

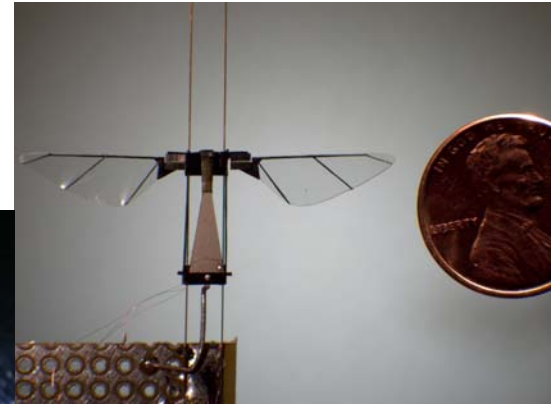
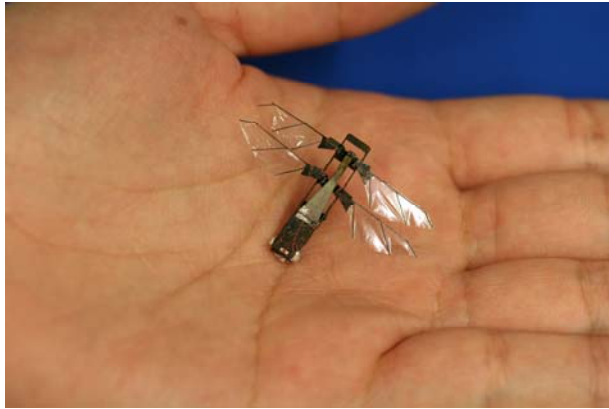
# Stiffness Control of Polymer Flexure Joints for Microrobotic Applications

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Adam Traina and Ben Finio

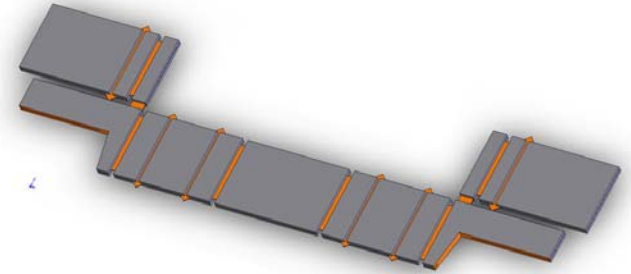
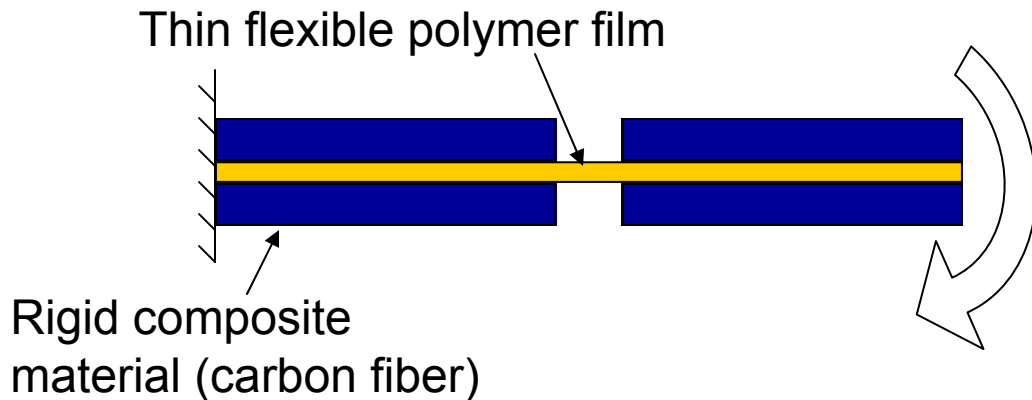
ES 240

1/10/2008



# Compliant Flexure Joints

- Problem: Large-scale “pint joints” don’t work with insect-scale robots
- Surface forces (friction) dominate inertial forces at small scale
- Traditional manufacturing techniques insufficient for such tiny robots
- Solution: Use flexible polymer hinges that act as revolute joints

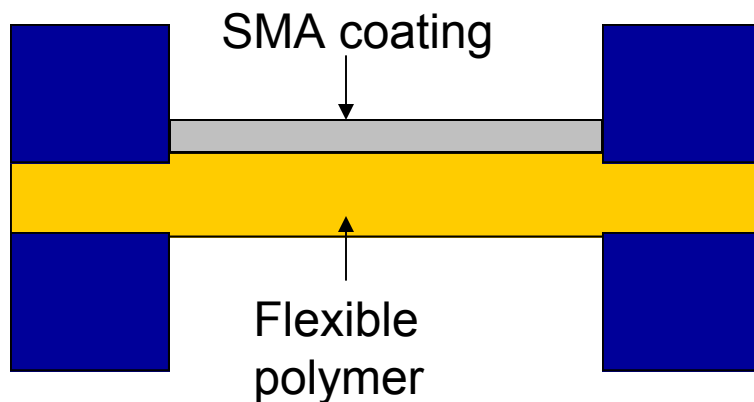


# Flexure Joint Stiffness

- Carbon fiber can be assumed to be rigid
- Each joint can be modeled as a cantilever beam with bending stiffness  $k = E \cdot I / L$  where  $E$  = elastic modulus,  $I$  = cross-sectional moment of inertia and  $L$  = flexure length
- Stiffness is thus a constant for a given flexure geometry and material
- Desirable to actively modulate the stiffness of the flexure joints – this can affect robot dynamics for control purposes
- To change stiffness, need to either vary flexure geometry or material properties

# Modulating Flexure Stiffness

- Idea: add material with variable elastic properties to flexure joint
- This allows change in the equivalent bending stiffness of the entire flexure
- Two-layer flexure can be modeled as a composite beam
- This can be accomplished with shape-memory alloy (SMA) materials, which have a very temperature-dependent elastic modulus

