

Elastic-Plastic Fracture Mechanics. Lecture 2

Lecture 1 described the Begley-Landes experiment, and the blunting of a crack due to large deformation. <http://imechanica.org/node/7900>. Lecture 2 is motivated by the following considerations.

When a rubber containing a crack is loaded, before the crack extends, strain everywhere in the rubber can be large. By contrast, when a metal containing a crack is loaded, before the crack extends, strain in the metal is typically small, except near the tip of the blunted crack. Consequently, to analyze deformation in the metal, at a distance a few times the crack opening displacement away from the crack tip, we can use the field theory of infinitesimal deformation.

When a metal strains far beyond the elastic limit, the stress-strain relation is often fit to a power law.

When a metal undergoes homogeneous deformation subject to proportional loading, the stress-strain behavior of plastic deformation is indistinguishable from that of elastic deformation.

The above idealizations are used to formulate a field theory of infinitesimal deformation and power-law material model. Within this theory, a large class of boundary-value problems has solutions of a remarkably simple form. This result, the Ilyushin theorem, generalizes the linearity in the linear elastic theory into a scaling relation for the power-law material.

The power-law material and the Ilyushin theorem are described in the present lecture. We then apply these ideas to elastic-plastic fracture mechanics. In particular, we will describe the HRR field.

Field theory of infinitesimal deformation. Let $u_i(x_1, x_2, x_3)$ be the field of displacement. The field of strain is

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

The field of stress $\sigma_{ij}(x_1, x_2, x_3)$ satisfies the condition of equilibrium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0.$$

On a surface with unit normal n_i , the traction t_i relates to the stress σ_{ij} as

$$\sigma_{ij} n_j = t_i.$$

The above equations are the same as those used in the linear elastic theory (<http://imechanica.org/node/205>). We next describe a material model of elastic-plastic deformation.

Ramberg-Osgood equation (1943). Pull a rod of a metal with an axial force, record the strain ε as a function of the stress σ . Ramberg and Osgood suggested that the data be fit to the equation

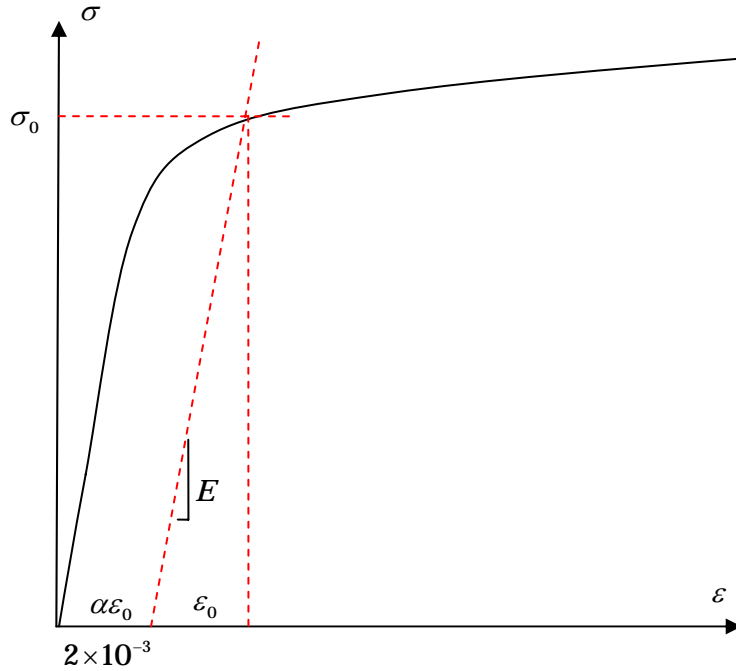
$$\varepsilon = \frac{\sigma}{E} + K \left(\frac{\sigma}{E} \right)^n.$$

The first term represents the elastic strain, and the second the plastic strain. The Ramberg-Osgood equation has three fitting parameters: E is Young's modulus, and K and n describe the strain-hardening behavior.

