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Final Project

Damage Identification of Cylindrical Shell Structures

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Layout

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2. Theoretical Analysis

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Introduction

The advent of smart materials

Electromechanical (EM) method

- Interaction relation between smart material transducer and host structure
- Impedance analyzer

Introduction

Smart Materials

↓

Piezoelectric Materials

↓

Lead Zirconate Titanate

(PZT)

Piezoelectricity

Direct Piezoelectric Effect

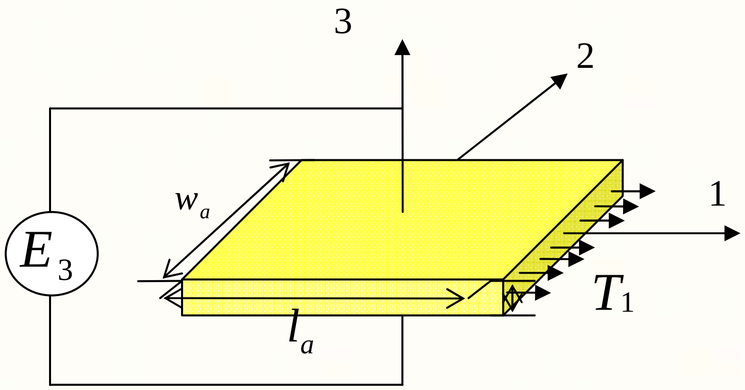
Stress field → Electric charges → Sensors

Converse Piezoelectric Effect

Electric Field → Strain field → Actuators

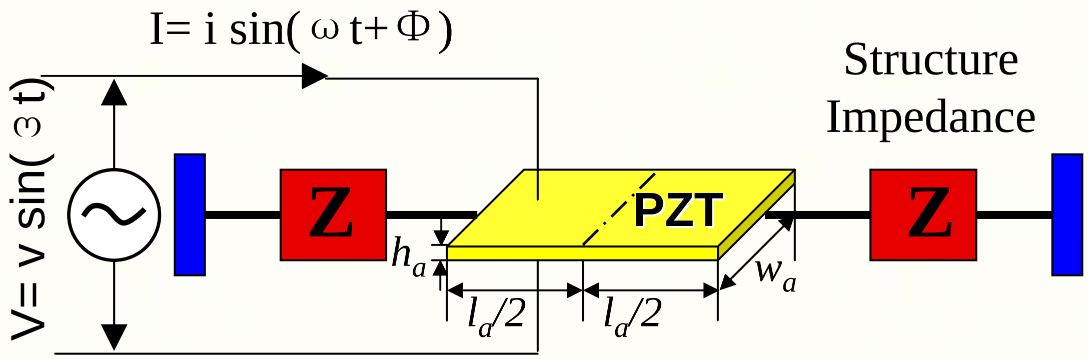
Introduction

Piezoelectric constitutive relations (IEEE standard, 1987)



$$\begin{cases} D_i = \varepsilon_{ik}^{T} E_k + d_{ip}^{d} T_q \\ S_p = d_{kp}^{c} E_k + s_{pq}^{E} T_q \end{cases}$$

EMI model (Liang et al., 1994)