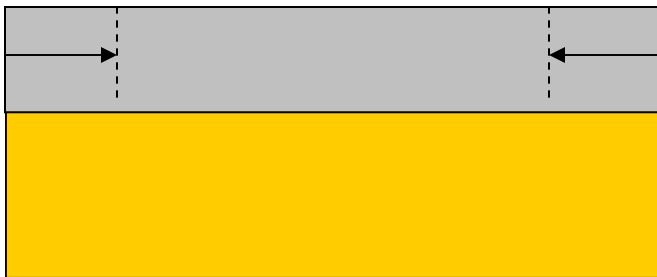


Beam cross-section

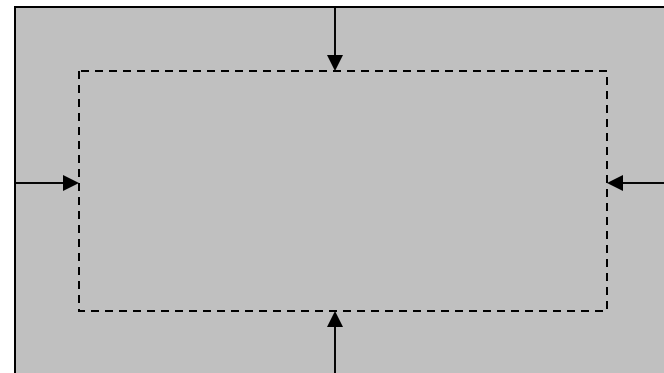


# But it's not that simple...

- Shape memory alloys also undergo a large strain when they are heated ( $\sim 7\%$ )
- Problem: thin film bonded to rigid substrate, film wants to contract
- Similar to plate theory homework problem
- Will result in deformation of the film and substrate, possible delamination or failure of the flexure



Side View



Top View

# Finite Element Analysis with COMSOL

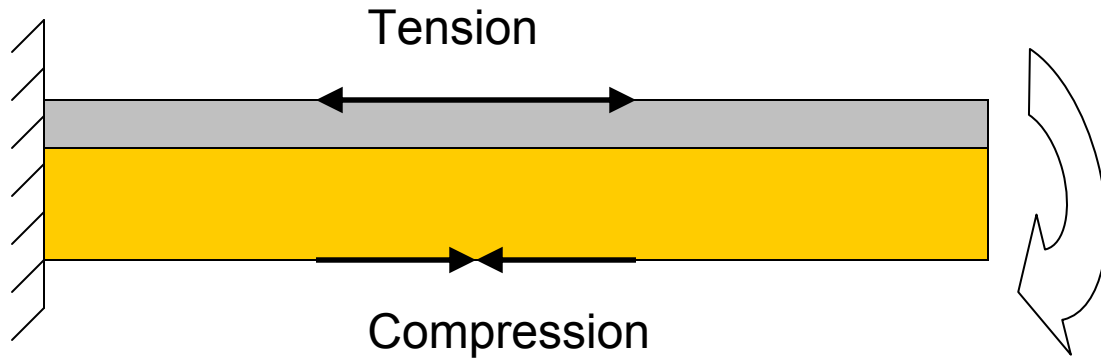
- Create two 3D plates rigidly fixed to each other on one side
- First model
  - Boundary conditions: one edge fixed, opposite edge has applied distributed moment, other edges free
  - Calculate moment with  $M = k_{eq} * \theta$  (analogous to  $F = k * x$ ), where  $\theta$  is desired angle of rotation of flexure joint
  - Does not account for large induced strain (~7%) when SMA is heated
- Second Model
  - Keep flexure flat, not concerned with rotation
  - Want to model contraction of SMA when heated and resulting stresses
- Third Model: Combination of loads from models 1 and 2

# Failure Criteria

- Criteria for delamination of film involves fracture mechanics and the amount of energy available to drive a crack – beyond scope of this project
- Instead interested in possible plastic deformation of either polymer layer or SMA
- This could occur due to either large-angle deformation of flexure or large strain induced in SMA when heated – will analyze both cases

# First FEA Model – Beam Bending

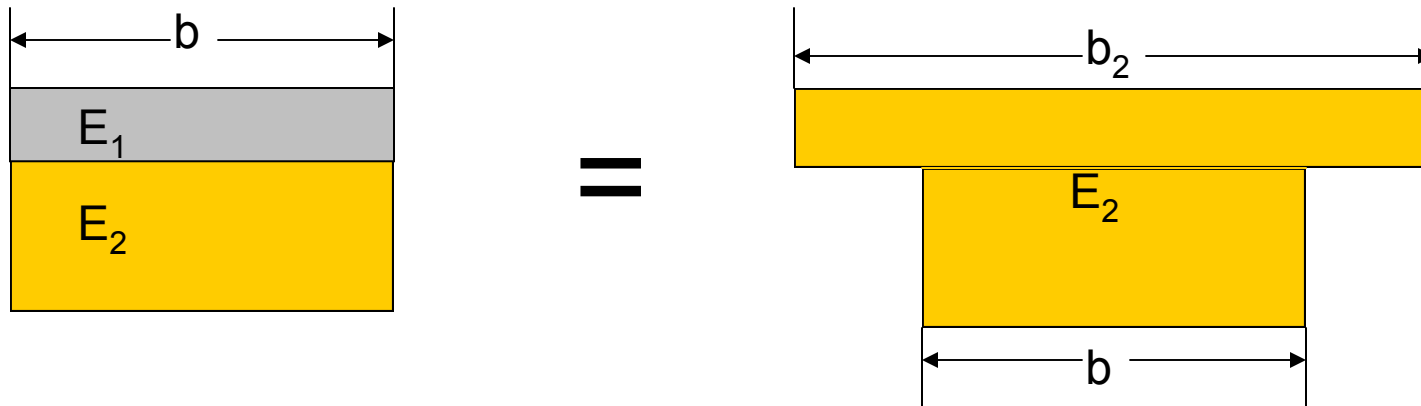
- BCs – one edge fixed, distributed moment on opposite edge, all other faces free. Run twice with moment in opposite directions, putting each side alternately in compression or tension





# Equivalent Modulus of a Composite Beam

- From undergraduate solids course



Where  $b_2 = nb$ ,  $n = E_1/E_2$

Therefore, if you can actively control  $E_1$ , you can change the equivalent stiffness and moment of inertia, and thus the entire bending stiffness  $k = E^*I/L$ .

