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Scaling dynamics of seismic activity fluctuations

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Abstract – We study the dynamics of the seismic activity in Mexico within a framework of dynamic scaling approach to time series fluctuations, recently suggested by Balankin (*Phys. Rev. E*, **76** (2007) 056120). We found that the relative seismic activity and the long-sampled fluctuations of seismic activity both display a self-affine invariance within a wide range of consecutive seismic evens. Furthermore, we found that the long-sampled fluctuations of seismic activity obey the dynamic scaling ansatz analogous to the Family-Vicsek dynamic scaling ansatz in the theory of kinetic roughening of moving interfaces. These findings imply that the records of recurrent seismic events possess hidden, long-term correlations associated with the scaling dynamics of seismic activity fluctuations.

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Introduction. – Seismicity is a complex spatio-temporal phenomenon obeying certain simple general laws which govern the statistics of their occurrences (for review, see [1–4]. Correlations in earthquake occurrence are prominent features of seismic dynamics investigated in many works [5–45]. Until recently, much research activity has been devoted to the study of the spatio-temporal correlations of seismic events in many parts of the world. A fundamental challenge of these studies concerns upscaling, that is, of determining the behavior of seismic activity at some large scale from those known at a lower scale. Turcotte [1] has shown that the epicenters within a seismic region follow a fractal distribution. The scaling laws for the temporal and spatial variability of the earthquakes have been obtained by the authors of [6] and Corral [7–10] which include various seismic regions with different tectonic properties. However, despite enormous efforts have been made by geologists and physicists to understand the earthquake phenomena since several decades, the dynamics of seismic activity remains poorly understood [3,4]. The essential point in the discussion about the possibility of earthquake predictions is the dependence of earthquake magnitude on the past seismicity [45]. Recently, it was noted that the occurrences of large earthquakes in California correlate with time intervals where fluctuations

in small earthquakes are suppressed relative to the long term average [46]. Further, it was demonstrated the existence of clustering in magnitude: earthquakes occur with higher probability close in time, space, and magnitude to previous events [47]. The authors of [47] have suggested a dynamical scaling relation between magnitude, time, and space distances which reproduces the complex correlation patterns observed in experimental seismic catalogs. More recently, the authors of [48] have demonstrated that the earthquakes are long-term correlated during periods of stationary seismic activity. These power law correlations show up in characteristic fluctuations in both magnitudes and interoccurrence times [48] and explain the memory in occurrence of earthquakes, early noted in [14], and the scaling form of the distribution function of the interoccurrence times in the seismic records, early suggested by Corral [8]. In this work, the fluctuations of seismic activity are analyzed within a framework of dynamic scaling approach to time series fluctuations (see ref. [49]).

Seismic data. – We used the time-series record of shallow (focus depth less or equal to 50 km) seismic events with magnitudes $M \geq 2.3$ (see fig. 1(a)) with epicenters located between the North America and the Pacific and Cocos plates [50]. The length of seismic record is of 17 years, starting from the earthquake at 03:26:55 p.m. 01/01/88 with magnitude $M(t_{i=1}=0) = 4.3$ up to the

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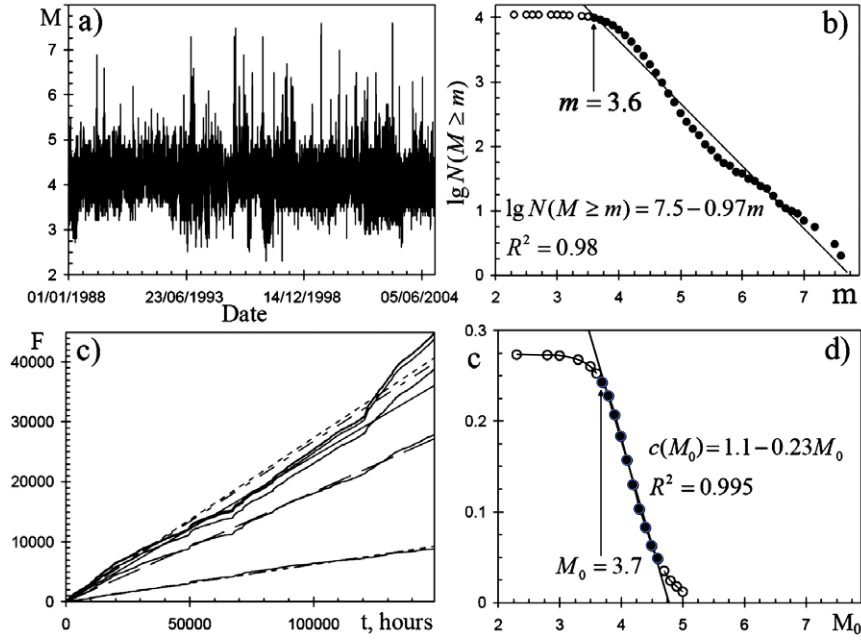