Theoretical Analysis

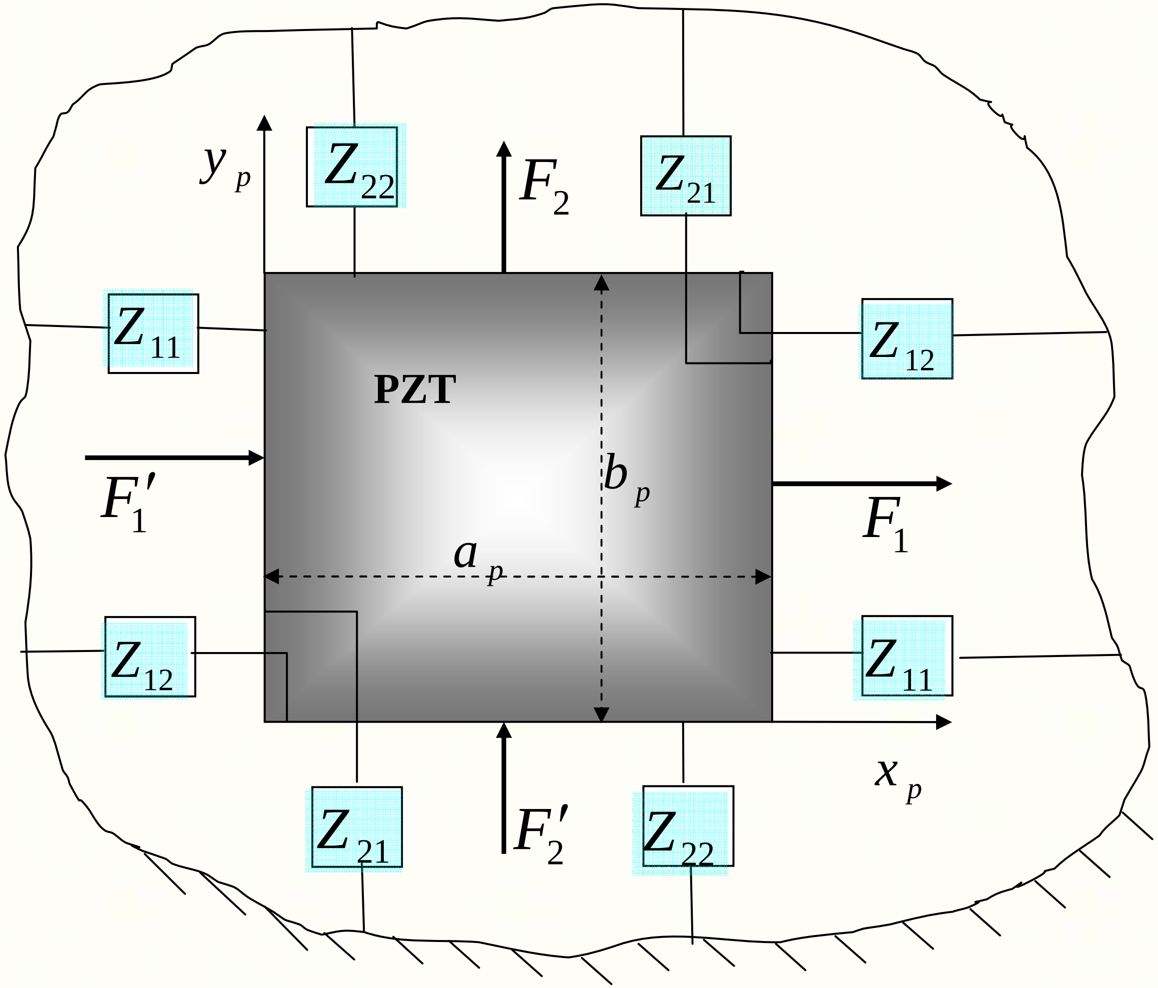
Modeling of PZT Transducers

- 2-D generic model

Modeling Of Host Structure

- Cylindrical shell
- Classical theory of thin shell
- P-Ritz Method

Modeling of PZT Transducer



(Yang, et al. 2005)

Theoretical Analysis

Motion equation

$$\frac{\tilde{Y}_p^E}{1-\mu_p^2}\frac{\partial^2 u_p}{\partial x_p^2} = \rho_p \ddot{u}_p$$

$$\frac{\tilde{Y}_p^E}{1-\mu_p^2}\frac{\partial^2 v_p}{\partial y_p^2} = \rho_p \ddot{v}_p$$

⇒

$$u_p = (A\sin K_p x_p + B\cos K_p x_p)e^{j\omega t}$$

$$v_p = (C\sin K_p y_p + D\cos K_p y_p)e^{j\omega t}$$

force applied on PZT patch

⇒ $$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = -\mathbf{Z}_{str}\begin{Bmatrix} \dot{u}_p \\ \dot{v}_p \end{Bmatrix}$$

stresses

⇒ $$\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

constitutive equation

⇒ $$D_3 = \begin{Bmatrix} d_{31} & d_{32} \end{Bmatrix}\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} + \tilde{\varepsilon}_{33}^T E_3$$

⇒

EM admittance of PZT transducers

$$Y=\frac{j\omega}{h_p}\left\{\frac{\tilde{Y}_p^E}{1-\mu_p^2}\left[\begin{aligned}&(d_{31}+\mu_p d_{32})\left(2A'b_p\tan\frac{K_p a_p}{2}-d_{31}a_p b_p\right)\\&+(\mu_p d_{31}+d_{32})\left(2C'a_p\tan\frac{K_p a_p}{2}-d_{32}a_p b_p\right)\end{aligned}\right]+\tilde{\varepsilon}_{33}^T a_p b_p\right\}$$

Where:

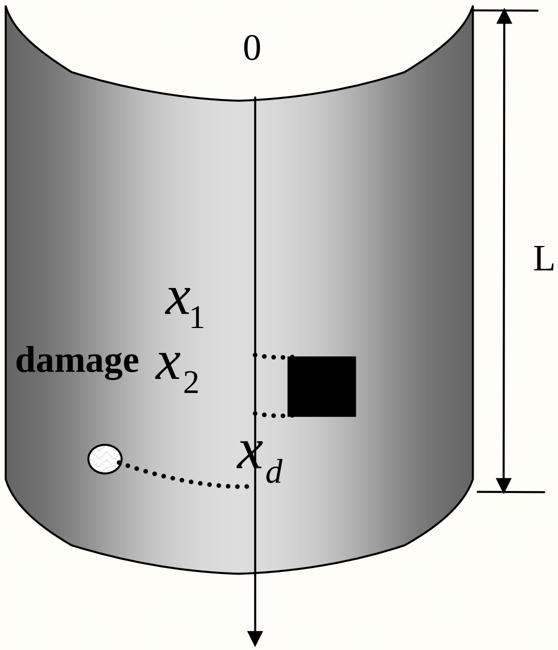
$$\begin{Bmatrix}A'\\C'\end{Bmatrix}=\mathbf{N}^{-1}\begin{Bmatrix}d_{31}\\d_{32}\end{Bmatrix}$$

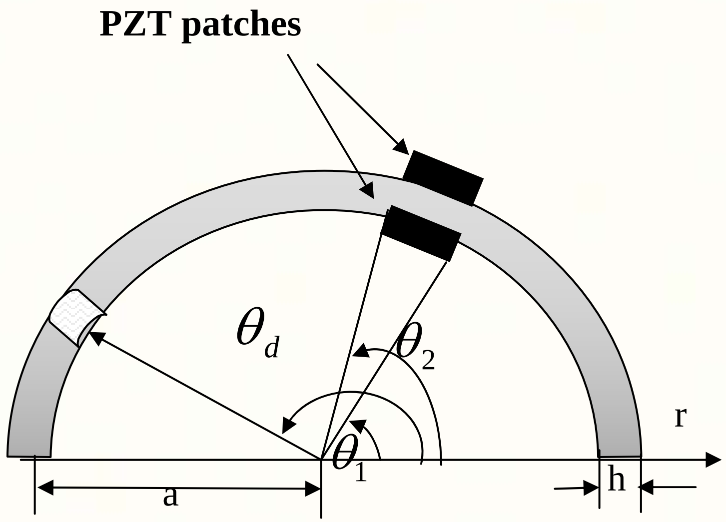
$$\mathbf{N}=\mathbf{I}-\frac{j\omega}{\tilde{Y}_p^E K_p h_p}\begin{bmatrix}\frac{Z_{11}}{b_p}-\mu_p\frac{Z_{21}}{a_p} & \frac{Z_{12}}{b_p}-\mu_p\frac{Z_{22}}{a_p}\\ -\mu_p\frac{Z_{11}}{b_p}+\frac{Z_{21}}{a_p} & -\mu_p\frac{Z_{12}}{b_p}+\frac{Z_{22}}{a_p}\end{bmatrix}\begin{bmatrix}-\tan\left(\frac{K_p a_p}{2}\right) & 0\\ 0 & -\tan\left(\frac{K_p b_p}{2}\right)\end{bmatrix}$$

$\mathbf{Z}_{str}$?

Theoretical Analysis

Cylindrical shell with a pair of PZT patches





Theoretical Analysis

Classical Theory of Thin Shell

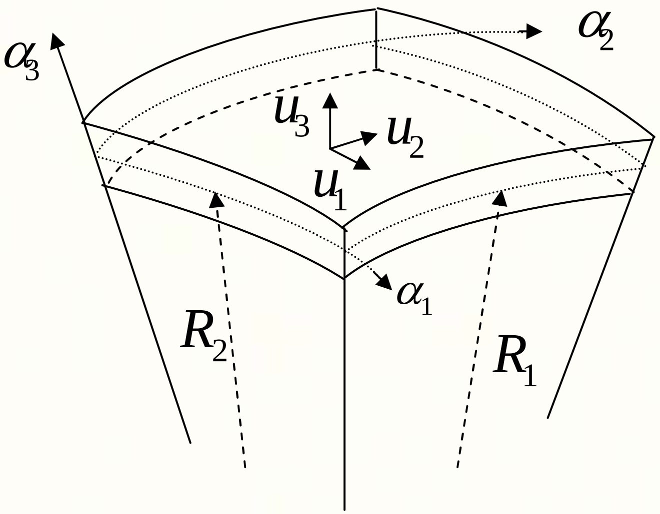
$$\varepsilon_{11} = \frac{1}{A_1}\frac{\partial}{\partial \alpha_1}(\bar{u}_1 + \alpha_3\beta_1) + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}(\bar{u}_2 + \alpha_3\beta_2) + \frac{\bar{u}_3}{R_1}$$