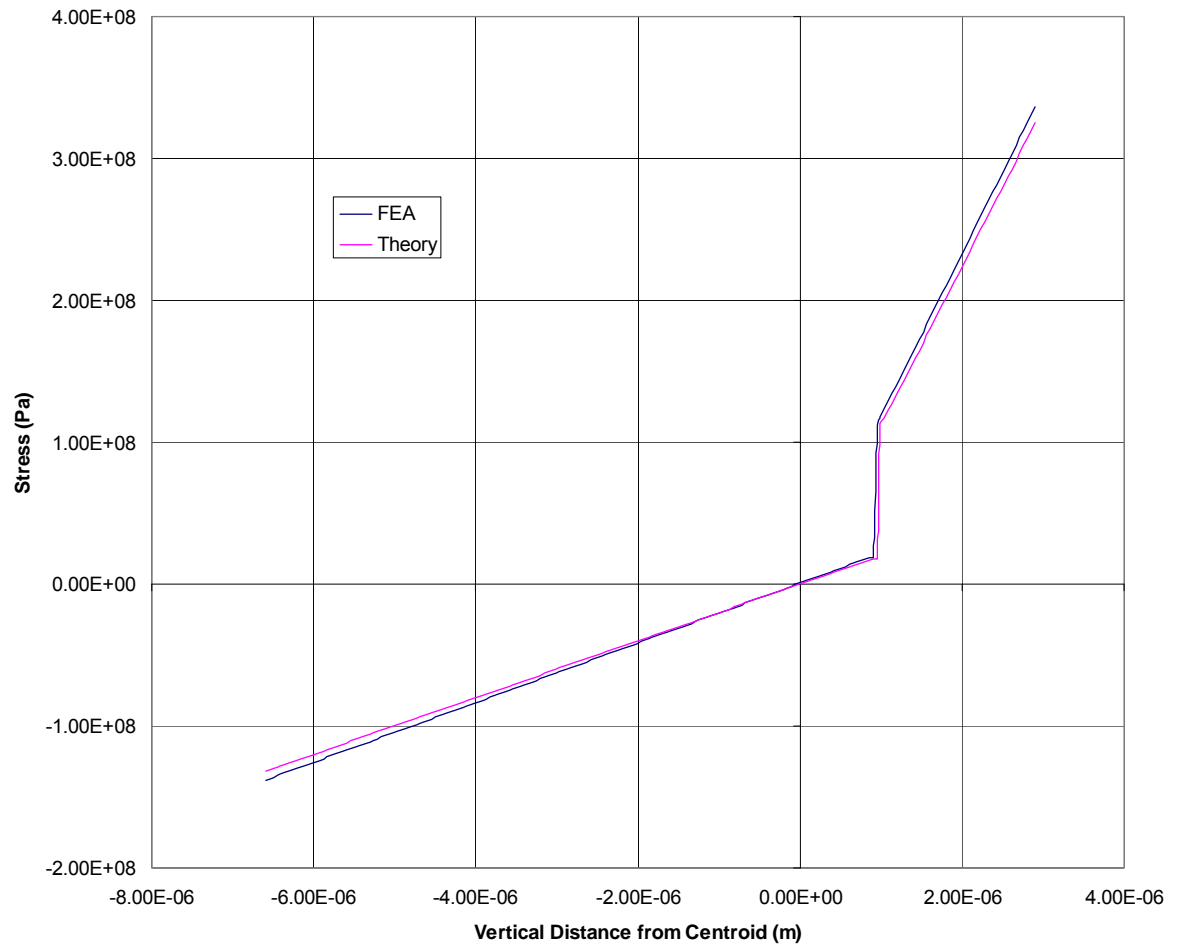
# 2-D Sanity Check

Theoretical and Finite Element Bending Stresses Due to Applied Moment

Beam Theory:

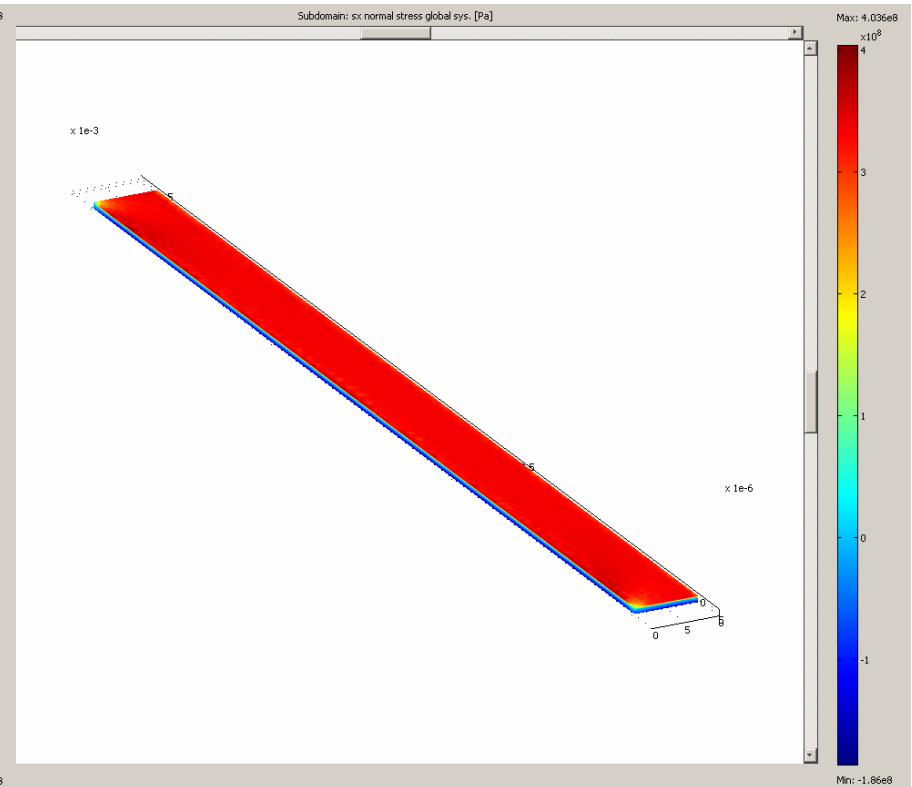
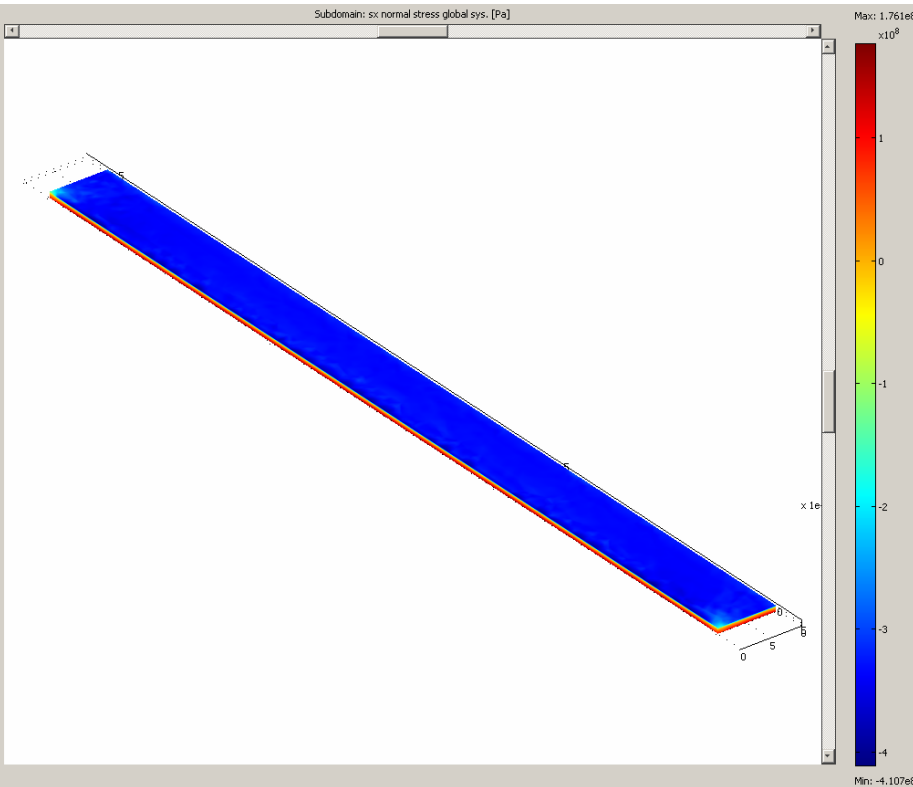
$$\sigma = \frac{My}{I}$$