Fig. 1: (a) The magnitude time series $M(t_i) \geq 2.3$ of shallow seismic events in Mexico from 01/01/1998 to 30/12/2004 [34] and (b) the logarithm of the number of earthquakes with magnitude above or equal to m vs. m for the seismic events shown in the panel (a); solid line is the Gutenberg-Richter law. (c) Time series of accumulated seismic activity $F(t_n)$ for magnitude time series with different threshold magnitudes (from top to bottom: $M_0 = 2.3, 3.3, 3.7, 4$, and 4.5); straight lines represent the best linear fits of $F(t_n)$. (d) Mean seismic activity rate c vs. threshold magnitude (M_0); circles: experimental data, solid line: best least square fitting of data denoted by full circles.

earthquake at 06:00:15 p.m. 30/12/2004 with magnitude $M(t_{i=N} = 137991.302 \text{ hours}) = 3.8$ [51]. The total number of seismic events, $N(M_0)$, in the time series $M(t_i) \geq M_0$ depends on the threshold magnitude M_0 . Specifically, $N(M_0 = 2.3) = 11053$, while $N(M_0 = 4.6) = 1385$. We found that the seismic record shown in fig. 1(a) obeys the Gutenberg-Richter law $\log_{10} N(M \geq m) = a - bm$ [52] for seismic events with magnitude $M \geq M_{GR} = 3.6$ only (see fig. 1(b)). This may indicate that some events with magnitudes $M < 3.6$ were not registered in the catalog.

Relative seismic activity. – To study the temporal variations in seismic activity, we analyzed the time series of cumulative seismicity defined as $F(t_n) = \sum_{i=1}^n M(t_i)$, where $1 \leq n \leq N(M_0)$. Figure 1(c) shows the graphs of cumulative seismicity for different threshold magnitudes. We noted that time series $F(t_n)$ fluctuate around their linear trends $F^*(t) = c(M_0)t$ defined by the linear least square fittings of $F(t_n)$. The mean seismic activity rate $c(M_0)$ linearly decreases with M_0 in the range $3.7 \leq M_0 \leq 4.6$, whereas for $M_0 < 3.7$ and $M_0 > 4.6$ the function $c(M_0)$ abruptly deviates from this behavior (see fig. 1(d)). We also noted that the time series $M(t_n) > 4$ are too short for scaling analysis. Therefore, in this work we analyzed the magnitude time series $M(t_i) \geq M_0$ with the threshold magnitudes in the range $M_{GR} = 3.6 \leq 3.7 \leq M_0 \leq 4$.

We defined the relative seismic activity as the difference $A(t_n) = F(t_n) - c(M_0)t_n$. The graphs of relative seismic activity are shown in fig. 2(a) for seismic records with

different threshold magnitude. To avoid the effect of interoccurrence times correlations on the scaling properties of relative seismic activity, the last is treated as a function of consecutive events, $A_n = A(t_n)$, presented in fig. 2(b). Accordingly, we studied the scaling behavior of the q -order structure functions, defined as [53–55]

$$G_q(\delta n) = \left(\frac{1}{N - \delta n} \sum_{n=1}^{N - \delta n} |A_{n+\delta n} - A_n|^q \right)^{1/q}, \quad (1)$$

