Temperature

Temperature is an idea so embedded in everyday experience that we will first abstract the idea from one kind of such experience: thermal contact. We will then show that the empirical observations about thermal contact are manifestations of the fundamental postulate. The latter relates temperature to two other quantities: the energy and the number of quantum states. We can experimentally determine the number of quantum states of a glass of wine.

An essential step to "understand" thermodynamics is to get to know temperature: how temperature comes down as an abstraction from empirical observations, and how it rises up as a consequence of the fundamental postulate.

By contrast, entropy is a simple idea: for the time being entropy is shorthand for the phrase "the logarithm of the number of quantum states of an isolated system". The fundamental postulate implies the second law of thermodynamics. The latter is an incomplete expression of the former.

Thermal contact. Consider a glass of wine and a piece of cheese. They have been kept as two separate isolated systems for such a long time that each by itself is in equilibrium. However, the two isolated systems are not in equilibrium with each other. We now allow the two systems to interact in a particular way: energy from one system can go to the other system. Will energy go from the wine to the cheese, or the other way around?

This mode of interaction, which re-allocates energy between the two systems, is called *thermal contact*. The energy that goes from one system to the other is called *heat*. To focus on thermal contact, we block all other modes of interaction: the two systems do not exchange volume, molecules, etc. In reality, any contact of two systems does more than just re-allocating energy. For example, when two bodies touch each other, they may form atomic bonds. As another example, the presence of two bodies in space modifies the electromagnetic field. For the time being, we assume that such effects are negligible, and focus exclusively on a single mode of interaction: the two systems can only exchange energy.

Mechanisms of heat transfer. The process by which energy "goes" from one place to another is called *heat transfer.* Heat transfer takes place by several familiar mechanisms:

- *Conduction*. Energy can go through a material. At a macroscopic scale, the material remains stationary. At a microscopic scale, energy is carried by flow of electrons and vibration of atoms.
- *Convection*. Energy can go with the flow of a fluid. This way of energy transfer is not analyzed now because it involves mass transfer between systems.
- *Radiation*. Energy can be carried by electromagnetic waves. Because electromagnetic waves can propagate in vacuum, two systems in thermal contact need not be in proximity.

Thermal equilibrium. When all other modes of interaction are blocked, two systems in thermal contact for a long time will cease to exchange energy, a condition known as *thermal equilibrium*. Our everyday experience with

thermal contact and thermal equilibrium may be distilled in terms of several salient observations.

Observation 1: hotness is a property independent of system.   If two systems are separately in thermal equilibrium with a third system, the two systems are in thermal equilibrium with each other. This observation is known as *the zeroth law of thermodynamics*.

This observation shows us that all systems in thermal equilibrium possess one property in common: *hotness*. The procedure to establish a level of hotness is empirical. We bring two systems into thermal contact, and check if they exchange energy. Two systems in thermal equilibrium are said to have the same level of hotness. Two systems not in thermal equilibrium are said to have different levels of hotness.

In thermodynamics, the word "hot" is used strictly within the context of thermal contact. It makes no thermodynamic sense to say that one movie is hotter than the other, because the two movies cannot exchange energy. The word hotness is synonymous to temperature.

Name a level of hotness. To facilitate communication, we give each level of hotness a name. As shown by the above empirical observation, levels of hotness are real: they exist in the experiment of thermal contact, regardless how we name them. The situation is analogous to naming streets in a city. The streets are real: they exist regardless how we name them. We can name the streets by using names of presidents, or names of universities. We can use numbers. We can be inconsistent, naming streets in one part of the city by numbers, and those in another part of the city by names of presidents. We can even give the same street several names, using English, Chinese, and Spanish.   Some naming schemes might be more convenient than others, but to call one scheme absolute is an abuse of language. We will give an example of such an abuse later. But for now, consider one specific naming scheme.

