Comment on "Method to analyze electromechanical stability of dielectric elastomers" [Appl. Phys. Lett. 91, 061921 (2007)]

Andrew N. Norris*
Mechanical and Aerospace Engineering, Rutgers University, Piscataway NJ 08854
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Explicit expressions are presented for the critical breakdown electric field value as a function of biaxial strain. Simplified results for uniaxial and for equi-biaxial stress provide further insight into the findings of Zhao and Suo [1].

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The paper of Zhao and Suo [1] describes the first complete electromechanical model for the phenomenon of electrical breakdown in thin elastomers. The purpose of this comment is to point out some analytical simplifications which provide further insight into their paper. The results here stem from the observation that the determinant of the Hessian $\mathbf{H}$ of eq. (4) in [1] may be factored, leading to semi-explicit formulas for the critical values of the electrical and mechanical parameters.

Let

$$z = \frac{\widetilde{D}^2}{\mu\epsilon} = \frac{\epsilon\widetilde{E}^2}{\mu}\lambda_1^4\lambda_2^4 = \frac{\epsilon E^2}{\mu}\lambda_1^2\lambda_2^2, \tag{1}$$

where the nondimensional parameter z is new, and all other notation follows that of [1]. It may be checked that the determinant reduces to a quadratic in z,

$$\det\mathbf{H} = \frac{\mu^2\epsilon^{-1}}{\lambda_1^8\lambda_2^8}\Big[5 + 3(\lambda_1^2+\lambda_2^2)\lambda_1^2\lambda_2^2 + \lambda_1^6\lambda_2^6 + [2-(\lambda_1^2+\lambda_2^2)\lambda_1^2\lambda_2^2]z - 3z^2\Big]. \tag{2}$$

The roots are real and of opposite sign. The critical value of z at which the Hessian is no longer positive definite is therefore

$$z_c(\lambda_1,\lambda_2) = \frac{1}{6}[2-(\lambda_1^2+\lambda_2^2)\lambda_1^2\lambda_2^2] + \frac{1}{6}\sqrt{[8+(\lambda_1^2+\lambda_2^2)\lambda_1^2\lambda_2^2]^2 + 4\lambda_1^2\lambda_2^2[3\lambda_1^4\lambda_2^4 + 4(\lambda_1^2+\lambda_2^2)]}. \tag{3}$$

The critical value of the electrical field at breakdown and the corresponding stresses are

$$\sqrt{\frac{\epsilon}{\mu}}E_c = \frac{z_c^{1/2}(\lambda_1,\lambda_2)}{\lambda_1\lambda_2}, \tag{4a}$$

$$\frac{s_j}{\mu} = \lambda_j - \frac{(1+z_c(\lambda_1,\lambda_2))}{\lambda_j\lambda_1^2\lambda_2^2}, \quad j=1,2. \tag{4b}$$

States of uniaxial and equi-biaxial stress are particularly simple. Under equal biaxial stress we obtain

$$\sqrt{\frac{\epsilon}{\mu}}E_c = \big(\frac{1}{3}\lambda^2 + \frac{5}{3}\lambda^{-4}\big)^{1/2}, \tag{5a}$$

$$\frac{s}{\mu} = \frac{2}{3}(\lambda - 4\lambda^{-5}). \tag{5b}$$

These parameterize the critical electrical and mechanical fields in terms of $\lambda \ge \lambda_c = 2^{1/3} \approx 1.26$. The minimum value of $\sqrt{\frac{\epsilon}{\mu}}E_c$ is 1.038 and occurs at $\lambda = 10^{1/6} \approx 1.47$.

Under uniaxial stress, eq. (4b) with $j=2$ and $s_2=0$ yields the following relation between the stretches:

$$\lambda_1^2 = \frac{3\lambda_2^2}{\lambda_2^6 - 1}. \tag{6}$$

Hence, we can parameterize the critical values in terms of $1 < \lambda_2 \le \lambda_c$:

$$\sqrt{\frac{\epsilon}{\mu}}E_c = \big(\frac{2}{3}\lambda_2^2 + \frac{1}{3}\lambda_2^{-4}\big)^{1/2}, \tag{7a}$$

$$\frac{s_1}{\mu} = \frac{\lambda_2}{\sqrt{3}}\frac{(4-\lambda_2^6)}{\sqrt{\lambda_2^6-1}}. \tag{7b}$$

In this case E_c is a monotonically decreasing function of the stress s_1, and $\sqrt{\frac{\epsilon}{\mu}}E_c \to 1$ in the limit of large uniaxial stress. Figure 3(b) in [1] indicates that this is the smallest achievable value of the critical electric field strength.

[1] X. Zhao and Z. Suo, Appl. Phys. Lett. **91** (2007).