$$\varepsilon_{22} = \frac{1}{A_2}\frac{\partial}{\partial \alpha_2}(\bar{u}_2 + \alpha_3\beta_2) + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}(\bar{u}_1 + \alpha_3\beta_1) + \frac{\bar{u}_3}{R_2}$$

$$\varepsilon_{12} = \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left(\frac{\bar{u}_2 + \alpha_3\beta_2}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left(\frac{\bar{u}_1 + \alpha_3\beta_1}{A_1}\right)$$

$$\beta_1 = \frac{\bar{u}_1}{R_1} - \frac{1}{A_1}\frac{\partial \bar{u}_3}{\partial \alpha_1}$$

$$\beta_2 = \frac{\bar{u}_2}{R_2} - \frac{1}{A_2}\frac{\partial \bar{u}_3}{\partial \alpha_2}$$



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Theoretical Analysis

Displacement relations for cylindrical shell under pure bending

$$u_x = -(r-a)\frac{\partial w}{\partial x}$$

$$v_\theta = -\frac{(r-a)}{a}\frac{\partial w}{\partial \theta}$$

$$w = \overline{w}(x,\theta)$$

Strain-displacement relations

$$\varepsilon_{xx} = -(r-a)\frac{\partial^2 \overline{w}}{\partial x^2}$$

$$\varepsilon_{\theta\theta} = \frac{1}{a}\overline{w} - \frac{1}{a^2}(r-a)\frac{\partial^2 \overline{w}}{\partial \theta^2}$$

$$\varepsilon_{x\theta} = -\frac{2}{a}(r-a)\frac{\partial^2 \overline{w}}{\partial x \partial \theta}$$

Stress-strain relations

$$\sigma_{xx} = \frac{E}{1-\mu^2}(\varepsilon_{xx} + \mu\varepsilon_{\theta\theta})$$

$$\sigma_{\theta\theta} = \frac{E}{1-\mu^2}(\varepsilon_{\theta\theta} + \mu\varepsilon_{xx})$$

$$\sigma_{x\theta} = G\varepsilon_{x\theta}$$

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Hamilton's principle

$$\delta \int_{t_0}^{t_1} (T - U + W_{nc}) dt = 0$$

Kinetic energy of cylindrical shell

$$T = \frac{1}{2} \rho \iiint (\dot{u}_x^2 + \dot{v}_\theta^2 + \dot{w}^2) a d\theta dr dx$$

The strain energy of cylindrical shell

$$U = \iiint \frac{1}{2} (\sigma_{xx}\varepsilon_{xx} + \sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{x\theta}\varepsilon_{x\theta}) a d\theta dx dr$$

- **non-dimensionalization** $\xi = \frac{2x}{L} - 1 \qquad \eta = \frac{2\theta}{\pi} - 1$

- **displacement** $W = \sum_{q=0}^{N} \sum_{i=0}^{q} c_m \phi_m$

ϕ_m ***satisfies the following conditions***

- ***linearly independent***
- ***Form a complete system of functions***
- ***Satisfy the geometrical boundary conditions***

- **The polynomial shape functions**

$$\phi_m = \xi^i \eta^{q-i} \phi_0 \qquad \text{m=1,2,……,} \ \frac{(N+1)(N+2)}{2}$$