Must be adjusted slightly for composite beam, then compared with FEA results

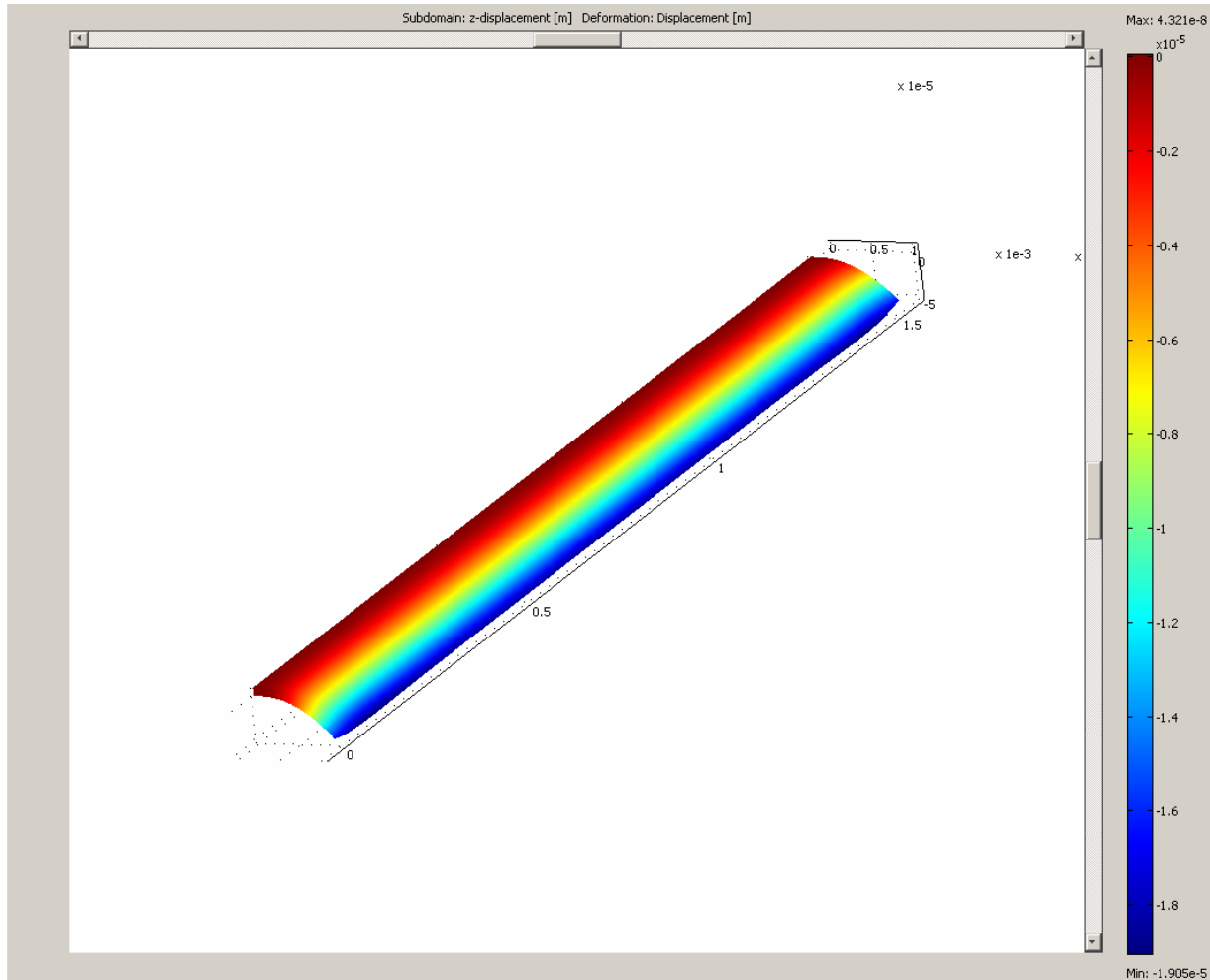


# Results of first model – low temp SMA phase

- Max tensile stress in Kapton  $\sim 176$  MPa, just over yield strength of 172 MPa
- Max tensile stress in SMA  $\sim 400$  MPa, well over low-temp phase yield strength of 100 MPa – SMA will plastically deform



# Deformed Shape Plot

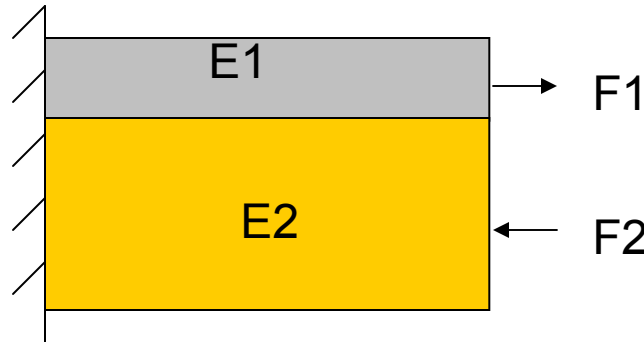
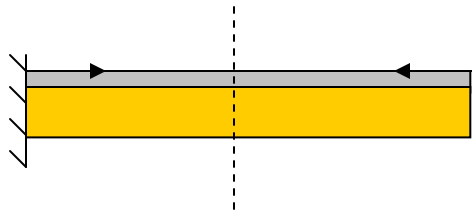


# Second FEA Model – SMA Contraction

- Previous models did not take into account large strain SMA undergoes when heated
- SMAs have a *negative* coefficient of thermal expansion – they contract with increasing temperature
- This can be modeled with FEA program, assuming thermal expansion coefficient of kapton is  $\sim 0$  compared to the SMA
- Fixed-fixed boundary conditions at the beam edge – not worried about rotation yet, just effects of contracting SMA

# Reality Check: 1D Thermal Strain Analysis

## 1) Force Balance:



$$\sigma_1 A_1 = \sigma_2 A_2$$

## 2) Set strains equal

$$\frac{\sigma_1}{E_1} + \alpha_1 \Delta T = \frac{\sigma_2}{E_2} + \alpha_2 \Delta T$$

- Two equations, two unknowns  $\sigma_1$  and  $\sigma_2$
- Thermal expansion coefficient for kapton negligibly small compared to SMA
- Analytical results:

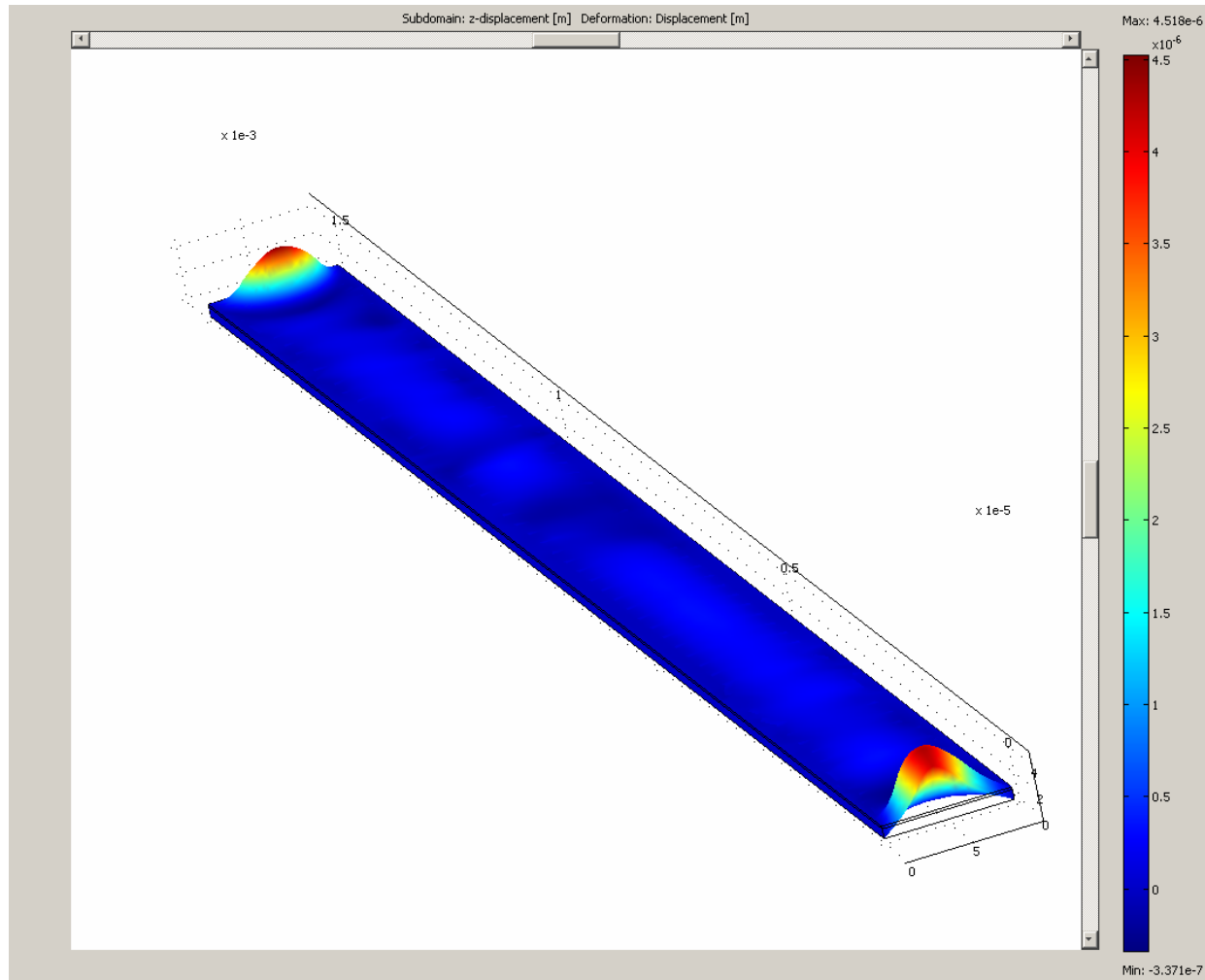
- $\sigma_{\text{kapton}} = -.393 \text{ GPa}$ ,  $\sigma_{\text{SMA}} = 1.47 \text{ GPa}$

