There is an example in that book as follows:

EXAMPLE 4.15 *Find Green's function for three-dimensional Poisson's equation,*

$$\nabla^2 G + \delta(\mathbf{x} - \mathbf{x}') = 0 \quad \Rightarrow \quad G_{,ii} + \delta(\mathbf{x} - \mathbf{x}') = 0 \qquad (4.61)$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x_i \partial x_i}, i = 1, 2, 3$ *and* $\delta(\mathbf{x} - \mathbf{x}') = \delta(x_1 - x_1')\delta(x_2 - x_2')\delta(x_3 - x_3')$

After some assumptions and Algebra operations finally comes to

$$G_{,ii}(\mathbf{x} - \mathbf{x}') = -\int_{-\infty}^{\infty} \bar{G}(\boldsymbol{\xi})\xi_i\xi_i \exp(i\boldsymbol{\xi} \cdot (\mathbf{x} - \mathbf{x}'))d\boldsymbol{\xi}$$

And Prof, Li have given elegant solution to this problem.

My puzzlements are as follows:

During solving this integration, Prof. Li use the coordinate transformation,

$$\begin{aligned} G(\mathbf{x} - \mathbf{x}') &= \frac{1}{(2\pi^3)} \int_{-\infty}^{\infty} \frac{1}{\xi^2} \exp(i\boldsymbol{\xi} \cdot (\mathbf{x} - \mathbf{x}'))d\boldsymbol{\xi} \\ &= \frac{1}{(2\pi^3)} \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \frac{1}{\xi^2} \exp(i\boldsymbol{\xi} \cdot (\mathbf{x} - \mathbf{x}'))\boxed{\xi^2 d\xi \sin\theta d\theta d\phi} \end{aligned}$$

This is the illustration in Li's book

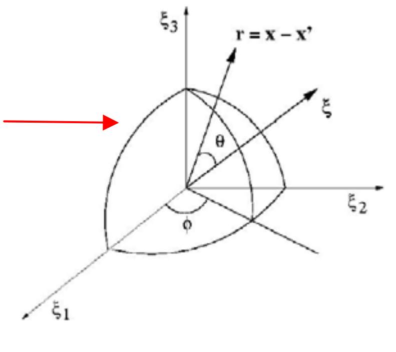
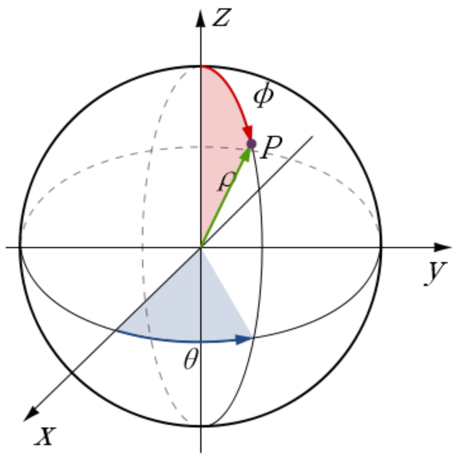


Figure 4.3. Inversion of three-dimensional Fourier transform



But to my understand, the relation between Cartesian coordinates and Spherical Coordinates is as follows

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$

And the volume fragment relation in these two coordinate is

$$dV_{xyz} = \rho^2 \sin\phi dV_{\rho\phi\theta} = \rho^2 \sin\phi d\rho d\phi d\theta$$

From these two pictures, we may note the difference, in Li's book, the angle θ is not the angle between the ξ and vertical axis ξ_3, why he could get the same transformation?