We can name each level of hotness after a physical event. For example, we can name a level of hotness after a substance that melts at this hotness. This practice is easily carried out because of the following empirical observation: at the melting point, a substance is a mixture of solid and liquid, and the hotness remains unchanged as the proportion of the solid and liquid changes. Thus, a system is said to be at the level of hotness WATER when the system is in thermal equilibrium with a mixture of ice and water at the melting point. Here are four distinct levels of hotness: WATER, LEAD, ALUMINUM, GOLD. We can similarly name other levels of hotness.

Observation 2: levels of hotness are ordered. When two systems of different levels of hotness are brought into thermal contact, energy goes only in one direction from one system to the other, but not in the opposite direction. This observation is known as *the second law of thermodynamics*.

This observation allows us to order any two levels of hotness. When two systems are brought into thermal contact, the system losing energy is said to have a higher level of hotness than the system gaining energy. For example, we say that hotness "LEAD" is lower than hotness "ALUMINUM" because, upon brining melting lead and melting aluminum into thermal contact, we observe that that

the amount of solid lead decreases, while the amount solid aluminum increases. Similar experiments show us the order of the four levels of hotness as follows:

WATER, LEAD, ALUMINUM, GOLD.

Observation 3: levels of hotness are continuous. Between any two levels of hotness A and B there exists another level of hotness. The experiment may go like this. We have two systems at hotness A and B, respectively, where hotness A is lower than hotness B. We can always find another system, which loses energy when in thermal contact with A, but gains energy when in thermal contact with B.

This observation allows us to name all levels of hotness using a single real variable. Around 1720, Fahrenheit assigned the number 32 to the melting point of water, and the number 212 to the boiling point of water. What would he do for other levels of hotness? Mercury is a liquid within this range of hotness and beyond, sufficient for most purposes for everyday life. When energy is added to mercury, mercury expands. The various volumes of mercury could be used to name the levels of hotness.

What would he do for a high level of hotness when mercury is a vapor, or a low level of hotness when mercury is a solid? He could switch to materials other than mercury, or phenomena other than thermal expansion.

In naming levels of hotness using a real variable, in essence Fahrenheit chose a one-to-one function, whose domain was various levels of hotness, and whose range was a real number. Any choice of such a function is called a *temperature scale*. While the order of various levels of hotness is absolute, no absolute significance is attached to the number that names each level of hotness. In particular, using numbers to name levels of hotness does not authorize us to apply arithmetic rules: the addition of two levels of hotness has no more empirical significance than the addition of the numbers of two houses on a street.

As illustrated by the melting-point scale of hotness, a non-numerical naming scheme of hotness perfectly captures all we care about everyday experience of hotness. Naming levels of hotness by using numbers makes it easier to memorize that hotness 80 is hotter than hotness 60. Our preference to a numerical scale reveals more about the prejudice of our brains than the nature of hotness.

Once a scale of hotness is set up, any monotonically increasing function of the scale gives another scale of hotness. For example, in the Celsius scale, the freezing point of water is set to be 0C, and the boiling point of water 100C. We further set the Celsius (C) scale to be linear in the Fahrenheit (F) scale. These prescriptions give the transformation between the two scales:

$$C = \frac{5}{9}(F - 32) .$$

In general, however, the transformation from one scale of hotness to another need not be linear. Any increasing function will preserve the order of the levels of hotness.

A system with variable energy is not an isolated system, but can be viewed as a family of isolated systems. We now wish to trace the above empirical observations of thermal contact to the fundamental postulate: *a system isolated for a long time is equally probable to be in any one of its quantum states.* For details of the fundamental postulate, see the notes on Isolated Systems (http://imechanica.org/node/290).

When a system can gain or lose energy by thermal contact, the system is not an isolated system. However, once the energy of the system is fixed at any particular value U, the system is isolated, and has a specific number of quantum states, Ω. Consequently, we can regard a system of variable energy as a family of isolated systems, characterized by the function $\Omega(U)$. Each member in the family is a distinct isolated system, with its own amount of energy and its own set of quantum states. The family has a single independent variable, the energy, because we choose to focus exclusively on thermal contact, and block all other modes of interaction between the system and the rest of the world.

