7] has been included, reads

$$\frac{A_{rep}}{A_0} = \text{erf}\left(\frac{\sqrt{\pi}}{2}\frac{\sigma_{rep}}{\sigma_{rough}}\right) \tag{12}$$

where $\sigma_{rough} = E^* h'_{rms}/2$ where h'_{rms} is the rms slope of the surface, and σ_{rep} can be estimate as a function of u from (9). We can plot some results for our model prediction (11) assuming $q_0 = 4096 a_0$ with $h'_{rms} = 0.1$ and $l_a/a_0 = 0.05$ (where $l_a = w/E^*$ and a_0 is atomic size, so this corresponds to the usual Lennard-Jones potential, $\Delta r \simeq a_0$ which has $\sigma_{th} \simeq 0.07E^*$) and therefore corresponding to some of the cases of Pastewka and Robbins [9] which we describe in more details later. We then choose various $h_{rms}/a_0 = 2, 3, \ldots, 10, 15, \infty$ (obviously, ∞ corresponds to Persson's adhesionless solution), see Fig. 3. The curve with $h_{rms}/a_0 = 10$ has been drawn red to facilitate reading of the plot. It is clear that for any numerical solution, the *assumption* that the curves are always linear near the origin to compute a slope, as done in PR, is not quite the best way forward—as there will be a limit to the resolution, for example $A_{rep}/A_0 = 0.02$ as already quite optimistic. As one reviewer suggested, *it makes a lot more sense to discuss adhesive tractions through the pull-off traction rather than through this slope.*

We predict the tangent near the origin to be very ill-defined near the threshold of stickiness, and the deviation from linearity a poor indication of pull-off decay, which follows an exponential trend with

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increasing amplitude of roughness as we will show in the next paragraph. Our prediction also shows that the repulsive contact area changes very little initially when there is small value of pull-off—and is close to the the value of the adhesionless Persson's solution (the last line in the plot of Fig. 3a, b for repulsive and total contact area, respectively). Notice that this implies that when we can happily use the standard contact mechanics solution, when we have roughness greater than few decades of atomic size.

It is clear also that in our model the "slope" of the area-load will depend on h_{rms}/a_0, and h'_{rms}, but not on h''_{rms} as in PR. Moreover, one could estimate the "secant" at for example $\frac{A_{rep}}{A_0} = 0.02$ from (9, 12) and get

$$\frac{u}{\gamma h_{rms}} = \log\left(\frac{\frac{3}{8} q_0 h_{rms}}{0.02 h'_{rms}}\right) \tag{13}$$

The "secant" will be vertical when $\frac{\sigma(u)}{\sigma_{th}} = 0$ in (11). As we don't believe this is a good indication of stickiness, we shall not further pursue this, and rather rely on pull-off as indication of stickiness.

5 Comparison with Pastewka-Robbins results

PR use a self-affine surface with power law PSD $A|\mathbf{q}|^{-2(1+H)}$ for wavevectors $q_r < |\mathbf{q}| < q_s$ $(q = 2\pi/\lambda)$ with roll off to a constant for $q_0 < |\mathbf{q}| < q_r$ (limited to $x = q_0/q_r = 1/2$) and zero otherwise. The surfaces have three values for the Hurst exponent $H = 0.3, 0.5, 0.8$, and magnification $\zeta = q_0/q_s = 512, \ldots 32$ resulting from a fixed roll-off wavelength $q_r = 2048 a_0$, where a_0 is atomic spacing, and varying small wavelength. The surfaces are generated in order to have two values of $h'_{rms} = 0.1, 0.3$.

PR do not discuss their values of h_{rms} as their entire paper is devoted to showing the small scale feature of surfaces are what count in adhesion. It turns out that their h_{rms} is very small, and indeed, despite their method of numerical solution seems to involve atoms, roughness is in fact introduced in the hard solid, and the counterface is simulating a perfect crystal with no deviation from flatness. Whether this is representative of atomic roughness is unclear, since Luan and Robbins [24] wrote a few years ago in a well known paper that atomic-scale surface roughness "is always produced by discrete atoms" and that it "leads to dramatic deviations from continuum theory". Hence, the fact that $h'_{rms} = 0.1, 0.3$ seems perhaps a choice motivated more by assuming realistic deformations. We shall see however that our result will seem to suggest an almost negligible effect of the small scale features, in contrast to what PR suggest, and this will be confirmed also by a reinterpretation of their own data.

