Applied Mechanics and Materials Vols. 174-177 (2012) pp 2115-2118
Online available since 2012/May/14 at www.scientific.net

doi:10.4028/www.scientific.net/AMM.174-177.2115

A New Element Model of Solid Bodies

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Keywords: solid body, element model, Elasticity, Hooke's law, truss element

Abstract. Based on the generalized Hooke's law and the superposition principle, a new element model of linearly elastic and orthotropic solid bodies is presented. Furthermore, the paper provides a new method for analyzing solid bodies. This new element is a simple truss, and a solid body is considered to consist of many such truss elements, then the solid body becomes a truss structure. Thus the internal forces and displacements of the solid body can be obtained by calculating this truss structure.

Introduction

For the solution of engineering problems, we usually assuming that the constitutive model of materials is linearly elastic in order to simplify the calculation, the theory of elasticity is a mechanics for studying the stress, strain and displacement of elastic solid bodies under the action of applied forces. The classical solution is analytical method, but to solve problems is extremely limited, the general method is the finite element method, but the two methods are very complicated [1-4]. Based on the generalized Hooke's law and the superposition principle, a new element model of linearly elastic and isotropic solid bodies is presented in the literature [5]. In order to expand the scope of application, the paper will popularize the new element model to orthotropic materials.

New Element Model

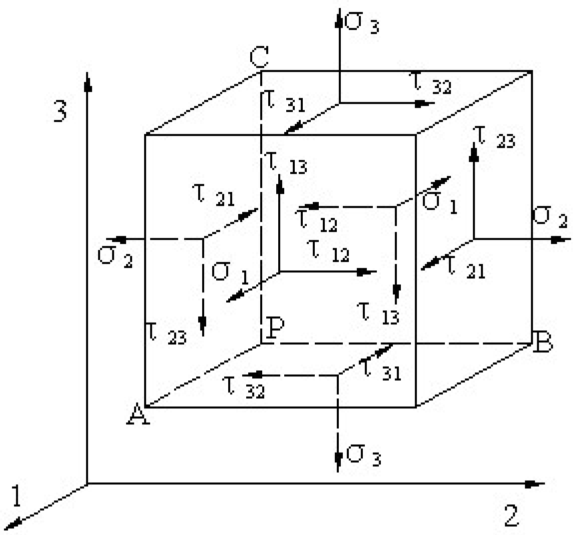

Fig. 1 Regular hexahedron element

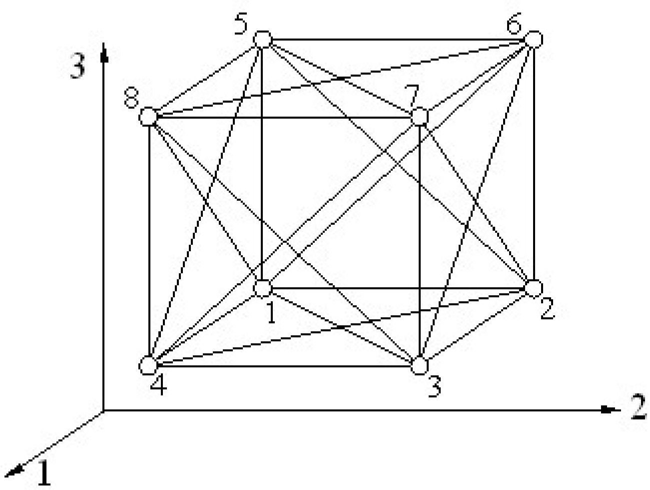

Fig. 2 Space truss element

The theory of elasticity studied elastic solid bodies from the static equilibrium, deformation and constitutive relations of the regular hexahedron element (Fig. 1). The method for setting up the new element model is completely same with the literature [5]. The new element includes the space truss element (Fig. 2) and the plane truss element (Fig. 3). The space truss element is adopted for spatial elastic problems, and the plane truss element is adopted for plane elastic problems. The Young's moduli of each bar of the space truss element are all E_1(Young's modulus in the 1-direction), the space truss element (Fig. 2) and the regular hexahedron element (Fig. 1) are the same in shape and dimension, and the deformations of the two elements under equivalent external forces are the same. Moreover, the deformation of the regular hexahedron element under an external force obeys the generalized Hooke's law. According to this condition, the cross-sectional area of each bar of the space truss element can be determined, and then the space truss element model is set up. The methods for setting up the plane truss element model and the space truss element model are similar.