The Ramberg-Osgood equation is often written in another form:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^n,$$

where σ_0 is the yield strength, $\varepsilon_0 = \sigma_0 / E$, and $\alpha = K(\sigma_0 / E)^{n-1}$. This equation is *not* a four-parameter fit. Rather, the yield strength σ_0 is introduced by a convention, such that the plastic strain is $\alpha\varepsilon_0 = 2 \times 10^{-3}$ when $\sigma = \sigma_0$. Representative values are $n = 5$, $E = 10^{11} \text{ N/m}^2$, and $\sigma_0 = 10^8 \text{ N/m}^2$.



Power law. When the total strain is much beyond the elastic strain, the latter is negligible, and the stress-strain curve is represented by a power law:

$$\frac{\varepsilon}{\varepsilon_0} = \alpha \left(\frac{\sigma}{\sigma_0} \right)^n.$$

It may be too clumsy to write the power law with four fitting parameters. Rather, we write

$$\varepsilon = \left(\frac{\sigma}{B} \right)^n.$$

The parameter B is of the dimension of stress. The value of B is the stress needed to pull the metal to the strain $\varepsilon = 1$, although in practice we seldom subject the metal to such a large strain. In any event, the power law only needs two fitting parameters.

Multiaxial stress-strain relation. The central aim of the theory of plasticity is to formulate the stress-strain relations under multiaxial stresses. Subject a material particle to multiaxial stresses, σ_{ij} , the strains in all directions will depend on the state of the stresses, as well as on the history in which the stresses are applied. We can experimentally record the evolution of the strains

under various histories of the stresses. Because there are infinitely many states and histories, this experimental approach is impractical. In the theory of plasticity, we prescribe flow rules of various kinds, which more or less fit experimentally recorded histories of evolution.

Here we will only consider proportional loading, namely, all the stress components increase proportionally to the final state. There is no unloading. Consequently, the loading history is completely specified by the final state of the stresses, σ_{ij} , and the strains in the final state will only depend on the stresses of the final state. That is, we model the elastic-plastic deformation by fictitious elastic deformation. In the following review, we will simply use the language of elasticity.

Energy density. When the strains vary by $\delta\varepsilon_{ij}$, the stresses do work $\sigma_{ij}\delta\varepsilon_{ij}$. For an elastic material, this work varies the free energy, δW , namely,

$$\delta W = \sigma_{ij}\delta\varepsilon_{ij}.$$

The energy density W is a function of the strains,

$$W = W(\varepsilon_{11}, \varepsilon_{12}, \dots).$$

The stresses are partial derivatives:

$$\sigma_{ij} = \frac{\partial W(\varepsilon_{11}, \varepsilon_{12}, \dots)}{\partial \varepsilon_{ij}}.$$

Thus, a model of an elastic material is specified by the energy-density function $W(\varepsilon_{11}, \varepsilon_{12}, \dots)$. Once this function is specified, the above equation gives the relations between the stresses and the strains.

When we say that an elastic-plastic material is modeled by a fictitious elastic material, we mean that we prescribe the function $W(\varepsilon_{11}, \varepsilon_{12}, \dots)$, and obtain the stress-strain relations by the above derivatives. Here W cannot be interpreted as the free-energy density. This prescription is just a material model.

In the above, the strains are taken to be the independent variables. To make the stresses as the independent variables, define another quantity:

$$\hat{W} = \sigma_{ij}\varepsilon_{ij} - W.$$

This equation, together with $\delta W = \sigma_{ij}\delta\varepsilon_{ij}$, leads to

$$\delta\hat{W} = \varepsilon_{ij}\delta\sigma_{ij}.$$

Thus, the \hat{W} is regarded as a function of the stresses, namely,

$$\hat{W} = \hat{W}(\sigma_{11}, \sigma_{22}, \dots).$$

Once this function is prescribed, the strains are related to the stresses by

$$\varepsilon_{ij} = \frac{\partial \hat{W}(\sigma_{11}, \sigma_{22}, \dots)}{\partial \sigma_{ij}}.$$

Now we have reduced the problem of prescribing six functions of six independent variables, $\varepsilon_{ij}(\sigma_{11}, \sigma_{22}, \dots)$, to a problem of prescribing a single function of six independent variables, $\hat{W}(\sigma_{11}, \sigma_{22}, \dots)$. We next make further simplifications, ultimately leading to a problem of prescribing a single function of a single independent variable.