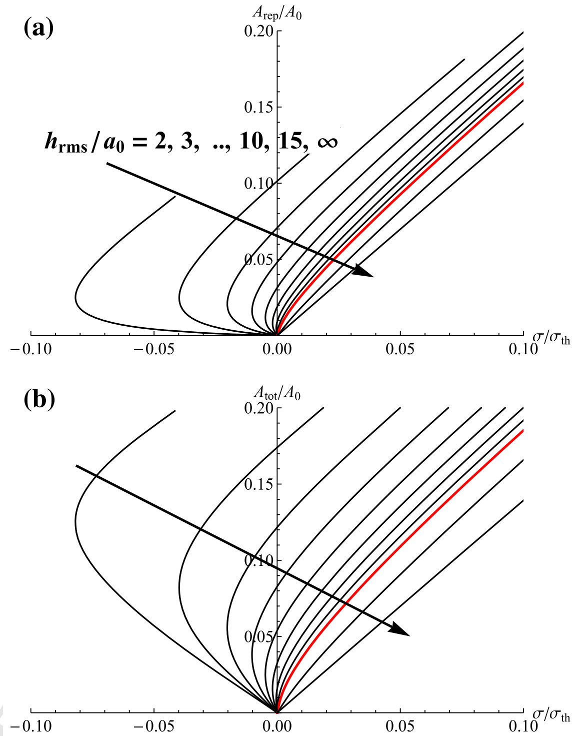

Fig. 3 Curves of **a** repulsive contact area A_{rep}/A_0 and **b** total contact area $A_{tot}/A_0 = (A_{rep} + A_{att})/A_0$ as a function of mean pressure σ/σ_{th}. For our "bearing area" model, with $h'_{rms} = 0.1$ and $l_a/a_0 = 0.05$ and various $h_{rms}/a_0 = 2, 3, \ldots, 10, 15, \infty$ (Persson's adhesionless solution) indicated by *arrow*. Also, $q_0 = 4096 a_0$ therefore corresponding to some of the cases of Pastewka and Robbins [9]

In Fig. S3 of PR, pull-off values are reported in a scale $4h_{rms}\sigma_{\min}/w$ which needs to be re-interpreted to extract $\sigma_{\min}$. From standard theory of Gaussian processes the moments of order n, can be computed as $n = 0, 2, 4$, $T(n) = 2\pi, \pi, 3\pi/4$,

$$m_n = A T(n) k_r^{n-2H} \left(\frac{1 - x^{n+2}}{n+2} + \frac{\zeta^{n-2H} - 1}{n - 2H}\right) \tag{14}$$

where $\zeta = q_s/q_r > > 1$ is the magnification, and knowing that $h_{rms} = \sqrt{m_0}$, and $h'_{rms} = \sqrt{2m_2}$, from the ratio of m_0 and m_2 we have

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$$h_{rms} \simeq \frac{h'_{rms}}{\sqrt{2}} \sqrt{(2-2H)\left(\frac{3}{4}+\frac{1}{H}\right)\frac{q_s}{2\pi}\zeta^H} \quad (15)$$