For example, a hydrogen atom is a system that can change its energy by absorbing photons. Thus, the hydrogen atom may be viewed as a family of isolated systems. According to quantum mechanics, this family of isolated systems is characterized by the function $\Omega(U)$:

$$\Omega(-13.6\mathrm{eV})= 2, \ \Omega(-3.39\mathrm{eV})= 6, \ \Omega(-1.51\mathrm{eV})=18,...$$

For the hydrogen atom, the gaps between energy levels happen to be large. For a complex system like a glass of wine, the gaps between energy levels are so small that we might as well regard the energy of the glass of wine as a continuous variable.

Two systems in thermal contact form a composite, which is an isolated system, with the partition of energy as an internal variable. The procedure to apply the fundamental postulate is as follows:
    (1) construct an isolated system;
    (2) dissect the set of all the quantum states of the isolated system into subsets, each subset being a macrostate of the isolated system; and
    (3) count the number of quantum states in each macrostate.
The probability for the isolated system to be in a macrostate is the number of quantum states in the macrostate divided by the total number of quantum states of the isolated system.

We next apply this procedure to the thermal contact of two systems. When the two systems are in thermal contact, energy may go from one system to the other, but the composite of the two systems is an isolated system. Consequently, the partition of energy between the two systems is an internal variable of the composite. The composite has a set of quantum state. This set is then dissected into a family of macrostates by using the partition of energy.

We call two systems A' and A''. Isolated at energy U', system A' has a total of $\Omega'(U')$ number of quantum states, labeled as $\{\gamma_1', \gamma_2',....,\}$. Isolated at energy U'', system A'' has a total of $\Omega''(U'')$ number of quantum states, labeled as $\{\gamma_1'', \gamma_2'',....,\}$.

Consider a particular partition of energy, i.e., a particular macrostate of the composite. The energy of the composite is partitioned as U' and U'' into the two systems, and a quantum state of the composite can be any combination of a quantum state chose from the set $\{\gamma_1', \gamma_2', ...\}$ and a quantum state chosen from the $\{\gamma_1'', \gamma_2'', ...\}$. For example, one quantum state of the composite is when system A' is in state γ_2' and system A'' is in state γ_3''. The total number of all such combinations is

$$\Omega'(U')\Omega''(U'').$$

This is the number of quantum states of the composite in the macrostate that energy is partitioned as U' and U'' between the two systems. In our idealization, we neglect any creativity of thermal contact: no fundamentally new quantum states are included.

Most probable partition of energy. Of all possible ways to partition energy between the two systems, which one is most probable?

Consider another macrostate of the composite: the energy of the composite is partitioned as $U' + dU$ and $U'' - dU$ between the two systems. That is, system A' gains energy dU at the expense of system A''. This macrostate of the composite consists of the following number of quantum states:

$$\Omega'(U' + dU)\Omega''(U'' - dU).$$

We assume that both systems are so large that energy transfer may be regarded as a continuous variable, and that the functions $\Omega'(U')$ and $\Omega''(U'')$ are differentiable. Consequently, the number of states in one macrostate differs from the number of states in the other macrostate by

$$\Omega'(U' + dU)\Omega''(U'' - dU) - \Omega'(U')\Omega''(U'') = \left(\Omega''(U'')\frac{\partial \Omega'}{\partial U'} - \Omega'(U')\frac{\partial \Omega''}{\partial U''}\right)dU.$$

According to the fundamental postulate, all states of the composite are equally probable, so that a macrostate consisting of more states is more probable. Consequently, it is more probable for energy to go from system A'' to A' than the other way around if

$$\Omega''(U'')\frac{\partial \Omega'(U')}{\partial U'} > \Omega'(U')\frac{\partial \Omega''(U'')}{\partial U''}.$$

When the two quantities are equal, energy is equally probable to go in either direction, and the two systems are said to be in *thermal equilibrium*.