- FEA Results:

- $\sigma_{\text{kapton}} = -.35 \text{ GPa}$ ,  $\sigma_{\text{SMA}} = 1.2 \text{ GPa}$

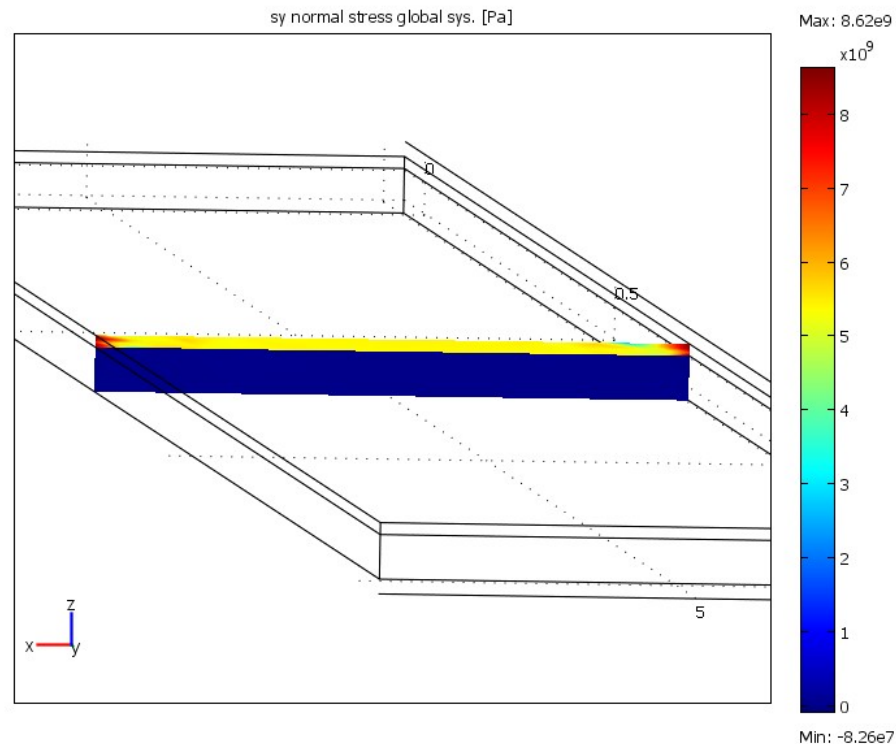
# Results of Second Model – Deformed Shape

- Largest deformation occurs at free edges of flexure



# Second Model - Stresses

- Stresses are rather high due to the large strain of the SMA, especially at the edges (roughly 2x stress concentration)
- This will likely result in failure of the SMA
- SMA may fracture since stress is above the ultimate tensile strength (960 MPa), not just the yield strength