which permits to estimate the rms amplitudes. In particular, for q_s/a_0 increasing from 4 to 64 in powers of 2 and $h'_{rms} = 0.1$, we obtain the values of h_{rms}/a_0 lay in the ranges $[0.7 \div 5]$ (one being subatomic, again of uncertain significance in particular, in view of Luan and Robbins [24]) , $[1.7, 6.7]$ and $[5.9, 10.3]$ for the cases of $H = 0.3, 0.5$, and 0.8, respectively. In other words, the high fractal dimensions are very small in amplitude, and this is very important in what we are going to discuss, because Persson's equation (9) was also assumed to be more appropriate to low fractal dimensions, and insensitive then on h'_{rms} and higher order moments which depend on the small wavelength truncation. Obviously values are 3 times higher for $h'_{rms} = 0.3$. Data are shown in PR paper with the same symbols as they will be shown here, so $h'_{rms} = 0.1, 0.3$ (closed, open symbols), and for $l_a/a_0 = 0.05, 0.005$ (red, blue)—we omit the change in size of the symbols since q_s/a_0 increasing from 4 to 64 also corresponds to an increase of h_{rms} which is easy to follow in the diagram.[3]

We assume $q_0 = 2q_r = 4096a_0$ (although the start of self-affine behaviour is at q_r) whereas $\frac{\sigma_{th}}{E^*} = 0.07$ and 0.025 for $\Delta r = 1.1a_0$ and $\Delta r = 0.35a_0$, respectively for PR choice of "truncated spline" potentials having $w/(E^* a_0) = l_a/a_0 = 0.05, 0.005$.

Results as a function of mean separation are shown in Fig. 4, for $l_a/a_0 = 0.05, 0.005$ respectively in Fig. 4a, b. In each plot, increasing values of h_{rms}/a_0 are considered, and $k = 2$ has been used. It is clear that for shorter range of adhesion, there are fewer curves as already for $h_{rms}/a_0 = 3$, there is negligible adhesion. The curves, in log scale, show that at very large h_{rms}/a_0 there may be a small error in that the asymptotic becomes slightly compressive again. For an approximate model, this is a very minor defect, compared to what can happen with other approximations, see Discussion.

Figure 5 reports a summary of the pull-off values obtained with our simple estimate, which are excellent for the case $l_a/a_0 = 0.05$ except perhaps the high fractal dimension cases, and the same occurs for $l_a/a_0 = 0.005$, since the high fractal dimension cases have so high rms amplitude that are in fact absent from the figure, as correctly our model predicts. Considering the simplicity of the approach, the results are clearly a very convenient solution. The improvements which could still be possible are for the high fractal dimension cases. In fact there is a single point at low fractal dimension (a red triangle) which is not well fit, but this is highly suspicious as it has a non-monotonic increase of pull-off with respect to its companion cases, and may be simply an error in the numerical results or in the reporting of them.

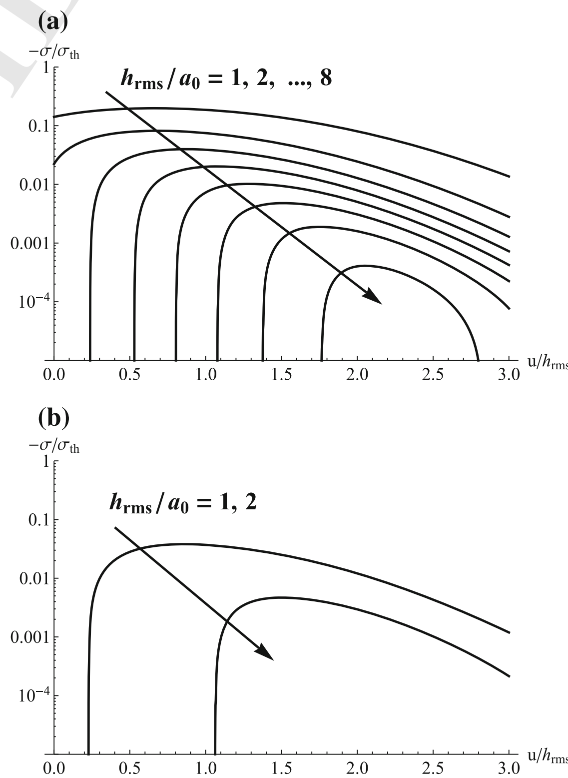

Fig. 4 The tensile part of traction (log scale) as a function of mean separation from the bearing-area DMT model, for PR cases with **a** $l_a/a_0 = 0.05$, **b** $l_a/a_0 = 0.005$

[3] Surprisingly, in Fig. S3 there are some blue closed symbols ($h'_{rms} = 0.1$ and $l_a/a_0 = 0.005$), which appear curious, as they appear as non-sticky in Fig. 4 of the paper. Also, the fractal dimension in Fig. S3 does not appear in correct order, as low H seem to have higher rms amplitude, whereas the opposite trend should occur. Probably there is an inversion of the data for $H = 0.3$ and $H = 0.8$, which is however irrelevant for the present scopes.