Once we know the two functions for the systems, $\Omega'(U')$ and $\Omega''(U'')$, the above analysis determines the more probable direction of energy transfer when the two systems are brought into thermal contact for a long time.

The absolute temperature. In the above inequality, either side involves quantities of the two systems. Divide both sides of the inequality by $\Omega''\Omega'$, and recall a formula in calculus, $\partial \log x / \partial x = 1/x$. We find that it is more probable for energy to flow from system A'' to A' if

$$\frac{\partial \log \Omega'(U')}{\partial U'} > \frac{\partial \log \Omega''(U'')}{\partial U''}.$$

We have just rewritten the inequality into a form that either side only involves quantities of one system. When the two quantities are equal, energy is equally probable to go in either direction, and the two systems are in thermal equilibrium.

The function $\Omega(U)$ is specific to a system in thermal contact, so is the quantity $\partial \log \Omega(U)/\partial U$. The previous paragraph, however, shows that the value $\partial \log \Omega(U)/\partial U$ is the same for all systems in thermal equilibrium. Consequently, any monotonically *decreasing* function of $\partial \log \Omega(U)/\partial U$ can be used to define a temperature scale. A particular choice of temperature scale T is given by

$$\frac{1}{T} = \frac{\partial \log \Omega(U)}{\partial U}.$$

The temperature scale defined above is called the *absolute temperature scale*. This name is misleading, for any monotonically decreasing function of $\partial \log \Omega(U)/\partial U$ is a temperature scale. All these scales reflect the same empirical reality of various levels of hotness. No one scale is more absolute than any other scales. So far as we are concerned, the phrase "absolute temperature" simply signifies a particular choice of a decreasing function of $\partial \log \Omega(U)/\partial U$, as given by the above equation.

Ideal gas. We can calculate the absolute temperature of a system by counting the number of states of the system as a function of energy, $\Omega(U)$. Such counting can be carried out for simple systems, but is prohibitively expensive for most systems. As an example, consider an ideal gas, in which the molecules are so far apart that their interaction is negligible. For the ideal gas, a counting of the number of states gives (see notes on Pressure, http://imechanica.org/node/885)

$$T = pV / N,$$

where p is the pressure, V the volume, and N the number of molecules. This equation relates the absolute temperature T to measurable quantities p, V and N.

Historically, this relation was discovered empirically before the fundamental postulate was formulated. As an empirical relation, it is known as the *ideal gas law*. The empirical content of the law is simply this. When the gas is thin, i.e., N/V is small, experiments indicate that flasks of ideal gases with the same value of pV/N do not exchange energy when brought into thermal contact. Consequently, we may as well call the combination pV/N hotness. This particular scale of hotness motivated the definition of the absolute temperature.

Experimental determination of temperature (thermometry). How does a doctor determine the temperature of a patient? Certainly she has no patience to count the number of quantum states of her patient. She uses a thermometer. Let us say that she brought a mercury thermometer into thermal contact with the patient. Upon reaching thermal equilibrium with the patient, the

mercury expands a certain amount, giving a reading of the temperature of the mercury, which is also the temperature of the patient.

The manufacturer of the thermometer must assign an amount of expansion of the mercury to a value of temperature. This he does by bringing the thermometer into thermal contact with a flask of an ideal gas. He determines the temperature of the gas by measuring its volume, pressure, and number of molecules. Also, by heating or cooling the gas, he varies the temperature and gives the thermometer a series of markings.

Any experimental determination of the absolute temperature follows these basic steps:

(1) Calculate the temperature of a simple system by counting the number of states. The result expresses the absolute temperature in terms of measurable quantities.
(2) Use the simple system to calibrate a thermometer by thermal contact.
(3) Use the thermometer to measure temperatures of any other system by thermal contact.

Steps (2) and (3) are sufficient to set up an arbitrary temperature scale. It is Step (1) that maps the arbitrary temperature scale to the absolute temperature scale.