$$\phi_0 = \prod_p \left[\Gamma_p(\xi, \eta)\right]^{\theta_p}$$

The Hamilton's principle

U D

$$(\mathbf{K}-\mathbf{M}\omega^2)\mathbf{C}=0$$

$$\left|\mathbf{K}-\mathbf{M}\omega^2\right|=0$$

$$\omega_k$$

$$\Phi_k=\sum_m c_{km}\phi_m$$

G

$$w=\sum_k \frac{G_k\Phi_k}{\omega_k^2-\omega^2}e^{j\omega t}$$

$\boldsymbol{u_x, v_\theta}$

$$u_x, v_\theta$$

$$\mathbf{Z}_{str}$$

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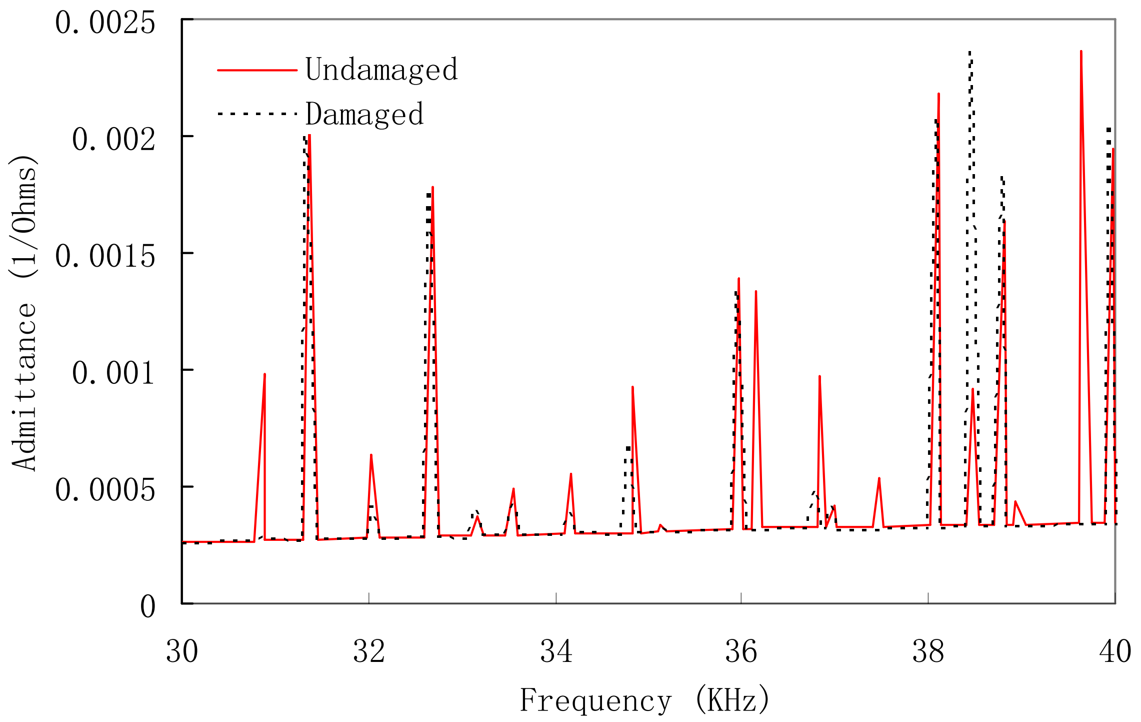
Results

Comparison of Theoretical Results for Damaged and Undamaged Shell Specimen



Undamaged	Damaged	Shift
30.891	30.889	0.002
31.36	31.313	0.047
32.009	32.002	0.007
32.649	32.627	0.022
33.562	33.555	0.007
34.168	34.124	0.044
35.95	35.946	0.004
36.137	36.137	0
36.807	36.796	0.011
38.097	38.083	0.014
38.451	38.412	0.039
38.796	38.76	0.036
39.619	39.617	0.002
39.956	39.926	0.03

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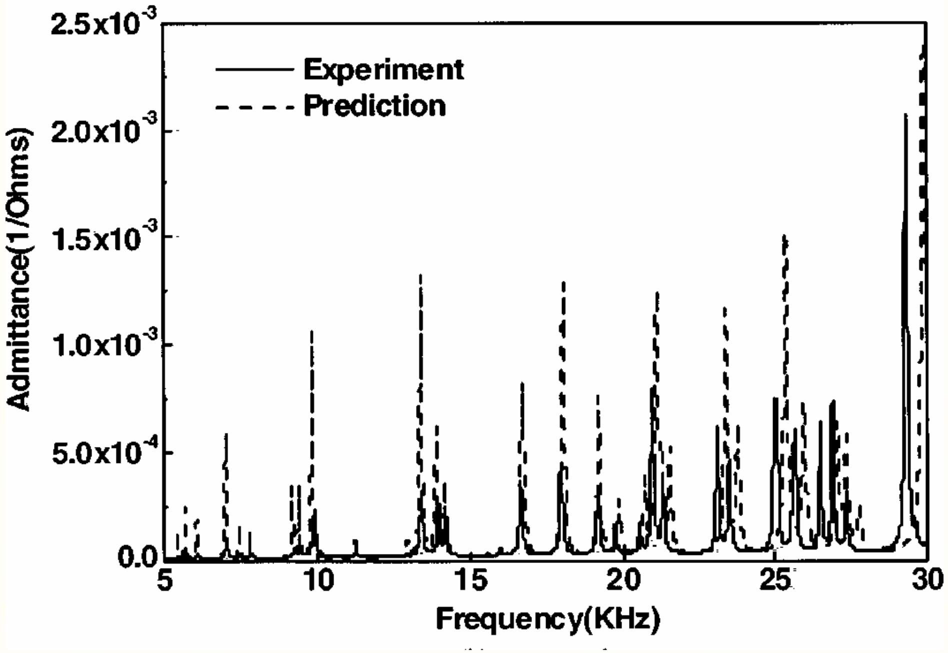