Isotropy. For an isotropic material, \hat{W} depends on the components of the stress tensor through the three invariants:

$$\sigma_{ii}, \quad \sigma_{ij}\sigma_{ij}, \quad \sigma_{ij}\sigma_{jl}\sigma_{li}.$$

All the subscripts are dummy, so that these invariants are scalars formed by the components of the stress tensor. These invariants are independent. Any of their combinations are also invariants.

Recall an analogous situation. For a vector f_i , its components form a single invariant $f_i f_i$. The invariant is a scalar with a simple interpretation: $(f_i f_i)^{1/2}$ is the magnitude of the vector.

Plastic deformation is unaffected by hydrostatic stress. Experiments have shown that plastic deformation is, to a good approximation, unaffected by a moderate hydrostatic stress, either applied alone or superimposed on any state of stress. As an idealization, we assume that superposing any hydrostatic stress will not affect plastic deformation. Consequently, \hat{W} is independent of the first invariant of stress. Furthermore, \hat{W} cannot depend on the second invariant of stress $\sigma_{ij}\sigma_{ij}$ by itself, because superposing a hydrostatic stress will change $\sigma_{ij}\sigma_{ij}$.

Instead, consider the general invariant quadratic in the components of the stress tensor combination:

$$\sigma_{ij}\sigma_{ij} + \alpha(\sigma_{kk})^2.$$

We wish to select a value of α such that the quadratic invariant $\sigma_{ij}\sigma_{ij} + \alpha(\sigma_{kk})^2$ is unchanged when any hydrostatic stress is superposed.

A state of stress, σ_{ij} , when superposed with a hydrostatic stress q , results in another state of stress $\sigma'_{ij} = \sigma_{ij} + q\delta_{ij}$. For this new state of stress, the quadratic invariant is

$$\begin{aligned} \sigma'_{ij}\sigma'_{ij} + \alpha(\sigma'_{kk})^2 &= (\sigma_{ij} + q\delta_{ij})(\sigma_{ij} + q\delta_{ij}) + \alpha(\sigma_{kk} + 3q)^2 \\ &= \sigma_{ij}\sigma_{ij} + 2\sigma_{kk}q + 3q^2 + \alpha(\sigma_{kk})^2 + 6\alpha\sigma_{kk}q + 9q^2 \end{aligned}$$

For the quadratic invariant to be unaffected by q , namely, to ensure that $\sigma'_{ij}\sigma'_{ij} + \alpha(\sigma'_{kk})^2 = \sigma_{ij}\sigma_{ij} + \alpha(\sigma_{kk})^2$, we must set $\alpha = -1/3$.

Thus, the quadratic invariant unaffected by hydrostatic stress takes the form

$$\sigma_{ij}\sigma_{ij} - \frac{1}{3}(\sigma_{kk})^2.$$

In the literature this invariant is written in a slightly different form:

$$\left(\frac{3}{2}\sigma_{ij}\sigma_{ij} - \frac{1}{2}(\sigma_{kk})^2 \right)^{1/2}.$$

The invariant is given notation σ_e , and is called the Mises equivalent stress. Under uniaxial stress, the Mises equivalent stress reduces to the applied stress.

Just as $(f_i f_i)^{1/2}$ is a scalar characterizing the magnitude of the vector f_i , the Mises equivalent stress σ_e is a scalar characterizing the magnitude of the

stress tensor, under the condition that this magnitude of stress tensor is unaffected by superposing any hydrostatic stress to the stress tensor.

Energy density as a function of a single scalar. As a further simplification, we assume that \hat{W} is independent of the third invariant of stress. Consequently, for an isotropic material whose deformation is unaffected by hydrostatic stress, we set the energy-density function as a function of a single variable, the Mises equivalent stress:

$$\hat{W} = \hat{W}(\sigma_e),$$

with

$$\sigma_e = \left(\frac{3}{2} \sigma_{ij} \sigma_{ij} - \frac{1}{2} (\sigma_{kk})^2 \right)^{1/2}.$$

Recall that

$$\varepsilon_{ij} = \frac{\partial \hat{W}(\sigma_{11}, \sigma_{22}, \dots)}{\partial \sigma_{ij}}.$$

Note that

$$\frac{\partial \sigma_e(\sigma_{11}, \sigma_{22}, \dots)}{\partial \sigma_{ij}} = \frac{3}{2\sigma_e} \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right).$$

We obtain that

$$\varepsilon_{ij} = \frac{3}{2\sigma_e} \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right) \frac{d\hat{W}(\sigma_e)}{d\sigma_e}.$$

Note that this material model indeed ensures incompressibility:

$$\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = 0.$$

In the above, we have used the stresses as the independent variables. Alternatively, we can use the strains as the independent variables, and make the energy density W as a function of an invariant quadratic in strains.