Our understanding of temperature now divides the labor of measuring absolute temperature among a doctor (Step 3), a manufacturer (Step 2), and a theorist (Step 1). Only the theorist needs to count the number of state, and only for a very few idealized systems, such as an ideal gas. The art of measuring temperature is called *thermometry*.

As with all divisions of labor, the goal is to improve the economics of doing things. One way to view any area of knowledge is to see how labor is divided and why. One way to make a contribution to an area of knowledge is to perceive a new economic change (e.g., computation is getting cheaper, or a new instrument is just invented) and devise a new division of labor.

Units of temperature. Recall the definition of the absolute temperature:

$$\frac{1}{T} = \frac{\partial \log \Omega(U)}{\partial U}.$$

Because the number of states of an isolated system is a pure number, the absolute temperature so defined has the same unit as energy.

An international convention, however, specifies the unit of energy by Joule (J), and the unit of temperature by Kelvin (K). The Kelvin is so defined that the triple point of pure water is 273.16K exactly. The conversion factor between the two units of temperature can be determined experimentally. For example, you can bring pure water in the state of triple point into thermal contact with a flask of an ideal gas. When they reach thermal equilibrium, a measurement of the pressure, volume and the number of molecules in the flask gives a reading of the temperature of the triple point of water in the unit of energy. Such experiments or more elaborate ones give the conversion factor between the two units of temperature:

$$1\text{K} \approx 1.38 \times 10^{-23}\,\text{J}.$$

This conversion factor is called Boltzmann's constant *k*. (Boltzmann had nothing to do with this conversion factor between two units.)

Another unit for energy is $1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$, giving a conversion factor:

$$1\text{K} = 0.863 \times 10^{-4}\,\text{eV}.$$

Any temperature scale is an increasing function of the absolute temperature *T*. For example, the Celsius temperature scale, *C*, relates to *T* by

$$C = T - 273.15\text{K} \quad (T \text{ in the Kelvin unit}).$$

The merit of using Celsius in everyday life is evident. It feels more pleasant to hear that today's temperature is 20C than 0.0253eV or 293K. It would be mouthful to say today's temperature is $404.34 \times 10^{-23}\,\text{J}$. By the same token, the hell would sound less hellish if we were told it burns at the temperature GOLD. Nevertheless, our melting-point scale of hotness can be mapped to the absolute temperature scale in either the Kelvin unit or the energy unit:

Melting-point scale	Kelvin unit	Energy unit
WATER	273.15 K	0.0236 eV
LEAD	600.65 K	0.0518 eV
ALUMINUM	933.60 K	0.0806 eV
GOLD	1337.78 K	0.1155 eV

Ideal gas law in irrational units. Because the Kelvin unit has no fundamental significance, if the temperature is used in the Kelvin unit, *kT* should be used whenever a fundamental result contains temperature. For example, if temperature is used with the Kelvin unit, the ideal gas law becomes

$$kT = pV / N \quad (T \text{ in the Kelvin unit}).$$

To muddy water even more, sophisticated people have introduced a unit to count numbers! The number of atoms in 12 grams of carbon is called a mole. (In this definition, each carbon atom used must be of the kind of isotope that has 6 neutrons.) Experimental measurements give

$$1 \text{ mole} \approx 6.022 \times 10^{23}.$$

This number is called Avogadro's number N_a.

When you express the number of molecules in units of mole, and the temperature in the Kelvin unit, the ideal gas law becomes

$$RT = pV / n \quad (T \text{ in the Kelvin unit}),$$

where $n = N / N_a$ is the number of moles, and

$$R = kN_a = 8.314\text{J}/(\text{K} \cdot \text{mole}).$$

Now you can join the sophisticated people and compound the silliness and triumphantly name *R*: you call *R* the universal gas constant. It makes you sound more learned than naming another universal gas constant: 1, as *R* should be if you measure temperature in the unit of energy, and count molecules by numbers.