where δn is a natural number on the interval $[1, N - 1]$ and $N = N(M_0)$ is the length of the time series $A_n (M \geq M_0)$. For time series with long-term correlations it is expected that $G_q(\delta n)$ displays a power law behavior, $G_q \propto (\delta n)^{H_q}$, where H_q is the generalized Hurst exponent [53–55]. For self-affine time series $H_q = H$ for all q , whereas for multi-affine time series, there is an infinite hierarchy of scaling exponents [54]. We found that q -order structure functions of relative seismic activity records A_n ($3.7 \leq M_0 \leq 4$) display the power law behavior with $H_q = H = 0.78 \pm 0.02$ at least for $1 \leq q \leq 10$ (see fig. 2(c),(d)). This indicates that the relative seismic activity possesses a self-affine scaling invariance within a wide range of consecutive seismic events. We also noted that the scaling exponent H coincides with the scaling exponent $\alpha = 0.8$ found in [48] by the detrended fluctuation analysis of the magnitude records within the periods of stationary seismic activity in Northern and Southern California.

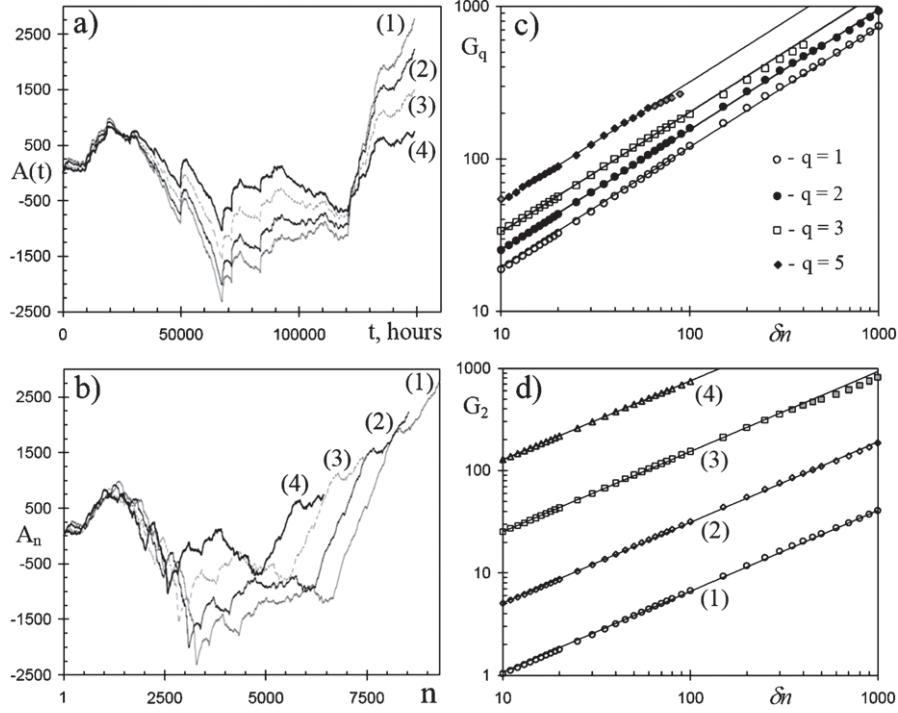


Fig. 2: (a) Time series of relative seismic activity $A(t)$ with threshold magnitude ($M_0 = 3.7$ (1), 3.8 (2), 3.9 (3), 4 (4)) and (b) the corresponding records A_n as functions of consecutive seismic events n (notice that for all records A_n the initial point $n = 1$ corresponds to $t_1 = 0$ for 03:26:55 p.m. 01/01/88). (c), (d) Log-log plots of: (c) G_q in arbitrary units *vs.* window size δn for relative seismic activity with threshold magnitude $M_0 = 3.8$ and (d) G_2 in arbitrary units *vs.* window size δn for relative seismic activity with threshold magnitude $M_0 = 3.7$ (1), 3.8 (2), 3.9 (3), and 4 (4). Notice that the graphs in the panels (c) and (d) are shifted along the y -axis for clarity.