Space Truss Element

The space truss element (Fig. 2) consists of 24 bars, it is a symmetric model, and the moduli of elasticity of all bars are all E_1. The bars including six types with different cross-sectional areas:

$A_1 = A_{14} = A_{23} = A_{67} = A_{58}$, $A_2 = A_{12} = A_{56} = A_{87} = A_{43}$, $A_3 = A_{15} = A_{48} = A_{37} = A_{26}$,

$A_4 = A_{13} = A_{24} = A_{57} = A_{68}$, $A_5 = A_{18} = A_{54} = A_{27} = A_{63}$, $A_6 = A_{16} = A_{25} = A_{47} = A_{38}$.

where, A =cross-sectional area of bar, for example, A_{58} is the cross-sectional area of bar 58, 5 and 8 being its two endpoints.

The dimensions in the 1-, 2-, and 3-directions of the space truss element (Fig. 2) are respectively:

$l_1 = l_{14} = l_{23} = l_{67} = l_{58}$, $l_2 = l_{12} = l_{56} = l_{87} = l_{43}$, $l_3 = l_{15} = l_{48} = l_{37} = l_{26}$.

where, l =length of bar, for example, l_{58} is the length of bar 58, 5 and 8 being its two endpoints.

According to the above method, when the engineering constants of material satisfy Equations (1),

$$G_{12} = \frac{(v_{12} + v_{13}v_{23}E_2/E_3)E_1}{(1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} ,$$

$$G_{31} = \frac{(v_{13} + v_{12}v_{23})E_1}{(1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} ,$$

$$G_{23} = \frac{(v_{23}E_2/E_1 + v_{12}v_{13})E_1}{(1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} \tag{1}$$

The cross-sectional area of bars can be written as

$$A_1 = \frac{l_2^2 l_3^2 (1 - v_{23}^2 E_2/E_3) - l_1^2 l_2^2 (v_{13} + v_{12}v_{23}) - l_1^2 l_3^2 (v_{12} + v_{13}v_{23}E_2/E_3)}{4 l_2 l_3 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} ,$$

$$A_2 = \frac{l_1^2 l_3^2 (1 - v_{13}^2 E_1/E_3) E_2/E_1 - l_1^2 l_2^2 (v_{23}E_2/E_1 + v_{12}v_{13}) - l_2^2 l_3^2 (v_{12} + v_{13}v_{23}E_2/E_3)}{4 l_1 l_3 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} ,$$

$$A_3 = \frac{l_1^2 l_2^2 (1 - v_{12}^2 E_1/E_2) E_3/E_1 - l_1^2 l_3^2 (v_{23}E_2/E_1 + v_{12}v_{13}) - l_2^2 l_3^2 (v_{13} + v_{12}v_{23})}{4 l_1 l_2 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} ,$$

$$A_4 = \frac{l_3 (l_1^2 + l_2^2)^{1.5} (v_{12} + v_{13}v_{23}E_2/E_3)}{4 l_1 l_2 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} ,$$

$$A_5 = \frac{l_2 (l_1^2 + l_3^2)^{1.5} (v_{13} + v_{12}v_{23})}{4 l_1 l_3 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} ,$$

$$A_6 = \frac{l_1 (l_2^2 + l_3^2)^{1.5} (v_{23}E_2/E_1 + v_{12}v_{13})}{4 l_2 l_3 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2v_{12}v_{13}v_{23}E_1/E_3)} \tag{2}$$

where, E_1, E_2, E_3= Young's moduli in the 1-, 2-, and 3-directions(Note that the 1-, 2-, and 3-directions shown in Fig. 1 and Fig. 2 are three principal material directions), v_{ij} =Poisson's ratio, i.e., the negative of the transverse strain in the i-direction over the strain in the j-direction when stress is applied in the j-direction, i.e., $v_{ij} = -\varepsilon_i / \varepsilon_j$ for $\sigma = \sigma_j$ and all other stresses are zero, G_{23}, G_{31}, G_{12}=shear moduli in the 2-3, 3-1, and 1-2 planes.

