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LETTERS IN FRACTURE AND MICROMECHANICS

CHARACTERISTIC LENGTH OF DAMAGE LOCALIZATION IN RUBBER

K.Y. Volokh
Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel

e-mail: cvolokh@technion.ac.il

Abstract. In this letter we calculate the characteristic length ~0.2 mm of natural rubber, which presents the width of a narrow zone where damage localizes initiating a crack. It is remarkable that our direct calculation is based on the results of the macroscopic experiments only.

Keywords: characteristic length, damage localization, rubber.

1. Introduction. It is convenient to consider a crack as an ideal separation of two adjacent material surfaces. In reality, however, the material surfaces emerge after the bulk material failure which is localized in a narrow zone of width h. This width is a special material property reflecting its internal structure and it is called the characteristic material length. We assume that the length is constant in quasi static and dynamic failure processes. No experimental data is available, unfortunately. The knowledge of the characteristic length is crucial for the finite element simulations of material failure because it allows regularizing the problem by suppressing the so-called pathological mesh sensitivity. The latter means that the characteristic length sets the size of the finite elements that should be used in the areas where failure propagates.

2. Characteristic length of natural rubber. The finding of the characteristic length is by no means trivial. It is often attempted to extract this length from the consideration of the microscopic material structure and then to fit it to the available results of macroscopic experiments. A procedure for fitting the characteristic length of DH36 steel can be found in Xue et al (2010), for example.

In the present letter we show how to directly calculate the characteristic length for natural rubber. The main idea behind the calculation is the following. Let us denote the *volumetric failure energy* by ω and the *surface failure energy* by γ. If failure localizes in a narrow zone of width h then the dissipated energy per volume $\sim h^3$ is $\sim \omega h^3$. On the other hand, the energy of the creation of two surfaces from the volume is $\sim \gamma h^2$. Equating the two energies, we get the characteristic length and, consequently, the finite element size

$$h \sim \frac{\gamma}{\omega}. \tag{1}$$

In the case of natural rubber, Rivlin and Thomas (1953) provided the value of the surface failure energy which can be used in calculations

$$\gamma = 1.3\,[\mathrm{J/cm}^2]. \tag{2}$$

Actually, there is a range of values for the surface failure energy because of the experimental inaccuracies. However, we identify the value given in (2) following Rivlin and Thomas (1953).

In order to find the volumetric failure energy, ω, we will use the concept of elasticity with energy limiters or softening hyperelasticity. For a detailed description of the concept we refer to Volokh (2007) or Volokh (2008), for example. Here we will motivate the concept qualitatively. Let us consider the interaction of two particles, which can be molecules or molecular clusters. The reference distance between them corresponds to zero interaction force and zero stored energy. The interaction passes three stages with the increase of the distance. At the first stage the force increases proportionally to the increasing distance: the linear stage. At the second stage the force-distance relationship deviates from the linear proportionality: the nonlinear stage. At the third stage the force drops with the increasing distance: the separation or failure stage. In the case of solids composed of many particles two first stages of the particle interaction are described by the linear and nonlinear theories of elasticity correspondingly where the changing distance between particles is averaged by a continuum strain measure and the energy of the particle interaction is averaged by a strain energy function. Remarkably, the third, failure, stage of the particle interaction is beyond the scope of the traditional elasticity theories. However, the failure description can still be introduced in elasticity by analogy with the failure description in the particle interaction. Indeed, the force of the pair interaction decreases with the increase of the interaction distance because the energy that can be stored during separation is limited by the constant of the bond energy. If the energy limiter exists for the pair interaction then it should exist in the multiple interactions. The latter means that we should limit the magnitude of the strain energy in order to describe material failure within the framework of elasticity. By introducing energy limiters we enforce the volumetric failure energy, ω, which is the maximum energy that can be stored and dissipated by an infinitesimal material volume. The physical meaning of ω is the average bond energy – see Section 8 of Volokh (2007) or Section 2 of Volokh (2008).