Experimental determination of heat (calorimetry). By definition, heat is the energy transferred from one system to the other during thermal contact. Although energy and temperature have the same unit, their experimental measurements can be completely independent of each other. For example, we can pass an electric current through a metal wire immersed in a pot of water. The Joule heating of the metal wire adds energy to the water, and a measurement of the electrical power, *IV*, over some time tells us the amount energy added to the water.

As another example, the energy absorbed by a mixture of ice and water is proportional to the volume of the ice melted. Consequently, once this proportionality constant is measured, e.g., by using the method of Joule heating, we can subsequently measure energy gained or lost by a system by placing it in thermal contact with a mixture of ice and water. The volume of the ice remaining in the mixture can be used to determine the energy transferred to the system. The art of measuring heat is called *calorimetry*.

Experimental determination of the number of quantum states. Once we know how to measure temperature and heat, we can determine the number of quantum states, Ω. We open the system in a single way by adding energy to the system, while blocking all other modes of interaction. For a small amount of energy dU added to the system, from the definition of temperature, we have

$$d\log\Omega = \frac{dU}{T}.$$

This method allows us to determine the function $\Omega(U)$ of a system up to a multiplicative factor. To fix the multiplication factor, we set $\Omega = 1$ as $T \to 0$.

Entropy of an isolated system. If you are tired of the phase "the logarithm of the number of quantum states of an isolated system", you may as well join the crowd and replace the phrase with a cryptic word: *entropy*. That is, you define the entropy *S* of an isolated system by

$$S = \log\Omega.$$

Entropy so defined is a pure number, and needs no unit.

When temperature is given in Kelvins, to preserve the relation $dS = T^{-1}dU$, one includes the conversion factor *k* in the definition and write

$$S = k\log\Omega.$$

This practice will give the entropy a unit J/K. Of course, this unit is as silly as a unit like inch/meter, or a unit like joule/calorie. Worse, this unit may give an impression that the concept of entropy is related to energy and temperature. This impression is wrong. Entropy is simply the shorthand for "the logarithm of the number of quantum states of an isolated system". The concept of entropy is more primitive than either energy or temperature. We happen to be interested in thermal contact, where energy and temperature are involved.

At a more elementary level, entropy is a pure number associated with any probability distribution, not just the probability distribution of quantum states of

an isolated system. For example, we can talk about the entropy of rolling a fair die ($S = \log 6$), or the entropy of tossing a fair coin ($S = \log 2$).

Statements associated with entropy. By changing a phrase you should not trick yourself into believing that you have learned anything new. When you speak of entropy, you simply paraphrase what you already know about "the logarithm of the number of quantum states of an isolated system", $\log \Omega$. The following statements are banal, but may sound profound if you forget what the word entropy means.

Entropy is well defined for an isolated system even when the system is not in equilibrium. Thus, entropy is a property of an isolated system, in or out of equilibrium.

The entropy has the limiting property that, as $T \to 0$, all systems have the same entropy, which we set to be zero.

Entropy is an extensive quantity. For a composite of several systems, the entropy of the composite is the sum of the entropies of the individual systems. This statement follows from a property of the logarithmic function:

$$\log(\Omega_1 \Omega_2) = \log \Omega_1 + \log \Omega_2 .$$

If we dissect the set of all the states of an isolated system into a family of macrostates, A, B, C... each macrostate has its own number of states, $\Omega_A, \Omega_B, \Omega_C, ...$, and its own value of entropy,

$$S_A = \log \Omega_A, S_B = \log \Omega_B, S_C = \log \Omega_C, ...$$

The probability for an isolated system in equilibrium to be in macrostate A is proportional to $\exp(S_A)$. The isolated system is more likely to be in a macrostate if the macrostate has a larger value of entropy.

When a system gains a small amount of energy *q*, and all other modes of interaction is blocked, the entropy of the system changes by

$$\Delta S = q / T .$$

The entropy of a substance. A substance is an aggregate of a large number of molecules (or atoms), homogeneous at a macroscopic scale. For example, diamond is a crystalline lattice of carbon atoms, water is a liquid of H_2O molecules, and oxygen is a gas of O_2 molecules. The entropy of a substance can be determined experimentally, just like the entropy of any other system.