Generalize a uniaxial stress-strain relation to multiaxial stress-strain relations. Under a state of uniaxial stress σ , The Mises equivalent stress σ_e equals the applied stress σ . Because $\hat{W}(\sigma_e)$ is a function of a single variable, this function can be obtained from the uniaxial stress-strain curve $\varepsilon(\sigma)$ by integrating

$$\delta \hat{W} = \varepsilon(\sigma) \delta \sigma.$$

To generalize a uniaxial stress-strain relation to multiaxial stress-strain relations, we follow these steps:

1. Determine the stress-strain relation $\varepsilon(\sigma)$ under the uniaxial loading. This is usually done by fitting experimentally measured stress-strain relation to a curve.
2. Integrate $\delta \hat{W} = \varepsilon(\sigma) \delta \sigma$ to obtain $\hat{W}(\sigma)$
3. In $\hat{W}(\sigma)$, replace σ with σ_e .
4. Calculate the multiaxial stress-strain relations from

$$\varepsilon_{ij} = \frac{3}{2\sigma_e} \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right) \frac{d\hat{W}(\sigma_e)}{d\sigma_e}.$$

Power law for multiaxial loading. The uniaxial stress-strain curve is fit to the power law:

$$\varepsilon(\sigma) = \left(\frac{\sigma}{B}\right)^n.$$

Integrating

$$\delta\hat{W} = \varepsilon(\sigma)\delta\sigma,$$

we obtain the energy-density function

$$\hat{W}(\sigma) = \frac{B}{n+1} \left(\frac{\sigma}{B}\right)^{n+1}.$$

Replace σ with σ_e , and we write

$$\hat{W}(\sigma_e) = \frac{B}{n+1} \left(\frac{\sigma_e}{B}\right)^{n+1}.$$

Inserting into

$$\varepsilon_{ij} = \frac{3}{2\sigma_e} \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right) \frac{d\hat{W}(\sigma_e)}{d\sigma_e},$$

we obtain that stress-strain relation under multiaxial loading:

$$\varepsilon_{ij} = \frac{3}{2} \left(\frac{\sigma_e}{B}\right)^{n-1} \left(\frac{\sigma_{ij}}{B} - \frac{\sigma_{kk} \delta_{ij}}{3B} \right).$$

J_2 -deformation theory. Perhaps you would like to see an alternative and entirely equivalent presentation given in textbooks. Define the mean stress as

$$\sigma_m = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}),$$

Define the deviatoric stress tensor as

$$s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}.$$

Define the Mises effective stress as

$$\sigma_e = \left(\frac{3}{2} s_{ij} s_{ij} \right)^{1/2}.$$

The second invariant of the deviatoric stress is given the symbol

$$J_2 = \frac{1}{2} s_{ij} s_{ij}.$$

The symbol gives the name to this material model: the J_2 -deformation theory. Needless to say, this J_2 unrelated to the J integral.

This textbook approach is entirely equivalent to the result in the previous paragraph. You can confirm that

$$s_{ij} s_{ij} = \sigma_{ij} \sigma_{ij} - \frac{1}{3} (\sigma_{kk})^2.$$

Ilyushin theorem (1946). Write the stress-strain relation in a generic form:

$$\varepsilon_{ij} = f_{ij} \left(\frac{\sigma_{11}}{B}, \frac{\sigma_{12}}{B}, \dots \right)$$

All we have done above is to make the function dimensionless by using B to normalize the stresses.