Dynamics of seismic activity fluctuations. – In spite of the dynamic scaling approach [49], we studied the seismic activity fluctuations defined as the time series of the standard deviations of A_n with the sampling interval Δ , *i.e.*,

$$h(n, \Delta) = \sqrt{\frac{1}{\Delta} \sum_{k=n-\Delta+1}^n (A_k - \langle A_k \rangle)^2}, \quad (2)$$

where $\Delta \leq n \leq N$ and $\langle \dots \rangle$ denotes the average over the sampling interval Δ .

The essential point of the dynamic scaling approach is to treat the dynamic of time series fluctuations as a kinetic roughening of moving interface $h(n, \Delta)$ [49,56]. In the theory of kinetic roughening (see [57]) it is expected that the mean plane of interface $h(x, t)$ with the spatial extent $0 \leq x \leq L$ moves as

$$\bar{h} = \langle h(x, t) \rangle_L \propto t^{\nu+1}, \quad (3)$$

where $\langle \dots \rangle_L$ denotes the spatial average and $-1 < \nu \leq 0$ is the velocity scaling exponent (see [49,57,58]). The dynamic of interface roughening can be characterized by the q -order structure functions $G_q(t, \delta x)$ which generally obey a dynamic scaling behavior characterized by two or more independent scaling exponents (see [57–61]). Admissible forms for the dynamic scaling ansatz of interface

roughening were discussed in [61]. The knowledge of the characteristic scaling exponents permits to construct the kinetic equation which reproduces the observed dynamics of kinetic roughening (see [57]).

In the case of a self-affine time series, such as relative seismic activity records $A_n(M_0)$ shown in fig. 2(b), from the definition of the Hurst exponent $0 \leq H \leq 1$ (see [57]) immediately follows that the averages of fluctuation records (2) over $N(M_0)$ consecutive events behave, as $\bar{h}(\Delta) = \langle h(n, \Delta) \rangle_N \propto \Delta^H$ (compare this scaling with eq. (3)). So, we can visualize the seismic activity fluctuations as a moving interface $h(n, \Delta)$, where n plays the role of spatial variable, while the sampling interval Δ plays the role of time (see figs. 3(a)–(c)), such that the mean plane of interface advances with the velocity scaling exponent $\nu = H - 1 = 0.2$.

To characterize the dynamic of seismic activity fluctuations, first of all we studied the scaling behavior of the structure function $G_2(\delta n, \Delta)$ for time series $h(n, \Delta)$ with different sampling intervals Δ . In this way, we found that the structure functions of fluctuation records with short sampling intervals $\Delta < \Delta_C = 100 \pm 10$ obey the power law scaling behavior $G_2 \propto (\delta n)^{\zeta_2}$ with the Δ -dependent scaling exponent $\zeta_2(\Delta)$, whereas all fluctuations records with $\Delta > \Delta_C$ are characterized by the same scaling exponent $\zeta_2 = 0.96 \pm 0.01$ (see fig. 4(a)). We also found that the

