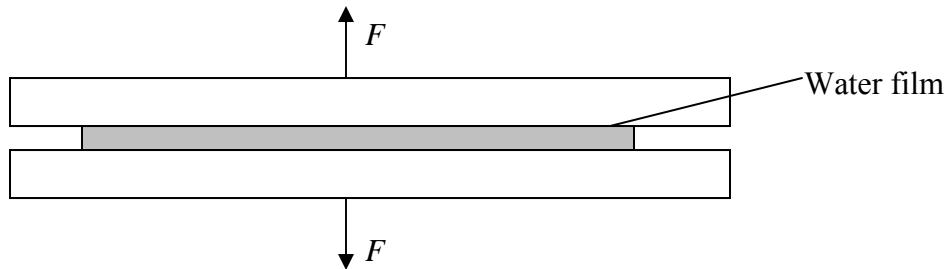


EM 388F: Fracture Mechanics

HW#3 (due Monday, March 3, 2008)

9. It is a common experience that, when, wet, two optical glass plates stick. It takes some force to separate them.
- Use the following picture to explain the phenomena.
 - Compute the force per unit area needed to maintain equilibrium using an energy method.
 - The edge of the water film is actually curved. This implied that pressure inside water differs from the atmospheric pressure. Calculate the pressure difference. Compare the results with (b).



10. Surface energy of glass

To verify his theory, Griffith had to independently measure the surface energy of glass. He did so at a number of high temperatures and deduced the value at room temperature by extrapolation. The following is an excerpt from his paper (A.A. Griffith, The phenomena of rupture and flow in solids, *Philosophical Transactions of the Royal Society* 221, 163-198, 1921).

Between 730°C and 900°C the method described below was found to be practicable. Fibers of glass about 2 inches long and from 0.002-inch to 0.01-inch diameter, with enlarged spherical ends, were prepared. These were supported horizontally in stout wire hooks and suitable weights were hung on their mid points. The enlarged ends prevented any sagging except that due to extension of fibers. The whole was placed in an electric resistance furnace maintained at the desired temperature. Under these conditions viscous stretching of the fibers occurred until the suspended weigh was just balanced by the vertical components of the tension in the fiber. The latter was entirely due, in the steady state, to the surface tension of the glass, whose value could therefore be calculated from the observed sag of the fiber. In the experiments the angle of sag was observed through a window in the furnace by means of a telescope with a rotating cross wire. If w is the suspended weight, d the diameter of the fiber, T the surface tension, and θ the angle at the point of suspension between the two halves of the fiber, then, evidently,

$$\pi d T \cos \frac{\theta}{2} = w .$$

For this method of determining the surface tension to be valid, it is evidently necessary that the angle of sag shall reach a steady value before the development of local contractions, arising from the instability of liquid cylinders, becomes appreciable.

Actions:

- Why he could not do this experiment at temperatures much higher than 900°C or much lower than 730°C?
- Derive his formula.
- For a 0.01-inch diameter fiber to reach 120° steady angle of sag, which is easily observable, how much weight should be suspended? Assume a reasonable magnitude of surface tension.

11. Griffith listed in his paper, for the glass he used, Young's modulus 9.01×10^6 psi, tensile strength 24,900 psi, and surface energy 0.0031 lb/in. Estimate the flaw size in the glass. Compare it with the typical molecular size and sample size.

12. **Griffith's Strength Model.** Griffith found that the strength of a material relates to the surface energy, elastic modulus, and flaw size:

$$S = \sqrt{\frac{2\gamma E}{\pi c}}$$

Consequently, the strength is not a material property. Rather, it depends on flaw size. Consider the glass used by Griffith, with surface energy $\gamma = 1.75 \text{ J/m}^2$ and Young's modulus $E = 62 \text{ GPa}$.

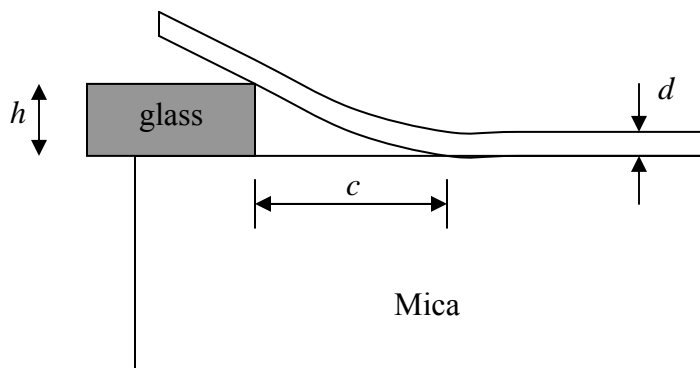
- Calculate the tensile strength of a large glass panel with defect size $c = 1 \text{ }\mu\text{m}$.
- The same glass panel now has a small circular hole of radius $a = 1 \text{ mm}$, assuming that the defect size remains to be $c = 1 \text{ }\mu\text{m}$. How much load can the panel carry?
- What if the circular hole is replaced by a crack-like notch with half-width $a = 1 \text{ mm}$?

13. Flow or fracture

A large thick plate of steel is detected to have an edge crack of depth 1 mm. The steel has a fracture energy of 10^4 J/m^2 and a yield strength of 1 GPa. The steel plate is loaded in tension. (a) Will the plate undergo general yield or fail by fast fracture? (b) What if the crack depth is 0.1 mm?

14. Obreimoff's experiments

Mica splits readily into flakes along a certain crystal plane. Obreimoff (The splitting strength of mica, Proc. Roy. Soc. Lond. A 127, 290-297, 1930) carried out the following experiments. A glass wedge is inserted beneath a thin flake of mica attached to a parent block and is made to drive along the cleavage plane.

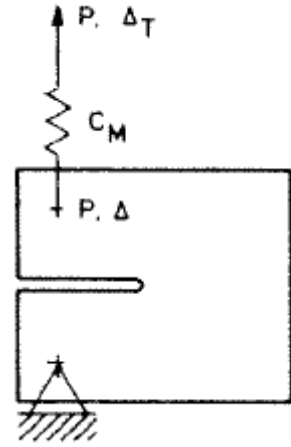


(a) Show that the energy release rate is: $G = \frac{3}{8} E d^3 h^2 c^{-4}$.

(b) Take $E = 200 \text{ GPa}$, $h = 480 \text{ }\mu\text{m}$, $d = 75 \text{ }\mu\text{m}$. The experimentally measured crack length is $c = 10 \text{ mm}$. Determine the surface energy of mica.

15. Consider a cylindrical tube with thickness t and radius R . The tube is made of a fibrous material so that it is prone to splitting along the axial direction. The tube is subject to a torque T . Compute the energy release rate that drives splitting. Assume a long crack that cuts across the tube wall and may spread along the axial direction.

16. In class we developed the compliance method that determines the energy release rate by measuring the compliance under the displacement control. Consider now that the loading device has a compliance, C_M , illustrated as a linear spring in the figure. Let Δ_T be the total displacement. Write down the total elastic energy of the system, including the specimen and the spring. Show that the relationship between the energy release rate and the compliance of the specimen is independent of the loading system (C_M). Note that $C_M \rightarrow 0$ corresponds to the displacement control test and $C_M \rightarrow \infty$ corresponds to load control.



17. Show that for a mode-III crack the stress intensity factor relates to the energy release rate as

$$G = \frac{K_{III}^2}{2\mu}.$$

18. Show that for a mode-III crack the J-integral around the crack tip relates to the stress intensity factor relates as

$$J = \frac{K_{III}^2}{2\mu}.$$