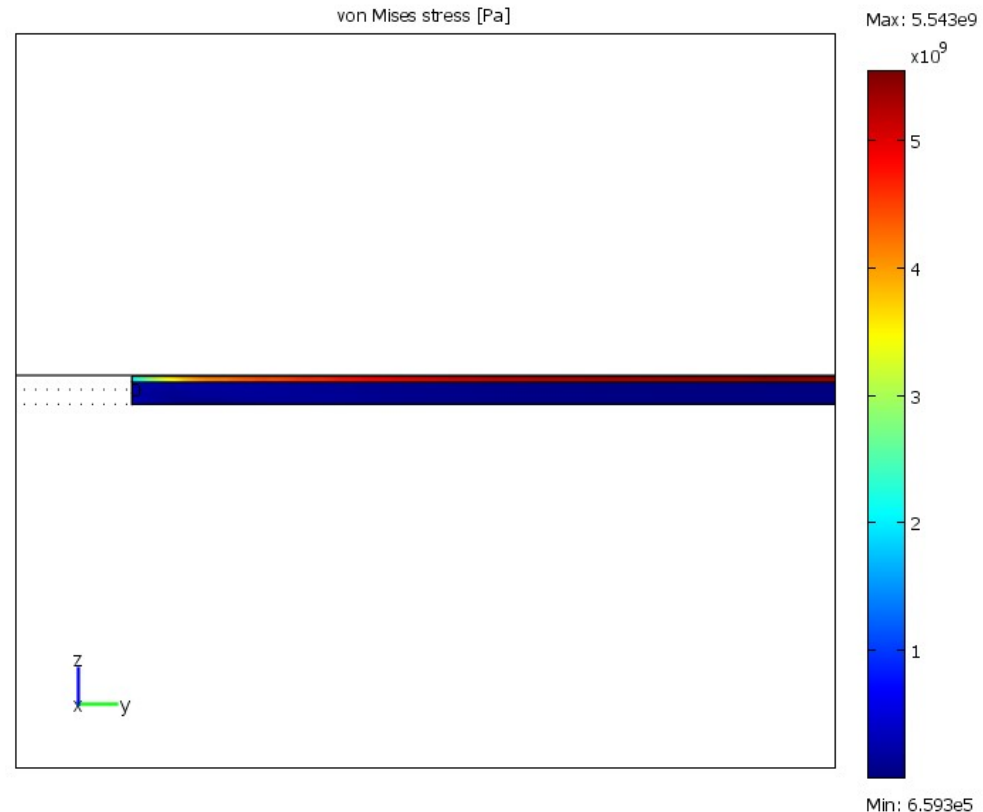




# Second model – stress (continued)

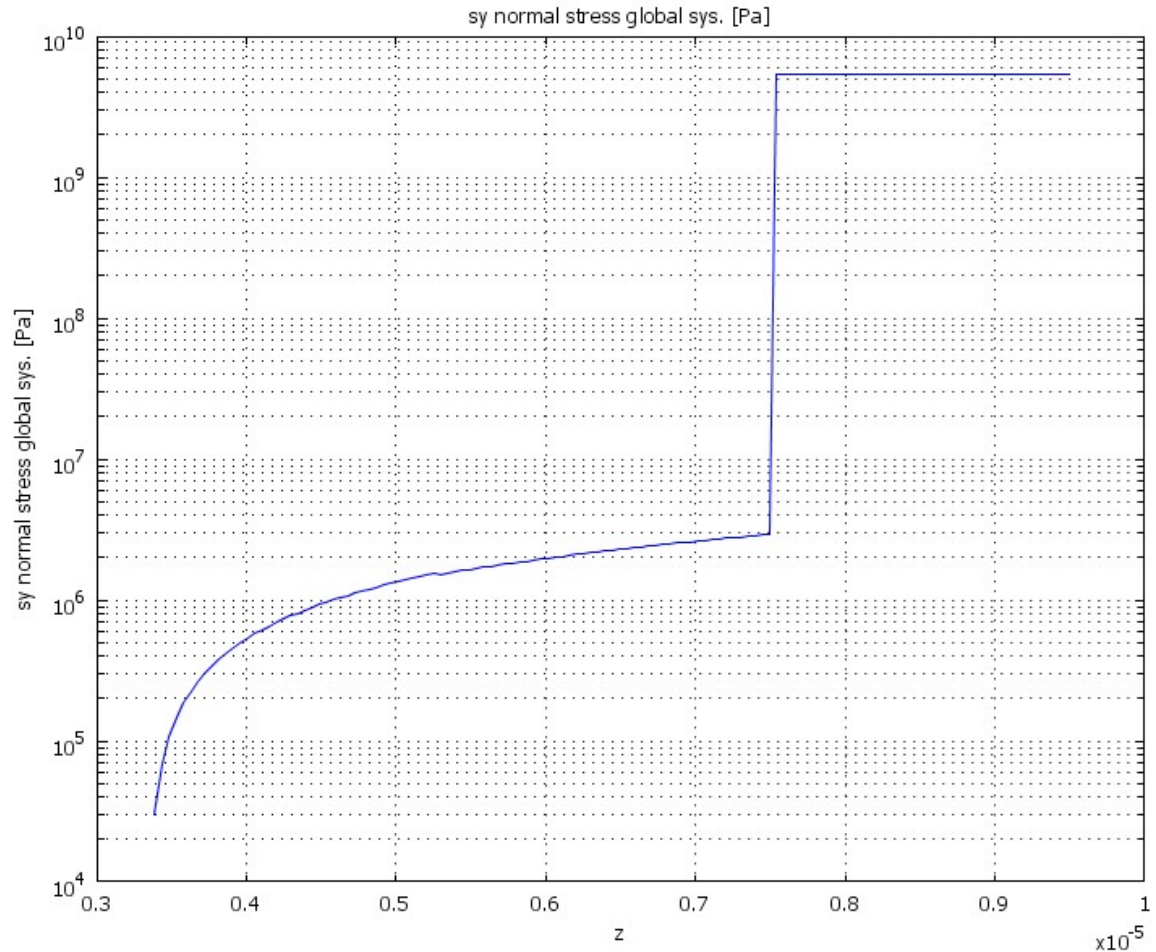
- Despite large deformations at edges, highest von Mises stresses actually occur in the center of the flexure

Side view of long side of flexure – note lower stress at the edge of SMA layer



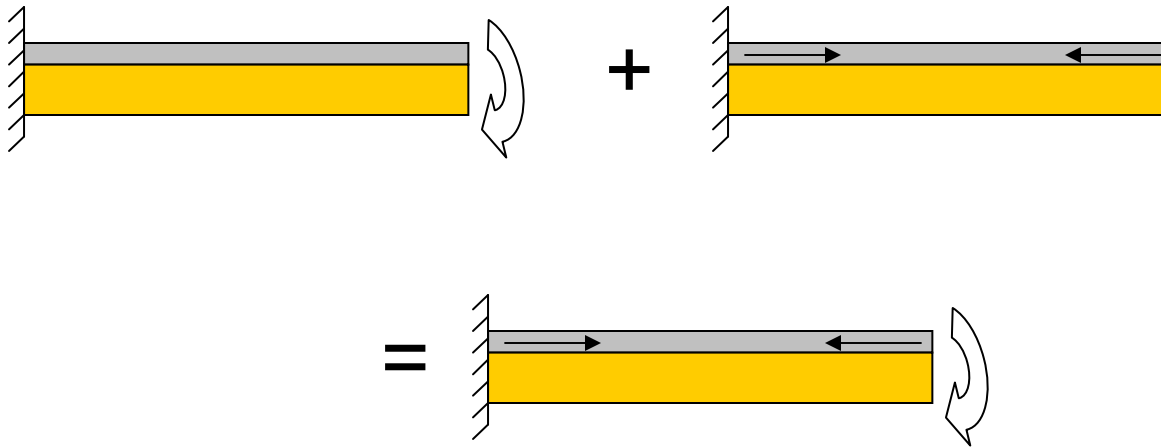
# Line plot of normal stress at center plane

