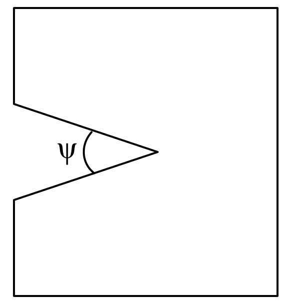
EM 388F: Fracture Mechanics

HW#2 (due Wednesday, February 6, 2008)

5. Stress concentration around a circular hole
Stress concentration at geometric discontinuities is the most important result in elasticity theory. In class, we derived the stress fields around a small circular hole in a large plate under remote equi-biaxial stress and shear. Use the method of linear superposition to derive the stress field around the hole when the plate is under a remote uniaxial stress.

(a) Under uniaxial tension, what is the highest tensile stress around the hole? And where?
(b) Under uniaxial compression, what is the highest tensile stress? And where?

6. Stress singularity at an angular corner. In class we derived the asymptotic elastic solutions for mode I and mode II cracks, both with a square root singularity for the stresses. Apply the same procedure for an angular corner in a homogeneous plate (see figure), and determine the exponents of stress singularity under both symmetric and antisymmetric modes for the angle $0 \le \psi \le \pi$.



7. Penny-shaped crack. A circular crack lies on region $x^2 + z^2 \le a^2$, in the plane $y = 0$ of an unbounded body. Subject to a remote stress field, $\sigma_{yy} = \sigma$ and $\tau_{xy} = \tau$. All other remote stress components vanish. The elasticity solution of this problem has been solved analytically, giving the displacements of the crack faces:

$$u_y^+ - u_y^- = \frac{4(1-\nu)\sigma}{\pi\mu}\sqrt{a^2 - x^2 - z^2}$$
$$u_x^+ - u_x^- = \frac{8(1-\nu)\tau}{(2-\nu)\pi\mu}\sqrt{a^2 - x^2 - z^2}$$
$$u_z^+ - u_z^- = 0$$

Determine the stress intensity factors of all three modes along the crack front.

8. The full-field solution for a finite crack in an infinite plate under remote tension is obtained by the complex variables method (see Alan Zehnder, *Lecture Notes on Fracture Mechanics*, http://hdl.handle.net/1813/3075). For a crack lying along the x axis ($y = 0, -a < x < a$), the stress components along the x axis is:

$$\sigma_x = \mathrm{Re}\left(\frac{\sigma_\infty x}{\sqrt{x^2 - a^2}}\right) - \sigma_\infty, \quad \sigma_y = \mathrm{Re}\left(\frac{\sigma_\infty x}{\sqrt{x^2 - a^2}}\right), \text{ and } \sigma_{xy} = 0$$

Compare the full-field solution to the asymptotic solution of the mode I crack tip field, to determine the stress intensity factor. Specify the region in which the error of the asymptotic field is less than 10%.