The stress-strain relation developed in the previous paragraph has a useful property. When all components of the stress tensor is multiplied by a factor λ , the components of the strain tensor is multiplied by a factor λ^n . That is,

$$f_{ij}\left(\lambda \frac{\sigma_{11}}{B}, \lambda \frac{\sigma_{12}}{B}, \dots\right) = \lambda^n f_{ij}\left(\frac{\sigma_{11}}{B}, \frac{\sigma_{12}}{B}, \dots\right).$$

In a boundary-value problem, this material model is combined with the strain-displacement relations

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),$$

and the condition of equilibrium

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0.$$

The boundary conditions can be prescribed by displacement or traction.

Consider a boundary value problem characterized by a length a and load σ_{appl} . The Ilyushin theorem says that the fields of stress, strain and displacement take the form

$$\begin{aligned} \sigma_{ij} &= \sigma_{\text{appl}} \hat{\sigma}_{ij}\left(\frac{\mathbf{x}}{a}, n\right), \\ \varepsilon_{ij} &= \left(\frac{\sigma_{\text{appl}}}{B}\right)^n \hat{\varepsilon}_{ij}\left(\frac{\mathbf{x}}{a}, n\right), \\ u_i &= a \left(\frac{\sigma_{\text{appl}}}{B}\right)^n \hat{u}_i\left(\frac{\mathbf{x}}{a}, n\right). \end{aligned}$$

Here $\hat{\sigma}_{ij}$, $\hat{\varepsilon}_{ij}$ and \hat{u}_i stand for dimensionless functions. The proof of this theorem is simple: the above form satisfies all the governing equations.

Thus, when a body is subject to an applied stress σ_{appl} , the field of stress is linear in σ_{appl} , and the field strain is proportional to $(\sigma_{\text{appl}})^n$. The Ilyushin theorem determines the dependence of the fields on the applied stress. The distribution of the fields still need be determined by solving the boundary-value problem.

The Ilyushin theorem generalizes the consideration of linearity in the linear elastic theory into a scaling relation for the power-law material.

Stress concentration around a cavity in a power-law material.

Consider a spherical cavity in a power-law material subject to a remote stress. The stress reaches maximum at the surface of the cavity. The stress concentration factor takes the form

$$\frac{\sigma_{\text{max}}}{\sigma_{\text{appl}}} = C(n).$$

The stress concentration factor C is a dimensionless number depending on n only. Some such boundary-value problems are described in Budiansky, Hutchison and Slutsky (1982).

<http://www.seas.harvard.edu/hutchinson/papers/362.pdf>

Energy release rate for a crack in a power-law material. Consider a crack of length $2a$ in a sheet subject to remote stress σ_{appl} . Because the scaling of the stress and strain fields, the energy release rate takes the form

$$G = aB \left(\frac{\sigma_{\text{appl}}}{B} \right)^{n+1} g(n).$$

The dimensionless number g depends on n , and must be determined by solving the boundary-value problem. In homework you will study such a boundary-value problem solved by He and Hutchinson (1981).

<http://www.seas.harvard.edu/hutchinson/papers/360.pdf>

HRR field (1968). Next consider the field around the tip of a crack in a power-law material. Let r be the distance from the tip of the crack. Near the tip of the crack, where r is much smaller than the length characteristic of the boundary-value problem, $r \ll a$, the field depends on the external boundary condition through the energy release rate G . Consequently, the problem has a single length scale: G/B . The dimensional considerations dictate that the field of stress near the tip of the crack should take the form

$$\sigma_{ij}(r, \theta) = BF_{ij} \left(\frac{r}{G/B}, \theta; n \right),$$

where F_{ij} are dimensionless functions.

According to the Ilyushin theorem, the field of stress $\sigma_{ij}(r, \theta)$ is linear in the applied stress σ_{appl} :

$$\sigma_{ij}(r, \theta) \propto \sigma_{\text{appl}}.$$

The energy release rate G scales with the applied stress as

$$G \propto (\sigma_{\text{appl}})^{n+1}.$$

Consequently, the field of stress scales with the energy release rate as

$$\sigma_{ij}(r, \theta) \propto G^{\frac{1}{n+1}}.$$

This scaling, together with the dimensional considerations, requires that the field of stress near the tip of the crack should take the form:

$$\sigma_{ij}(r, \theta) = B \left(\frac{G}{Br} \right)^{\frac{1}{n+1}} f_{ij}(\theta, n),$$

where $f_{ij}(\theta, n)$ are dimensionless functions.