Specifically, we employ the constitutive description of natural rubber (Volokh, 2010) where the strain energy function has the form

$$\psi = \frac{\Phi}{m}\{\Gamma(\frac{1}{m}, 0) - \Gamma(\frac{1}{m}, \frac{W^m}{\Phi^m})\}, \tag{3}$$

$$W = \sum_{k=1}^{3} C_{k0}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)^k, \quad J = \lambda_1\lambda_2\lambda_3 = 1, \tag{4}$$

where λ_i are the principal stretches and there are several fitting parameters: $C_{10} = 0.298\,\text{MPa}$; $C_{20} = 0.014\,\text{MPa}$; $C_{30} = 0.00016\,\text{MPa}$; $\Phi = 82\,\text{MPa}$; $m = 10$; and $\Gamma(s, x) = \int_x^\infty t^{s-1} \exp(-t)\, dt$ is the upper incomplete gamma function.

The constitutive model of rubber includes five constants. Three constants C_{k0} describe the intact Yeoh material (4) and they were calibrated by Hamdi et al (2006). Constant m is responsible for the steepness of failure and its meaning is illustrated in Fig. 1 (Volokh, 2010; Trapper and Volokh, 2010). For $m \geq 10$ the failure occurs quite abruptly. The only crucial failure parameter Φ was fitted to the critical failure stretch $\lambda_{cr} = 7.12$ found in uniaxial tension tests by Hamdi et al (2006).

The strain energy (3) nicely fits the biaxial failure tests by Hamdi et al (2006) – Fig. 1.

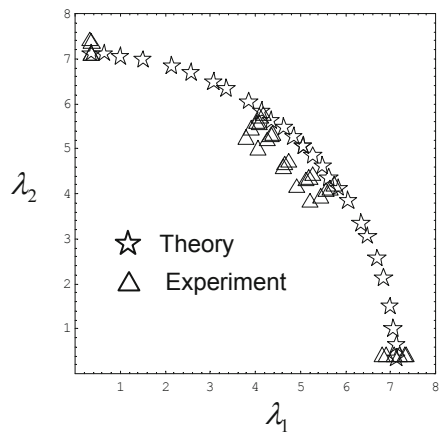

Fig. 1 Critical failure stretches in biaxial tension for natural rubber.

In the case of the local volumetric failure, i.e. rupture of the material bonds within the representative volume, the strain energy approaches its asymptotic value which can be called the energy limiter and interpreted as the volumetric energy dissipated during failure

$$\omega = \psi(W \to \infty) = \frac{\Phi}{m}\Gamma(\frac{1}{m}, 0)\,. \tag{5}$$

Substituting $m = 10$ and $\Phi = 82\,\text{MPa}$ in (5) we get

$$\omega = 78\,[\text{J/cm}^3]\,. \tag{6}$$

Substituting (2) and (6) in (1) we get the characteristic length and the element size

$$h \approx 0.2\,[\text{mm}]\,. \tag{7}$$

3. Discussion. The obtained characteristic length ~0.2 mm of natural rubber can be tackled in computations based on the strain energy function (3). However, this small size is only crucial for the finite elements in the areas where the crack propagation takes place. Non-fractured areas can be approximated arbitrarily. We do not have any experimental observations of the characteristic length yet we hope that our theoretical prediction will encourage experimentalists.

Finally, we should make two remarks on the use of the characteristic length in finite element simulations. First, we note that many commercial codes consider only the surface failure energy γ as a material parameter and they fit the constitutive law to the volumetric failure energy $\omega \sim \gamma / h$ which varies with the varying element size, h. Thus, the elements of different sizes have different constitutive equations and the finite element model as a whole lives its own life rather than approximates a boundary value problem. Second, regularization procedures based on nonlocal or gradient continua theories are more reasonable than the intuitive regularizations described in the first remark. Nonlocal and gradient theories incorporate characteristic lengths in their formulations. However, the accurate numerical analysis based on these theories still requires finite elements with the size of the order of the characteristic length. Besides, the interpretation of boundary conditions for nonlocal and gradient theories is not a trivial issue.

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