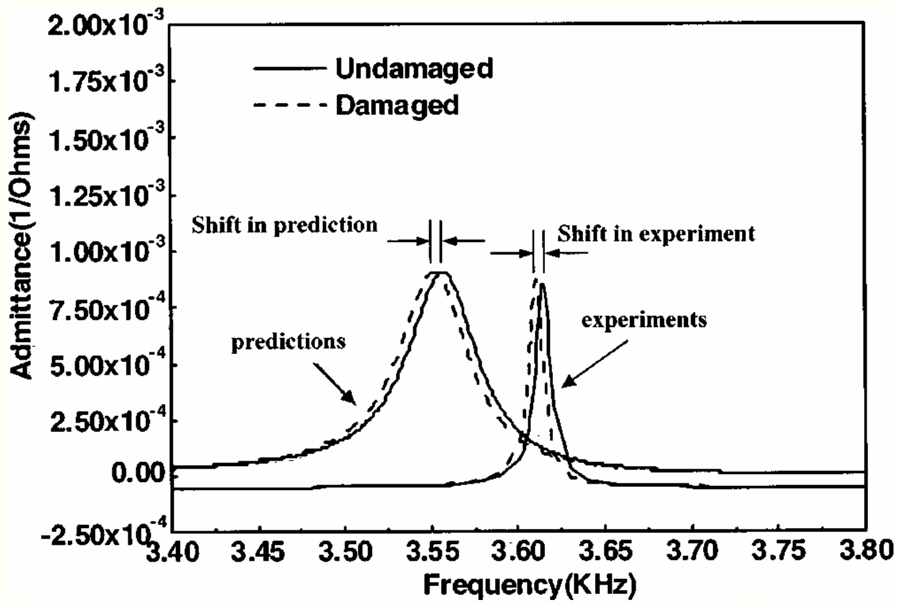

EM admittance of PZT bonded to plate (Xu, et al. 2004)

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Summary

Theoretical Analysis

- ***Modeling of PZT transducers***
  (2-D generic impedance model)
- ***Modeling of host structure***
  (Ritz method, polynomial shape functions, Hamilton Principle)

Summary

Assumptions

- Homogeneous and isotropic material
- Thin shell model
- The center plane is stress free
- The in-plane displacements of the center plane are neglected
- The line perpendicular to the center plane remains perpendicular to the center plane after deformation.

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THANK YOU

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Stiffness and mass matrices for undamaged shell

$$k_{mn} = \int_{-1}^{1}\int_{-1}^{1} \left\{ \begin{aligned} & \frac{Eh^3}{12(1-\mu^2)} \left[\frac{16}{L^4} \frac{\partial^2 \phi_m}{\partial \xi^2} \frac{\partial^2 \phi_n}{\partial \xi^2} + \frac{\mu}{a^2} \frac{16}{L^2 \pi^2} \frac{\partial^2 \phi_m}{\partial \eta^2} \frac{\partial^2 \phi_n}{\partial \xi^2} \right] \\ & + \frac{Eh^3}{12(1-\mu^2)} \left[\frac{1}{a^4} \frac{16}{\pi^4} \frac{\partial^2 \phi_m}{\partial \eta^2} \frac{\partial^2 \phi_n}{\partial \eta^2} + \frac{\mu}{a^2} \frac{16}{\pi^2 L^2} \frac{\partial^2 \phi_m}{\partial \xi^2} \frac{\partial^2 \phi_n}{\partial \eta^2} \right] \\ & + \frac{Eh}{1-\mu^2} \frac{1}{a^2} \phi_m \phi_n + \frac{Gh^3}{3a^2} \frac{16}{\pi^2 L^2} \frac{\partial^2 \phi_m}{\partial \xi \partial \eta} \frac{\partial^2 \phi_n}{\partial \xi \partial \eta} \end{aligned} \right\} a \frac{\pi L}{4} d\xi d\eta$$