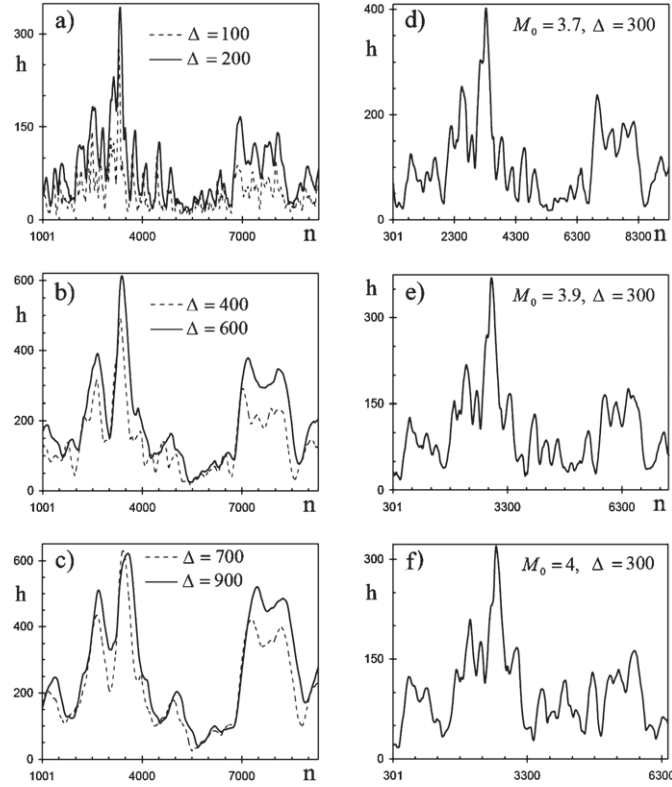


Fig. 3: (a)–(f) Records of seismic activity fluctuations (2) of magnitude time series: (a)–(c) with $M_0 = 3.8$ for different sampling intervals Δ and (d)–(f) with the sampling interval $\Delta = 300$ for different threshold magnitude M_0 . Notice that all graphs in panels (a)–(c) start from $n = 1001$ corresponding to $t_{1001} = 14645.355$ hours (03:39:26 p.m. 05/06/89), while for graphs of $h(n, \Delta = 300)$ shown in panels (d)–(f) the initial point $n = 301$ corresponds to different dates.

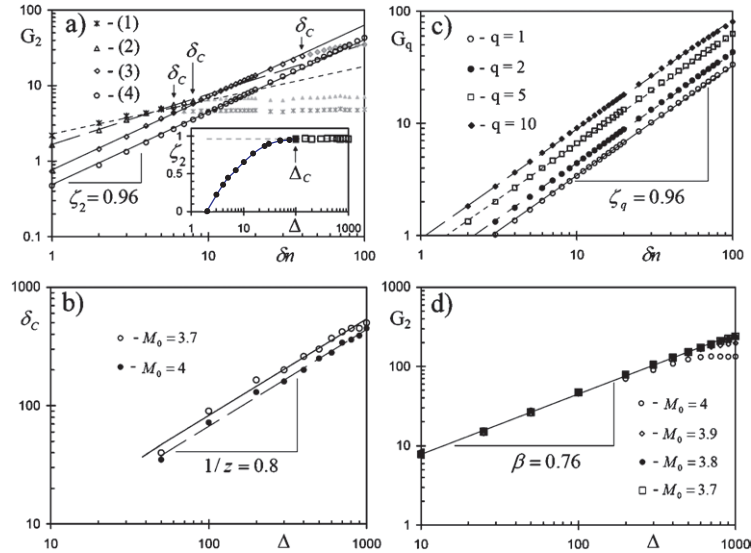


Fig. 4: (a) Log-log plots of G_2 in arbitrary units *vs.* window size δn for fluctuations of seismic activity ($M_0 = 3.7$) with different sampling intervals: $\Delta = 5$ (1), $\Delta = 10$ (2), $\Delta = 100$ (3), and $\Delta = 1000$ (4); notice that data with $\delta n > \delta_C$ (grey symbols) are excluded from power-law fittings represented by straight lines. Insert shows the graph of ζ_2 *vs.* Δ . (b) Log-log plot of correlation interval length δ_C *vs.* sampling interval size Δ for the fluctuations of seismic activity records with the threshold magnitudes $M_0 = 3.7$ and $M_0 = 4$. (c) Log-log plots of G_q in arbitrary units *vs.* window size δn for fluctuations of seismic activity ($M_0 = 3.7$) with the sampling interval $\Delta = 1000$ for different q ; and (d) log-log plots of $G_2(\delta n \gg \delta_C, \Delta)$ in arbitrary units *vs.* sampling interval length Δ for the fluctuations of seismic activity with different threshold magnitude M_0 . Notice that the graphs in the panels (c) and (d) are shifted along the y -axis for clarity.