The combination of the dimensional considerations and the Ilyushin theorem determine the form of the HRR field. Numerical calculation is needed to determine the angular distribution for each value of n , which was done in the HRR papers. For $n = 1$, the solution should recover the linear elastic solution, assuming incompressibility ($\nu = 1/2$).

Similar considerations show that the fields of strain and displacement around the tip of the crack take the following forms:

$$\varepsilon_{ij}(r, \theta) = \left(\frac{G}{Br} \right)^{\frac{n}{n+1}} g_{ij}(\theta, n),$$

$$u_i(r, \theta) = r \left(\frac{G}{Br} \right)^{\frac{n}{n+1}} h_i(\theta, n).$$

Why should G be the only parameter that represents the external boundary conditions? Rivlin and Thomas (1953) conjectured that the energy release rate G could be used as the driving force for the extension of a crack in rubber. They confirmed this conjecture experimentally, without theoretical justification. Indeed, the experimental confirmation surprised Rivlin himself (<http://imechanica.org/node/7773>). Why should G be the *only* parameter that represents the external boundary conditions?

Although subsequent experiments have amply confirmed their conjecture, there have been exceptions. For example, if a specimen, such as that tested in the “pure-shear” configuration, is subject to a large pre-extension in the direction of tearing, the measured fracture energy is much reduced from its un-prestrained value (A.N. Gent and H.J. Kim, Tear strength of stretched rubber. Rubber Chemistry and Technology 51, 34-44, 1978).

I am unaware of any theoretical analysis of this effect, but you should search the literature and let me know if you find related papers. I believe that the effect is related to the T -stress described in the literature of metal fracture. For rubbers, the substantial pre-extension in the direction of tearing substantially consumes part of the limiting stretch of the polymer chains. Of course, such a pre-extension may also motivate cavitation. The effect of the pre-extension on tearing will depend on the mechanism of tearing.

Significance of the HRR field. By contrast, the literature of metal fracture has devoted ample attention to the question whether G is the only parameter that represents the external boundary conditions. The Begley-Landes experiment was devoted to this question. As we discussed in Lecture 1, their approach parallels that of Rivlin and Thomas (1953).

In one theoretical formulation of the answer to this question, the analogy has been made between the HRR field in the Elastic-Plastic Fracture mechanics and the K -field in the Linear Elastic Fracture Mechanics (LEFM). That is, we ascertain if the HRR field may provide an annulus whose inner radius excludes the annulus from the process of separation, and whose outer radius excludes the annulus from the external boundary condition. Within this annulus, the field is the HRR field. This annulus is known as the J -dominant annulus. If such a J -dominant annulus exists, the energy release rate G , or the J integral, is the only messenger between the external boundary conditions and the process of separation.

But the J -dominant annulus may not exist for many reasons, such as

1. The presence of substantial stress or stretch in the direction of crack extension.
2. The actual deformation depends on the history of stress, and cannot be modeled by fictitious nonlinear elastic deformation.
3. The process of separation itself occupies a large zone, overlapping with the zone influenced by the external boundary conditions.

For an entry to the large body of literature on this ongoing field of research, see

- J.W. Hutchinson, Fundamentals of the phenomenological theory of nonlinear fracture mechanics. Journal of Applied Mechanics **50**, 1042-1051 (1983). <http://www.seas.harvard.edu/hutchinson/papers/369.pdf>

- N.P. O'Dowd and C.F. Shih, Family of crack-tip fields characterized by a triaxiality parameter—I. Structure of the fields. *Journal of the Mechanics and Physics of Solids*. 39, 989-1015 (1991).
- D. Carka and C.M. Landis, On the path-dependence of the J -integral near a stationary crack in an elastic-plastic material. 28 January 2010. See the paper and ongoing discussion at <http://imechanica.org/node/7468>

Explore the HRR field on your own. In homework you will explore aspects and some applications of the HRR field, including

- Using the J integral to determine the order of singularity of the HRR field
- The stress ahead of the crack tip
- The relation between the HRR field and finite deformation of a blunted crack (the Rice-Johnson paper)
- The Ritchie-Knott-Rice (RKR) model

References

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