Plane Truss Element

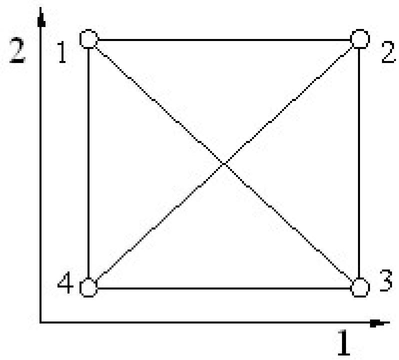

Fig. 3 Plane truss element

The plane truss element (Fig. 3) is a rectangular truss consisting of six bars, it is a symmetric model, and the moduli of elasticity of all bars are E_1. The dimensions in the 1-direction and 2-direction are $l_1 = l_{12} = l_{43}$, $l_2 = l_{14} = l_{23}$ respectively, where l is the length of bar. The bars include three types with different cross-sectional areas, i.e., $A_1 = A_{12} = A_{43}$, $A_2 = A_{14} = A_{23}$, $A_3 = A_{13} = A_{24}$, where A is the cross-sectional area of bar. Plane elastic problems include plane stress and plane strain problems, thus the plane truss element model is set up for the two problems respectively, assuming that the direction of thickness is in the 3-direction, and the thickness of solid body is h . For a plane elastic problem, the method for setting up the plane truss element model is similar with the space truss element model.

Plane Stress Problem. According to the above method, when the engineering constants of material satisfy Equations (3),

$$G_{12} = \frac{v_{12} E_1}{1 - v_{12}^2 E_1 / E_2} \tag{3}$$

The cross-sectional area of bars can be written as

$$A_1 = \frac{(l_2^2 - v_{12} l_1^2) h}{2(1 - v_{12}^2 E_1 / E_2) l_2}, \quad A_2 = \frac{(l_1^2 E_2 / E_1 - v_{12} l_2^2) h}{2(1 - v_{12}^2 E_1 / E_2) l_1}, \quad A_3 = \frac{v_{12} h (l_1^2 + l_2^2)^{1.5}}{2(1 - v_{12}^2 E_1 / E_2) l_1 l_2} \tag{4}$$

Plane Strain Problem. According to the above method, when the engineering constants of material satisfy Equations (5),

$$G_{12} = \frac{(v_{12} + v_{13} v_{23} E_2 / E_3) E_1}{1 - v_{12}^2 E_1 / E_2 - v_{13}^2 E_1 / E_3 - v_{23}^2 E_2 / E_3 - 2 v_{12} v_{13} v_{23} E_1 / E_3} \tag{5}$$

The cross-sectional area of bars can be written as

$$A_1 = \frac{l_2^2 (1 - v_{23}^2 E_2 / E_3) - l_1^2 (v_{12} + v_{13} v_{23} E_2 / E_3)}{2(1 - v_{12}^2 E_1 / E_2 - v_{13}^2 E_1 / E_3 - v_{23}^2 E_2 / E_3 - 2 v_{12} v_{13} v_{23} E_1 / E_3) l_2} h \quad ,$$

$$A_2 = \frac{l_1^2 (1 - v_{13}^2 E_1 / E_3) E_2 / E_1 - l_2^2 (v_{12} + v_{13} v_{23} E_2 / E_3)}{2(1 - v_{12}^2 E_1 / E_2 - v_{13}^2 E_1 / E_3 - v_{23}^2 E_2 / E_3 - 2 v_{12} v_{13} v_{23} E_1 / E_3) l_1} h \quad ,$$

$$A_3 = \frac{(l_1^2 + l_2^2)^{1.5} (v_{12} + v_{13} v_{23} E_2 / E_3)}{2(1 - v_{12}^2 E_1 / E_2 - v_{13}^2 E_1 / E_3 - v_{23}^2 E_2 / E_3 - 2 v_{12} v_{13} v_{23} E_1 / E_3) l_1 l_2} h \tag{6}$$

where, E_1, E_2, E_3= Young's moduli in the 1-, 2-, and 3-directions,v_{ij} =Poisson's ratio, i.e., the negative of the transverse strain in the i-direction over the strain in the j-direction when stress is applied in the j-direction, i.e., $v_{ij} = -\varepsilon_i / \varepsilon_j$ for $\sigma = \sigma_j$ and all other stresses are zero, G_{12}=shear modulus in the 1-2 plane.

Analysis

A solid body is considered to consist of many such truss elements, the neighboring truss elements being connected through their common nodes. In this way, the solid body becomes a truss structure, and its internal forces and displacements can be obtained by calculating this truss structure. The internal forces and displacements of the solid body approach the exact solutions when the dimensions of the truss element approach zero. Using the new method and the finite element method of Elasticity, some solid bodies have been analyzed, and the results of the two methods are in good agreement except in the neighborhood of the load application point and the constraint boundary. For a transversely isotropic material, it is a special subclass of orthotropic material, thus its new element model can be given easily by using the above equations of orthotropic material.

Conclusions

The paper provides a truss element model for analyzing linearly elastic and orthotropic solid bodies. On the basis, then a new method for analyzing a solid body is obtained, and it is very simple. Furthermore, it is useful for understanding the mechanical mechanism of a solid body under the action of applied forces.

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