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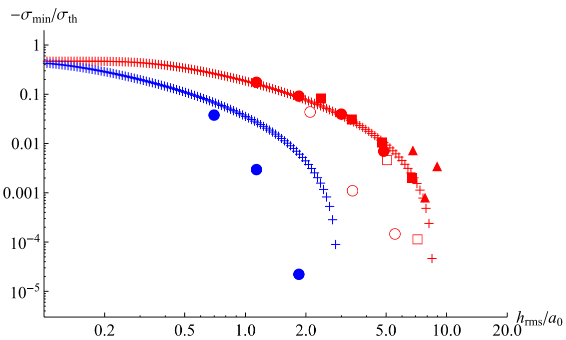

Fig. 5 Pull-off estimates with eqt. (11) for PR data with (*red colour*) $l_a/a_0 = 0.05$, (*blue colour*) $l_a/a_0 = 0.005$. *Symbols* are like in the original PR Fig. S3, and hence correspond to different short wavelength truncation, different H or fractal dimension, and different $h'_{rms} = 0.1$ (*closed symbols*) and $h'_{rms} = 0.3$ (*open symbols*). Some effect of h'_{rms} may appear but is not marked, and similarly, even if our DMT "bearing area model" is meant for low fractal dimensions, no effect of fractal dimension appears clear

The cases which may require some correction if one aims at a more quantitative estimate are those with high fractal dimension which show lower adhesion—but notice that from the data the indication is not so clear: the low slope case for $l_a/a_0 = 0.05$ (closed red circles) is still within our approximation, whereas the case $l_a/a_0 = 0.005$ is not (closed blue circle). The high slopes cases with $l_a/a_0 = 0.005$ are not in the figure, in very good agreement with our expectation, since they would have 3 times higher rms amplitude than the $h'_{rms} = 0.1$ slopes.

This is therefore partly expected from our model when we used Persson's equation (9) in the form for low fractal dimension. This is the case of practical interest as the majority of natural surfaces so far measured have indeed fractal dimension $D < 2.5$ and rather close to $D = 2.2$ (see Persson et al. [5]). Therefore, not only the additional corrective factors which we would need (some estimates we have more repulsive code suggest that they not be what Persson [16] suggest) are more complicated, as they depend on truncation wavelength, but are not really worth the effort, as they would only serve to fit more accurately the numerical results of academic interest. We are happy enough to have found very good agreement for *all* the case of low and intermediate fractal dimensions, with our simple closed form result.

6 Discussion

It is clear that our theory shows no dependence at all of slopes and curvatures, and, we expect, this is especially true at low fractal dimensions, which is the case of practical interest, but anyway is in strong contrast with PR conclusions about the slope of the area-load equation: this indicates either that their criterion is purely on "loading" conditions, or else that they did not measure accurately the area-load near the origin where the load can indeed very small, but still significant. That their model supports the linearity of the area-load relationship suggests indeed this may be the case—they perhaps looked at this due to their use a DMT equation where the attractive area is further simplified by taking only the asymptotic expression for separation: if we were to use their estimates of the attractive area, see "Appendix", we would end up with the quite paradoxical result on the effect of amplitude of roughness, due to the fact that they looked at cases where the contact area is significant, and during loading.

Naturally, we don't expect our simple model can predict the effects of roughness in every possible regime, as we obtain a purely non-hysteretic behaviour and therefore we probably obtain a "lower bound" to pull-off.

