

## Transversely isotropic material

The direction of fibers at a point ' $\mathbf{X}$ ' in the undeformed co-ordinate system is  $\mathbf{a}_o$ . Strain energy density,  $\psi$ , is expressed in terms of right Cauchy-Green tensor,  $\mathbf{C}$ , and also  $\mathbf{a}_o$ . The following is the function

$$\psi = \psi \left( \mathbf{C}, \mathbf{a}_o \otimes \mathbf{a}_o \right)$$

I have the following doubts.

- 1.**  $\mathbf{a}_o$  is a vector, but, this is expressed as a second order tensor. Why? Is it to make the function,  $\psi$ , uniform in terms of its arguments? When do we convert a first order tensor into a second order (or above) tensors?
- 2.**  $\psi$  is expressed in terms of five invariants. Among five, three are regular invariants as we see in case of isotropic materials. Rest of the two are ( $I_4$  and  $I_5$ ) in terms of  $\mathbf{C}$  and  $\mathbf{a}_o$ . What is the basis for these two (why only two?) invariants and how do we get the expressions for  $I_4$  and  $I_5$  invariants.