$$m_{mn} = \int_{-1}^{1}\int_{-1}^{1} \left[\frac{\rho h^3}{3L^2} \frac{\partial \phi_m}{\partial \xi} \frac{\partial \phi_n}{\partial \xi} + \frac{\rho h^3}{3a^2 \pi^2} \frac{\partial \phi_m}{\partial \eta} \frac{\partial \phi_n}{\partial \eta} + \rho h \phi_m \phi_n \right] a \frac{\pi L}{4} d\xi d\eta$$

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Stiffness and mass matrices for damaged shell

$$k_{mn}=\frac{Eh^3a\pi L}{3(1-\mu^2)}\int_{-1}^{1}\int_{-1}^{1}\left[\begin{array}{l}\frac{1}{L^4}\frac{\partial^2\phi_m}{\partial\xi^2}\frac{\partial^2\phi_n}{\partial\xi^2}+\frac{\mu}{a^2\pi^2L^2}\frac{\partial^2\phi_m}{\partial\xi^2}\frac{\partial^2\phi_n}{\partial\eta^2}+\frac{\mu}{a^2\pi^2L^2}\frac{\partial^2\phi_m}{\partial\eta^2}\frac{\partial^2\phi_n}{\partial\xi^2}\\+\frac{1}{a^4\pi^4}\frac{\partial^2\phi_m}{\partial\eta^2}\frac{\partial^2\phi_n}{\partial\eta^2}+\frac{3}{4h^2a^2}\phi_m\phi_n+\frac{2(1-\mu)}{a^2\pi^2L^2}\frac{\partial^2\phi_m}{\partial\xi\partial\eta}\frac{\partial^2\phi_n}{\partial\xi\partial\eta}\end{array}\right]d\xi d\eta$$

$$-\frac{4(E-E_d)h^3}{3(1-\mu^2)}\left[\begin{array}{l}\frac{1}{L^4}\frac{\partial^2\phi_m}{\partial\xi^2}\frac{\partial^2\phi_n}{\partial\xi^2}+\frac{\mu}{a^2\pi^2L^2}\frac{\partial^2\phi_m}{\partial\xi^2}\frac{\partial^2\phi_n}{\partial\eta^2}+\frac{\mu}{a^2\pi^2L^2}\frac{\partial^2\phi_m}{\partial\eta^2}\frac{\partial^2\phi_n}{\partial\xi^2}\\+\frac{1}{a^4\pi^4}\frac{\partial^2\phi_m}{\partial\eta^2}\frac{\partial^2\phi_n}{\partial\eta^2}+\frac{3}{4h^2a^2}\phi_m\phi_n+\frac{2(1-\mu)}{a^2\pi^2L^2}\frac{\partial^2\phi_m}{\partial\xi\partial\eta}\frac{\partial^2\phi_n}{\partial\xi\partial\eta}\end{array}\right]_{\substack{\xi=\xi_d\\ \eta=\eta_d}}\cdot A_d$$

$$m_{mn}=\int_{-1}^{1}\int_{-1}^{1}\left[\frac{\rho h^3}{3L^2}\frac{\partial\phi_m}{\partial\xi}\frac{\partial\phi_n}{\partial\xi}+\frac{\rho h^3}{3a^2\pi^2}\frac{\partial\phi_m}{\partial\eta}\frac{\partial\phi_n}{\partial\eta}+\rho h\phi_m\phi_n\right]a\frac{\pi L}{4}d\xi d\eta$$