For a substance composed of only one species of molecules, the entropy of a piece of the substance is proportional to the number of molecules in the piece. Consequently, we may report the entropy per molecule of the substance, namely, the entropy of the piece of the substance, *S*, divided by the number of molecules in the piece, *N*. This ratio S / N is specific to the substance, and is independent of the size and shape of the piece.

At the room temperature and atmospheric pressure, the entropy of diamond, lead and water are 0.3, lead 7.8 and 22.70, respectively. A strong substance, such as diamond, has a small value of entropy, because individual atoms are held together by strong chemical bonds, which limit the number of quantum states.

Complex substances generally have larger entropies than simple substances. For example, the entropies for O, O_2 and O_3 are 19.4, 24.7, 28.6, respectively.

When a substance melts, the substance transforms from a crystal to a liquid. Associated with this transformation, the substance acquires more states, and the entropy typically increases by a number between 1 to 1.5.

Latent heat. We often use T as the independent variable. This choice can be tricky because temperature and the members of the family of isolated systems may have a one-to-many mapping. For example, when ice is melting into water, energy is absorbed, but temperature does not change, so that associated with the melting temperature are many members in the family of isolated systems.

The amount of energy absorbed or released when a substance transforms from one phase to the other is known as the *latent heat.* Associated with the latent heat is a jump in entropy

$$\Delta S = \frac{\Delta U}{T_c},$$

where ΔU is the latent heat (i.e., the difference in the energy between the co-existent phases), and T_c is the phase-transition temperature.
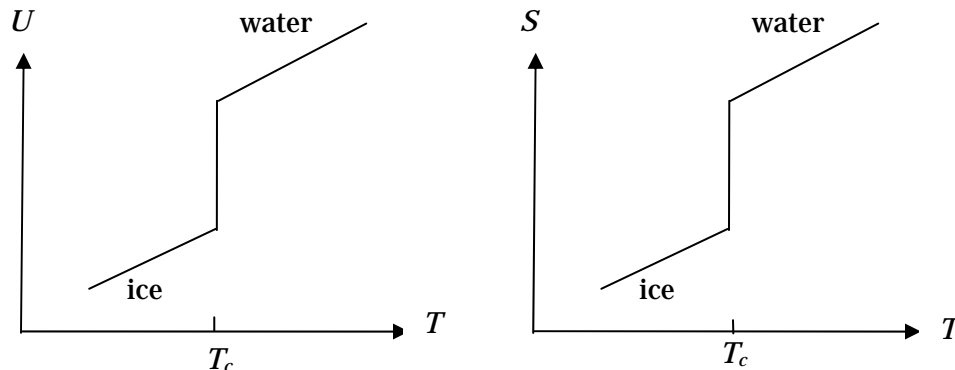
Heat capacitance. Beside such a phase transition, U and T do have a 1-to-1 relation. Associated with a small change in the energy of the system, dU, the temperature of the system changes by dT. Define the *heat capacitance* of the system by

$$C = \frac{\partial U(T)}{\partial T}.$$

Because the temperature has the unit of energy, heat capacitance is dimensionless. If you'd like to use a different unit for temperature, kT has the unit of energy, and the heat capacitance is given in units of k. The heat capacitance is also a function of temperature, $C(T)$.

In this experimental record of $U(T)$, the slope is the heat capacitance, and the jump the latent heat.

Once the function $U(T)$ is measured, recall the definition of the absolute temperature,

$$dS = \frac{dU}{T}.$$

We write

$$dS = \frac{C(T)dT}{T}.$$

Integrating, we obtain the entropy is obtained as a function of temperature:

$$S(T) = \int_0^T \frac{C(T)dT}{T}.$$

We set $S = 0$ at $T = 0$. To this integral we also add the jump in entropy upon passing a phase transition.