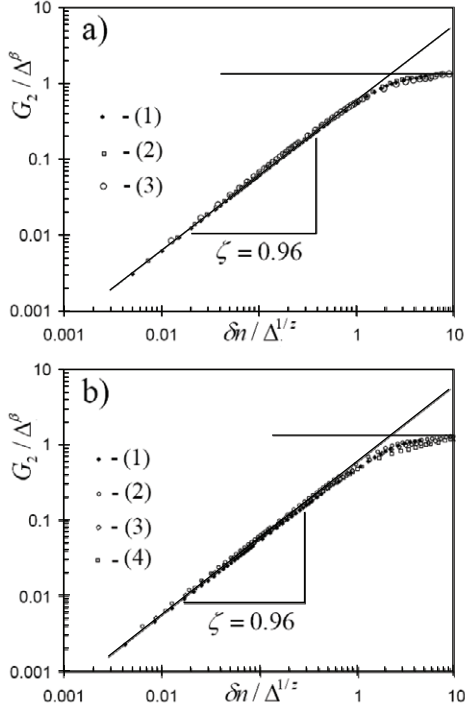


Fig. 5: Data collapse according to dynamic scaling ansatz in the coordinates G_2/Δ^β vs. $\delta n/\Delta^{1/z}$ for the fluctuations of seismic activity with: (a) $M_0 = 3.7$ and $\Delta = 800$ (1), 500 (2), and 250 (3); (b) $M_0 = 3.7$, $\Delta = 1000$ (1), $M_0 = 3.9$, $\Delta = 400$ (2), and $M_0 = 4$, $\Delta = 300$ (3).

self-affine correlation length δ_C (see fig. 4(a)) displays the power law scaling behavior

$$\delta_C \propto \Delta^{1/z_2} \quad (4)$$

with the dynamic scaling exponent $z_2 = 1.25 \pm 0.05 = z$ (see fig. 4(b)); notice the slight dependence of δ_C on the threshold magnitude M_0 , which may be attributed to the decrease of time series length $N(M_0)$ with increasing M_0 .

Further, we found that the long-sampled fluctuations ($\Delta \geq \Delta_C = 100 \pm 10$) are self-affine, *i.e.*, $G_q \propto (\delta n)^{\zeta_q}$ with $\zeta_q = 0.96 \pm 0.01 = \zeta$ at least for $1 \leq q \leq 10$ (see fig. 4(c)). Moreover, the values of $G_2(\Delta, \delta n \gg \delta_C)$ are found to be independent of M_0 and scale with the sampling interval as $G_2(\Delta, \delta n \gg \delta_C) \propto \Delta^\beta$ (see fig. 4(d)), where the fluctuation growth exponent $\beta = 0.76 \pm 0.02$ fulfills the dynamic scaling relation $z = \zeta/\beta$ (see [57]). Therefore, the data collapse for the long-sampled fluctuations of seismic records with different threshold magnitudes, shown in fig. 5 in coordinates $f = G_2/\Delta_n^\beta$ vs. $\delta n/\Delta_n^{1/z}$, suggests that the structure function of seismic activity fluctuations obeys the dynamic scaling behavior

$$G_2(\Delta, \delta n) \propto \Delta^\beta f_2(\delta n/\delta_C), \quad (5)$$

where the scaling function $f_2(y)$ behaves as

$$f_2 \propto \begin{cases} y^\zeta, & \text{if } y < 1, \\ \text{const}, & \text{if } y \gg 1. \end{cases} \quad (6)$$

In the theory of kinetic roughening an analog of the scaling behavior (4)–(6) is known as the Family-Vicsek dynamic scaling ansatz characterized by two independent scaling exponents ζ and $\beta = z\zeta$ (see ref. [57]), the values of which are determined by the dimensionality, the conservation laws, the symmetry of the kinetic equation, the range of the interactions, and the coupling of the order parameter to conserved quantities [54,62]. Hence, the knowledge of universality class allows us to understand the fundamental processes ruling the system dynamics (see [57,62]) and so, one can construct the kinetic equation governed the fluctuation dynamics (see [54,57]).

Conclusion. – In summary, our findings suggest that the long-term correlations in the seismic activity fluctuations are not restricted by the periods of stationary seismic activity and are robust with respect to the threshold magnitude used for seismic catalog filtering. This permits us to use the powerful tools of kinetic roughening theory to model the fluctuations of seismic activity using the seismic catalogs around the world.

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