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$$G_k = \frac{\int_0^L \int_0^\pi -\left[\frac{\partial \overline{M}_1}{\partial x} + \frac{1}{a}\frac{\partial \overline{M}_2}{\partial \theta}\right]\Phi_k a d\theta dx}{\mathbf{C}_k^T \mathbf{M} \mathbf{C}_k} = G_k^1 + G_k^2$$

$$G_k^1 = \frac{\int_0^L \int_0^\pi -\left(\frac{\partial \overline{M}_1}{\partial x}\right)\Phi_k a d\theta dx}{\mathbf{C}_k^T \mathbf{M} \mathbf{C}_k} = \frac{\frac{h+h_p}{b_p}\frac{a\pi}{L}\sum_m \left[c_{mk}\int_{\eta_1}^{\eta_2}\left(\left.\frac{\partial \phi_m}{\partial \xi}\right|_{\xi=\xi_2} - \left.\frac{\partial \phi_m}{\partial \xi}\right|_{\xi=\xi_1}\right)d\eta\right]}{\mathbf{C}_k^T \mathbf{M} \mathbf{C}_k} \cdot \overline{F}_1 = P_k \overline{F}_1$$

$$G_k^2 = \frac{\int_0^L \int_0^\pi -\left(\frac{1}{a}\frac{\partial \overline{M}_2}{\partial \theta}\right)\Phi_k a d\theta dx}{\mathbf{C}_k^T \mathbf{M} \mathbf{C}_k} = \frac{\frac{h+h_p}{a_p}\frac{L}{a\pi}\sum_m \left[c_{mk}\int_{\xi_1}^{\xi_2}\left(\left.\frac{\partial \phi_m}{\partial \eta}\right|_{\eta=\eta_2} - \left.\frac{\partial \phi_m}{\partial \eta}\right|_{\eta=\eta_1}\right)d\xi\right]}{\mathbf{C}_k^T \mathbf{M} \mathbf{C}_k} \cdot \overline{F}_2 = Q_k \overline{F}_2$$

$$u_x(x,\theta,t) = -\frac{h+h_p}{2}\frac{\partial \overline{w}(x,\theta,t)}{\partial x}$$

$$v_\theta(x,\theta,t) = -\frac{h+h_p}{2a}\frac{\partial \overline{w}(x,\theta,t)}{\partial \theta}$$

Force-velocity response relation

$$\begin{Bmatrix} \dot{u}_x\big|_{x=x_1} - \dot{u}_x\big|_{x=x_2} \\ a\dot{v}_\theta\big|_{\theta=\theta_1} - a\dot{v}_\theta\big|_{\theta=\theta_2} \end{Bmatrix} = \mathbf{Q}\begin{Bmatrix} \overline{F}_1 \\ \overline{F}_2 \end{Bmatrix}$$

$$\mathbf{Z}_{str} = \frac{1}{2}\mathbf{Q}^{-1}$$

Admittance of host structure

$$Q_{11} = j\omega \frac{h+h_p}{L} \sum_k \left\{ \frac{P_k}{\omega_k^2 - \omega^2} \sum_k \left[c_{mk} \left(\left. \frac{\partial \phi_m}{\partial \xi} \right|_{\substack{\xi=\xi_2 \\ \eta=\eta_c}} - \left. \frac{\partial \phi_m}{\partial \xi} \right|_{\substack{\xi=\xi_1 \\ \eta=\eta_c}} \right) \right] \right\}$$

$$Q_{12} = j\omega \frac{h+h_p}{L} \sum_k \left\{ \frac{Q_k}{\omega_k^2 - \omega^2} \sum_k \left[c_{mk} \left(\left. \frac{\partial \phi_m}{\partial \xi} \right|_{\substack{\xi=\xi_2 \\ \eta=\eta_c}} - \left. \frac{\partial \phi_m}{\partial \xi} \right|_{\substack{\xi=\xi_1 \\ \eta=\eta_c}} \right) \right] \right\}$$

$$Q_{21} = j\omega \frac{h+h_p}{a\pi} \sum_k \left\{ \frac{P_k}{\omega_k^2 - \omega^2} \sum_k \left[c_{mk} \left(\left. \frac{\partial \phi_m}{\partial \eta} \right|_{\substack{\eta=\eta_2 \\ \xi=\xi_c}} - \left. \frac{\partial \phi_m}{\partial \eta} \right|_{\substack{\eta=\eta_1 \\ \xi=\xi_c}} \right) \right] \right\}$$

$$Q_{22} = j\omega \frac{h+h_p}{a\pi} \sum_k \left\{ \frac{Q_k}{\omega_k^2 - \omega^2} \sum_k \left[c_{mk} \left(\left. \frac{\partial \phi_m}{\partial \eta} \right|_{\substack{\eta=\eta_2 \\ \xi=\xi_c}} - \left. \frac{\partial \phi_m}{\partial \eta} \right|_{\substack{\eta=\eta_1 \\ \xi=\xi_c}} \right) \right] \right\}$$