Adhesion for "soft" bodies, under the so called JKR regime (Johnson et al. [20]), shows instabilities like in the simple case of a single sinusoid (Johnson [25]) which leads to very strong adhesion after a sufficiently high pressure has been applied, or even (for sufficiently high work of adhesion) to spontaneous full contact and strong adhesion. This effect is also seen clearly in numerical experiments *assuming* JKR conditions (for a 1D form of roughness) of Carbone et al. [26], and cannot be modelled with asperities, nor with any other present theory, except in rather special cases (Guduru [27]).

7 Conclusion

We have provided a very simple model for pull-off of hard elastic solids (presumably, for low Tabor parameters, for which the DMT solution is approximately valid. We have given a very simple estimate from the bearing-area of the area of attraction, and this seems to give reasonable results.

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Appendix: The PR DMT-like approximation

We shall investigate what happens when estimating the area of attraction with the simplified DMT equations provided by PR [9]. They write $A_{rep} = Pd_{rep}/\pi$ where P is the perimeter of the contact which grows proportionally to load, and $A_{att} = Pd_{att}$, so thatd_{rep}, d_{att} are representative diameter of repulsive region, and size of the lateral strip of attraction. As they find by that $d_{rep} = 4h'_{rms}/h''_{rms}$

$$\frac{d_{att}}{d_{rep}} = \left(\frac{3}{4}\frac{\Delta r}{h'_{rms} d_{rep}}\right)^{2/3} = \left(\frac{3}{16}\frac{h''_{rms}\Delta r}{h'^2_{rms}}\right)^{2/3} \tag{16}$$

then

$$A_{att} = \pi\left(\frac{3}{16}\frac{h''_{rms}\Delta r}{h'^2_{rms}}\right)^{2/3} A_{rep} \tag{17}$$

which is their eqt.6. This is supported by some log-log plots (the insets of Figs. 3 and 4) which however show each quite significant deviations of factors larger than 2-3, so that the result in terms of actual A_{rep}/A_{att} could change by an order of magnitude. As pull-off occurs at large rms amplitudes for very small A_{rep}, this may explain the very large effect this approximation has.

Now we can use A_{rep} from Persson's solution (12), and hence the tensile mean traction is

$$\frac{\sigma_{att}}{\sigma_{th}} = \frac{A_{att}}{A_0} = \pi\left(\frac{3}{16}\frac{h''_{rms}\Delta r}{h'^2_{rms}}\right)^{2/3} \frac{3}{4} q_0 \frac{h_{rms}}{h'_{rms}} \exp\left(\frac{-u}{\gamma h_{rms}}\right) \tag{18}$$

Summing the repulsive and attractive contributions leads to a single negative exponential dependence on mean gap

$$\frac{\sigma(u)}{E^*} \simeq \frac{3}{8\gamma} q_0 h_{rms} \exp\left(\frac{-u}{0.45 h_{rms}}\right) \times \left[1 - \frac{\sigma_{th}}{E^*}\frac{2\pi}{h'_{rms}}\left(\frac{3}{16}\frac{h''_{rms}\Delta r}{h'^2_{rms}}\right)^{2/3}\right] \tag{19}$$

and therefore there is no longer a threshold on h_{rms}. If the surfaces are such that

$$\frac{\sigma_{th}}{E^*}\frac{2\pi}{h'_{rms}}\left(\frac{3}{16}\frac{h''_{rms}\Delta r}{h'^2_{rms}}\right)^{2/3} > 1$$

then there is (pull-off) stickiness, which is of course their result. However, clearly this result in the "sticky" range is completely absurd: pull-off would then be obtained at very low values of u/h_{rms} (in principle, for our asymptotic Persson's equation, we would reach $u = 0$) which is quite counterintuitive.: we could truncate at realistic values of $u/h_{rms} = 0.1$ as Persson [16] shows that the repulsive pressure increases there much more than his asymptotic solution. But the main effect of (19) would be quite paradoxical: a pull-off which actually increases with h_{rms}! Now, as we can certainly not believe this paradox, and as we have already shown that their criterion doesn't satisfy the values of pull-off obtained [28–30], we would also tend to think that their result has too many approximations and results from inaccurate fits in log-log plots which hide important deviations.

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