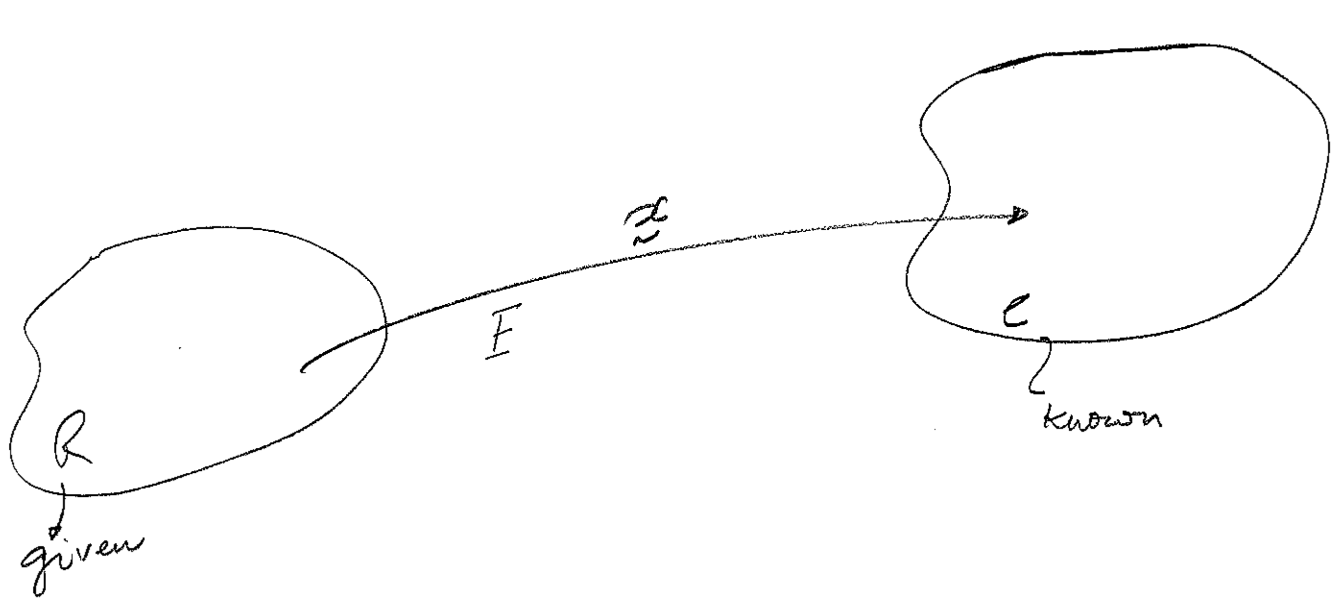
(1)



Problem: Given R an C find a field F^e on C such that

$$\underline{F}^{e^T} \underline{F}^e =: C^e \quad \text{[require } F^e \text{ to be invertible tensor valued]}$$

$$\underline{F}^e \underline{S}(\underline{C}^e) \underline{F}^{e^T} = \underline{\sigma}$$

$$\operatorname{div} \underline{\sigma} = \underset{\sim}{0} \qquad \text{on } C$$

$$\& \quad \underline{\sigma} \underset{\sim}{n} = \underset{\sim}{0} \qquad \text{on } \partial C.$$

where $\underline{S}$ is a given symmetric tensor valued function of $\underline{C}^e$ and div is divergence operator on C. (NOTE: $\underline{S}$ need not be an isotropic tensor valued fn).

<u>Step 1</u>: choose <u>any</u> symmetric second order tensor field $\underline{X}$ on C. W.r.t. rectangular Cartesian coordinates define

$$\sigma^*_{ij} = e_{ike} e_{jrs} X_{es,rk} \qquad \text{(Beltrami)}$$

Note $\sigma^*_{ij,j} = 0$ & $\sigma^*_{ij} = \sigma^*_{ji}$ [In the way I define the curl this would be $\underline{\sigma}^* = \operatorname{curl}[(\operatorname{curl} \underline{X})^T]$]

Step 2: Solve a fictional linear elasticity (Neumann) problem on the configuration $\mathcal{C}$

$$\operatorname{div}\left[(\operatorname{grad}\underset{\sim}{v})_{sym}\right] = \underset{\sim}{0}$$

$$\& \ (\operatorname{grad}\underset{\sim}{v})_{sym}\, \underset{\sim}{n} = -\underline{\sigma}^{*}\underset{\sim}{n}.$$

Note: since $\operatorname{div}\underline{\sigma}^{*} = 0$ on $\mathcal{C}$

$$\int_{\partial \mathcal{C}} \underline{\sigma}^{*}\underset{\sim}{n}\, da = \underset{\sim}{0}$$ so no problem with existence of solutions to $\underset{\sim}{v}$.

Step 3: Now define $\underline{\sigma} := \underline{\sigma}^{*} + (\operatorname{grad}\underset{\sim}{v})_{sym}$

& check that

$$\left.\begin{aligned} \operatorname{div}\underline{\sigma} &= \underset{\sim}{0} \text{ on } \mathcal{C} \\ \& \quad \underline{\sigma}\,\underset{\sim}{n} &= \underset{\sim}{0} \text{ on } \partial\mathcal{C} \end{aligned}\right\} \text{by construction.}$$

Note: An easy collection of such stress states are generated by choosing $\underline{X}$ to be of compact support on $\mathcal{C}$ & $\underset{\sim}{v} \equiv \underset{\sim}{0}$.

Step 4: Now point-wise solve the nonlinear algebraic equations $\underline{F}^{e}(\underset{\sim}{x})\, S\left(\underline{C}^{e}(\underset{\sim}{x})\right)\underline{F}^{e^T}(\underset{\sim}{x}) = \underline{\sigma}(\underset{\sim}{x})$ ① $\forall\, \underset{\sim}{x}$.

for $\underline{F}^{e}$. Note that this is a set of six equations for 9 variables $\underline{F}^{e}$. One looks at each point $\underset{\sim}{x}$ for invertible $\underline{F}^{e}(\underset{\sim}{x})$ solutions. Uniqueness is not to be expected in general. Depending upon the form of $\underline{S}(\cdot)$

it is also possible that solutions may not exist.

Practically, it is clear several different approaches may be adopted to define a minimum error solution.

Step 5: We have essentially completed the problem. Now it depends on how you define "eigenstrains" in this nonlinear case. Basically, we have characterized a large class of elastic distortion fields that produce (self equilibrated) residual stress fields on $\mathcal{C}$.

The characterization consists of a "free" symmetric tensor valued field $\underline{X}$ that can be chosen at will.

There may be additional non-uniqueness from the "algebraic part" of the soln.; this part may also serve as a partial selection criterion, by ruling out those $\underline{X}$ fields for which at some point there is no soln. to (1).

If you define "eigenstrains" in this context through a multiplicative decomposition

$\underline{F}^{e^{-1}}\underline{F} =: \underline{F}^{p}$ where $\underline{F}$ is the deformation gradient between $\mathcal{R}$ & $\underline{C}$.

then $\underline{F}^{p}$ is easily calculated as shown above.