- log plot shows that stress in kapton layer is not constant, as it appears in contour plot on previous slide



# Third Model: Combination of Loading Cases

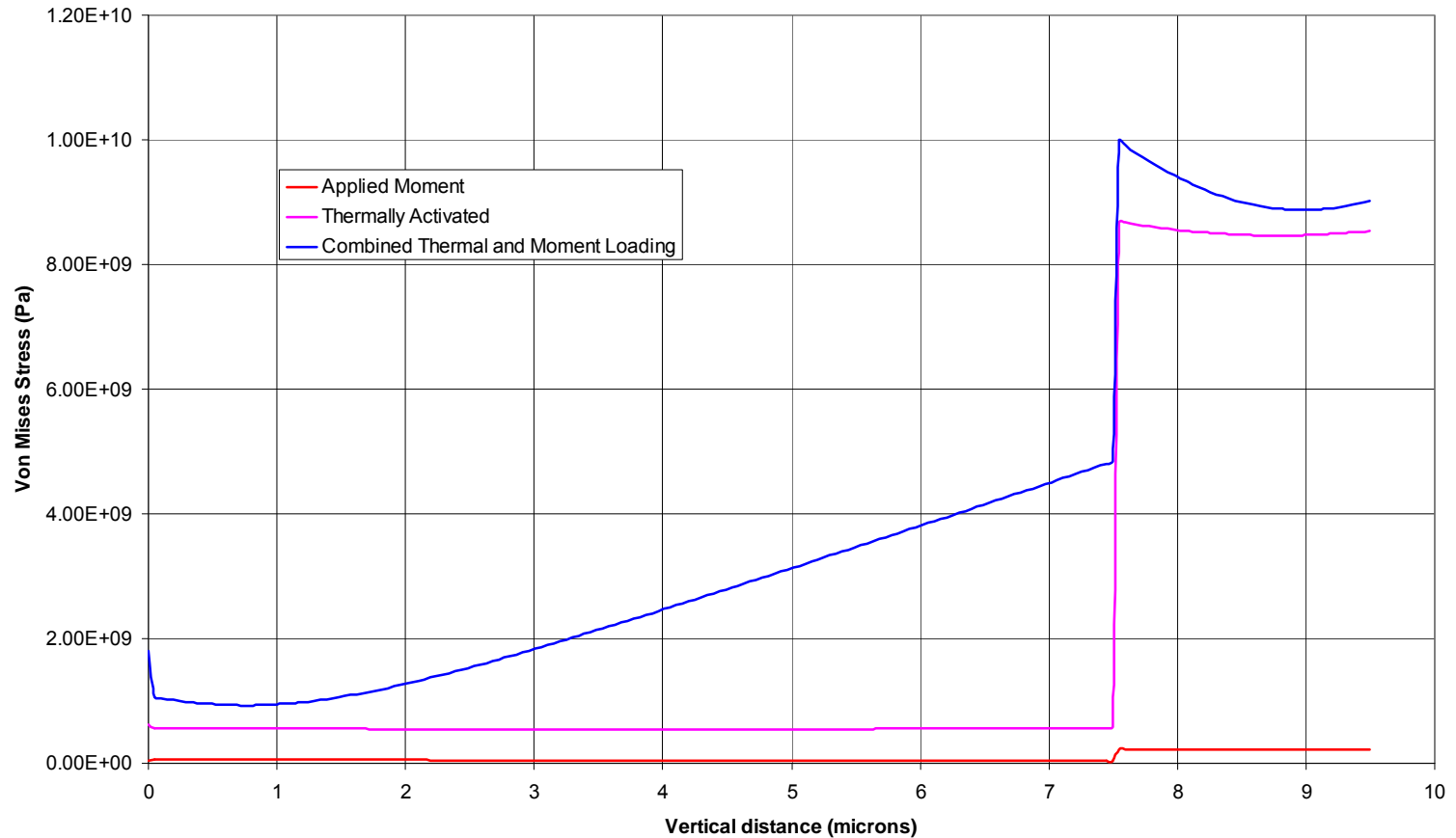
- Account for both applied moment and contraction of SMA due to thermal actuation
- Resulting stresses are higher
- Stress concentrations occur at corners in all three models



# Combination Loading: Corner Stresses

Stress concentration about  
2x along corners

Von Mises Stress at Corner of Flexure for Three Loading Cases



# Conclusion

- This flexure design will fail under the imposed loading
- Shape memory alloy may be too stiff for this purpose
- Possible solution: use of shape-memory polymer, which is more flexible
- Questions?

# Appendix: References

- For details on composite beam calculations, see R.C. Hibbeler, “Mechanics of Materials”
- For more information on the Harvard Microrobotics Lab, visit the group’s website:  
<http://www.micro.seas.harvard.edu/>
- Material properties were taken from [www.matweb.com](http://www.matweb.com) – searchable online database that includes brand-name materials (Such as Kapton and Nitinol)

# Appendix: Physical Parameters

- Geometry:
  - Flexure length = 100  $\mu\text{m}$ , width = 1500  $\mu\text{m}$ , thickness = 7.5  $\mu\text{m}$
  - SMA coating thickness = 2  $\mu\text{m}$
- Material Properties
  - Kapton <sup>®</sup> Polymer Film from Dupont Corporation:
    - $E = 5 \text{ GPa}$
    - $\nu = .34$
    - Tensile strength = 172 MPa
  - Nitinol Shape Memory Alloy (Low Temp/High Temp phases)
    - $E = 28 \text{ GPa} / 75 \text{ GPa}$
    - $\nu = .3 / .3$
    - Tensile strength = 100 MPa / 560 MPa
    - undergoes  $\sim 5\%$  linear strain (contraction) when heated