Stiffness Control of Polymer Flexure Joints for Microrobotic Applications

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Compliant Flexure Joints

- Problem: Large-scale “pint joints” don’t work with insect-scale robots
- Surface forces (friction) dominate inertial forces at small scale
- Traditional manufacturing techniques insufficient for such tiny robots
- Solution: Use flexible polymer hinges that act as revolute joints

![Diagram showing compliant flexure joint]

- Thin flexible polymer film
- Rigid composite material (carbon fiber)
Flexure Joint Stiffness

- Carbon fiber can be assumed to be rigid
- Each joint can be modeled as a cantilever beam with bending stiffness \( k = \frac{E*I}{L} \) where \( E \) = elastic modulus, \( I \) = cross-sectional moment of inertia and \( L \) = flexure length
- Stiffness is thus a constant for a given flexure geometry and material
- Desirable to actively modulate the stiffness of the flexure joints – this can affect robot dynamics for control purposes
- To change stiffness, need to either vary flexure geometry or material properties
Modulating Flexure Stiffness

• Idea: add material with variable elastic properties to flexure joint
• This allows change in the equivalent bending stiffness of the entire flexure
• Two-layer flexure can be modeled as a composite beam
• This can be accomplished with shape-memory alloy (SMA) materials, which have a very temperature-dependent elastic modulus
But it’s not that simple…

- Shape memory alloys also undergo a large strain when they are heated (~7%)
- Problem: thin film bonded to rigid substrate, film wants to contract
- Similar to plate theory homework problem
- Will result in deformation of the film and substrate, possible delamination or failure of the flexure
Finite Element Analysis with COMSOL

• Create two 3D plates rigidly fixed to each other on one side

• First model
  – Boundary conditions: one edge fixed, opposite edge has applied distributed moment, other edges free
  – Calculate moment with $M = k_{eq} \ast \theta$ (analogous to $F = k\ast x$), where $\theta$ is desired angle of rotation of flexure joint
  – Does not account for large induced strain (~7%) when SMA is heated

• Second Model
  – Keep flexure flat, not concerned with rotation
  – Want to model contraction of SMA when heated and resulting stresses

• Third Model: Combination of loads from models 1 and 2
Failure Criteria

• Criteria for delamination of film involves fracture mechanics and the amount of energy available to drive a crack – beyond scope of this project

• Instead interested in possible plastic deformation of either polymer layer or SMA

• This could occur due to either large-angle deformation of flexure or large strain induced in SMA when heated – will analyze both cases
First FEA Model – Beam Bending

• BCs – one edge fixed, distributed moment on opposite edge, all other faces free. Run twice with moment in opposite directions, putting each side alternately in compression or tension
Equivalent Modulus of a Composite Beam

• From undergraduate solids course

\[
\begin{align*}
E_1 & = 0 \\
E_2 & = 0 \\
\text{Where } b_2 &= nb, n = E_1/E_2
\end{align*}
\]

Therefore, if you can actively control \( E_1 \), you can change the equivalent stiffness and moment of inertia, and thus the entire bending stiffness \( k = E*I/L \).
2-D Sanity Check

Beam Theory:

\[ \sigma = \frac{M_y}{I} \]

Must be adjusted slightly for composite beam, then compared with FEA results.
Results of first model – low temp SMA phase

- Max tensile stress in Kapton \( \sim 176 \) MPa, just over yield strength of 172 MPa

- Max tensile stress in SMA \( \sim 400 \) MPa, well over low-temp phase yield strength of 100 MPa – SMA will plastically deform
Deformed Shape Plot
Second FEA Model – SMA Contraction

• Previous models did not take into account large strain SMA undergoes when heated
• SMAs have a *negative* coefficient of thermal expansion – they contract with increasing temperature
• This can be modeled with FEA program, assuming thermal expansion coefficient of kapton is ~0 compared to the SMA
• Fixed-fixed boundary conditions at the beam edge – not worried about rotation yet, just effects of contracting SMA
Reality Check: 1D Thermal Strain Analysis

1) Force Balance:

\[ \sigma_1 A_1 = \sigma_2 A_2 \]

2) Set strains equal

\[ \frac{\sigma_1}{E_1} + \alpha_1 \Delta T = \frac{\sigma_2}{E_2} + \alpha_2 \Delta T \]

• Two equations, two unknowns \( \sigma_1 \) and \( \sigma_2 \)
• Thermal expansion coefficient for kapton negligibly small compared to SMA
• Analytical results:
  • \( \sigma_{\text{kapton}} = -0.393 \text{ GPa} \), \( \sigma_{\text{SMA}} = 1.47 \text{ GPa} \)
• FEA Results:
  • \( \sigma_{\text{kapton}} = -0.35 \text{ GPa} \), \( \sigma_{\text{SMA}} = 1.2 \text{ GPa} \)
Results of Second Model – Deformed Shape

• Largest deformation occurs at free edges of flexure
Second Model - Stresses

- Stresses are rather high due to the large strain of the SMA, especially at the edges (roughly 2x stress concentration).
- This will likely result in failure of the SMA.
- SMA may fracture since stress is above the ultimate tensile strength (960 MPa), not just the yield strength.
Second model – stress (continued)

- Despite large deformations at edges, highest von Mises stresses actually occur in the center of the flexure.

Side view of long side of flexure – note lower stress at the edge of SMA layer.
Line plot of normal stress at center plane

- log plot shows that stress in kapton layer is not constant, as it appears in contour plot on previous slide
Third Model: Combination of Loading Cases

- Account for both applied moment and contraction of SMA due to thermal actuation
- Resulting stresses are higher
- Stress concentrations occur at corners in all three models
Combination Loading: Corner Stresses

Stress concentration about 2x along corners

Von Mises Stress at Corner of Flexure for Three Loading Cases

- Applied Moment
- Thermally Activated
- Combined Thermal and Moment Loading

Vertical distance (microns)

Von Mises Stress (Pa)
Conclusion

- This flexure design will fail under the imposed loading
- Shape memory alloy may be too stiff for this purpose
- Possible solution: use of shape-memory polymer, which is more flexible
- Questions?
Appendix: References

- For details on composite beam calculations, see R.C. Hibbeler, “Mechanics of Materials”
- For more information on the Harvard Microrobotics Lab, visit the group’s website: http://www.micro.seas.harvard.edu/
- Material properties were taken from www.matweb.com – searchable online database that includes brand-name materials (Such as Kapton and Nitanol)
Appendix: Physical Parameters

• Geometry:
  – Flexure length = 100 \( \mu \)m, width = 1500 \( \mu \)m, thickness = 7.5 \( \mu \)m
  – SMA coating thickness = 2 \( \mu \)m

• Material Properties
  – Kapton ® Polymer Film from Dupont Corporation:
    • \( E = 5 \) GPa
    • \( \nu = .34 \)
    • Tensile strength = 172 MPa
  – Nitinol Shape Memory Alloy (Low Temp/High Temp phases)
    • \( E = 28 \) GPa / 75 GPa
    • \( \nu = .3 / .3 \)
    • Tensile strength = 100 MPa / 560 MPa
    • undergoes \(~5\%\) linear strain (contraction) when heated