Applied Mechanics and Materials Vols. 204-208 (2012) pp 4418-4421
Online available since 2012/Oct/26 at www.scientific.net
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doi:10.4028/www.scientific.net/AMM.204-208.4418

A New Model of Orthotropic Bodies

Jiang Ke

School of Civil Engineering and Architecture, Shaanxi University of Technology,

Hanzhong, shaanxi, 723001, China

kj5525@sina.com

Keywords: Elasticity, new model, orthotropic body, truss, generalized Hooke's law

Abstract. The solution of linearly elastic and orthotropic solid bodies is very complicated, thus a new element model for solving linearly elastic and orthotropic bodies is presented by using the generalized Hooke's law and the superposition principle. This new model is a simple truss consisting of some bars, and then the solution of orthotropic body becomes simple. Furthermore, this new model reveals the physical mechanism of a solid body under the action of applied forces.

Introduction

In the field of solid mechanics, a method of solving problems in isotropic and linear elasticity by means of a lattice analogy was presented by Hrenikoff at the University of British Colombia in Vancouver in 1941 and by McHenry at the U.S. Bureau of Reclamation in 1943. The solution of linearly elastic and orthotropic solid bodies is very complicated due to some difficulties in mathematics [1-8]. Based on the generalized Hooke's law and the superposition principle, a new element model of isotropic and linearly elastic solid bodies is presented in the literature [9]. The paper will extend the new element model to orthotropic materials.

New Model

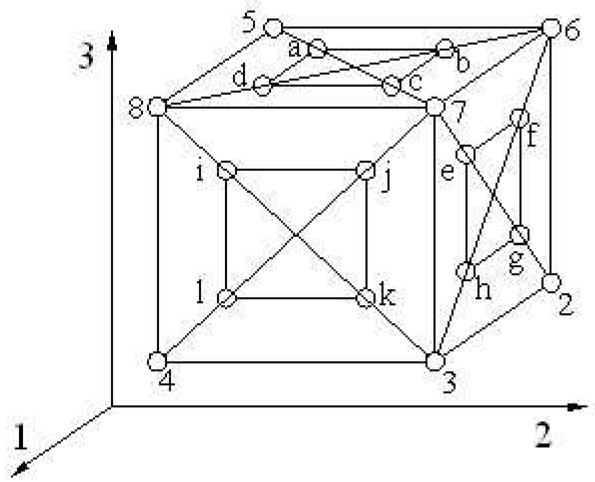

Fig. 1 Space truss element

The new element model includes the space truss element (Fig.1) and the plane truss element (Fig. 2(A)). The space truss element is adopted for spatial elastic problems, and the plane truss element is adopted for plane elastic problems. The method for setting up the new element model is same with the literature [9]. The truss element and the regular hexahedron element are the same in shape and dimension, and the deformations of the two elements under equivalent external forces are the same. Moreover, the deformation of the regular hexahedron element under an external force obeys the generalized Hooke's law. According to this condition, the cross-sectional area of each bar of the truss element can be determined, and then the truss element model is set up.

Space Truss Element

The space truss element (Fig. 1) consists of seventy-two bars, it is a symmetric model, and the moduli of elasticity of all bars are all E_1 (Young's modulus in the 1-direction). The dimensions in the 1-, 2-, and 3-directions of the space truss element are respectively: $l_1 = l_{23} = l_{67} = l_{58}$, $l_2 = l_{56} = l_{87} = l_{43}$, $l_3 = l_{48} = l_{37} = l_{26}$, where, l =length of bar, for example, l_{23} is the length of bar 23, 2 and 3 being its two endpoints. Moreover, $l_{ad} = l_{bc} = m_1 l_1$, $l_{ab} = l_{cd} = m_1 l_2$, $l_{ef} = l_{gh} = m_2 l_1$, $l_{fg} = l_{eh} = m_2 l_3$, $l_{ij} = l_{kl} = m_3 l_2$, $l_{il} = l_{jk} = m_3 l_3$, where, the ranges of values for m_1, m_2 and m_3 are all (0.01,0.99). The bars including fifteen types with different cross-sectional areas:

$A_1 = A_{23} = A_{67} = A_{58}$, $A_2 = A_{56} = A_{87} = A_{43}$, $A_3 = A_{48} = A_{37} = A_{26}$,

$A_4 = A_{57} = A_{68}$, $A_5 = A_{27} = A_{36}$, $A_6 = A_{47} = A_{38}$,

$A_7 = A_{5a} = A_{6b} = A_{7c} = A_{8d}$, $A_8 = A_{7e} = A_{6f} = A_{2g} = A_{3h}$, $A_9 = A_{8i} = A_{7j} = A_{3k} = A_{4l}$,

$A_{10} = A_{ad} = A_{bc}$, $A_{11} = A_{ab} = A_{cd}$, $A_{12} = A_{ef} = A_{gh}$,

$A_{13} = A_{fg} = A_{eh}$, $A_{14} = A_{ij} = A_{kl}$, $A_{15} = A_{il} = A_{jk}$.

where, A =cross-sectional area of bar, for example, A_{47} is the cross-sectional area of bar 47, 4 and 7 being its two endpoints.

According to the above method, the cross-sectional area of bar can be written as

$$A_1 = \frac{l_2^2 l_3^2 (1-v_{23}^2 E_2/E_3) - l_1^2 l_2^2 (v_{13} + v_{12} v_{23}) - l_1^2 l_3^2 (v_{12} + v_{13} v_{23} E_2/E_3)}{4 l_2 l_3 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2 v_{12} v_{13} v_{23} E_1/E_3)} ,$$

$$A_2 = \frac{l_1^2 l_3^2 (1-v_{13}^2 E_1/E_3) E_2/E_1 - l_1^2 l_2^2 (v_{23} E_2/E_1 + v_{12} v_{13}) - l_2^2 l_3^2 (v_{12} + v_{13} v_{23} E_2/E_3)}{4 l_1 l_3 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2 v_{12} v_{13} v_{23} E_1/E_3)} ,$$

$$A_3 = \frac{l_1^2 l_2^2 (1-v_{12}^2 E_1/E_2) E_3/E_1 - l_1^2 l_3^2 (v_{23} E_2/E_1 + v_{12} v_{13}) - l_2^2 l_3^2 (v_{13} + v_{12} v_{23})}{4 l_1 l_2 (1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2 v_{12} v_{13} v_{23} E_1/E_3)} ,$$

$$A_4 = \frac{l_3 (l_1^2 + l_2^2)^{1.5} G_{12}}{4 l_1 l_2 E_1} , \quad A_5 = \frac{l_2 (l_1^2 + l_3^2)^{1.5} G_{31}}{4 l_1 l_3 E_1} , \quad A_6 = \frac{l_1 (l_2^2 + l_3^2)^{1.5} G_{23}}{4 l_2 l_3 E_1} ,$$

$$A_7 = \left(\frac{v_{12} + v_{13} v_{23} E_2/E_3}{1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2 v_{12} v_{13} v_{23} E_1/E_3} - \frac{G_{12}}{E_1} \right) \frac{l_3 (l_1^2 + l_2^2)^{1.5}}{4 l_1 l_2} ,$$

$$A_8 = \left(\frac{v_{13} + v_{12} v_{23}}{1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2 v_{12} v_{13} v_{23} E_1/E_3} - \frac{G_{31}}{E_1} \right) \frac{l_2 (l_1^2 + l_3^2)^{1.5}}{4 l_1 l_3} ,$$

$$A_9 = \left(\frac{v_{23} E_2/E_1 + v_{12} v_{13}}{1 - v_{12}^2 E_1/E_2 - v_{13}^2 E_1/E_3 - v_{23}^2 E_2/E_3 - 2 v_{12} v_{13} v_{23} E_1/E_3} - \frac{G_{23}}{E_1} \right) \frac{l_1 (l_2^2 + l_3^2)^{1.5}}{4 l_2 l_3} ,$$

$$A_{10} = \frac{l_1 A_7}{(l_1^2 + l_2^2)^{0.5}} , \quad A_{11} = \frac{l_2 A_7}{(l_1^2 + l_2^2)^{0.5}} , \quad A_{12} = \frac{l_1 A_8}{(l_1^2 + l_3^2)^{0.5}} ,$$

$$A_{13} = \frac{l_3 A_8}{(l_1^2 + l_3^2)^{0.5}} , \quad A_{14} = \frac{l_2 A_9}{(l_2^2 + l_3^2)^{0.5}} , \quad A_{15} = \frac{l_3 A_9}{(l_2^2 + l_3^2)^{0.5}} \tag{1}$$

where, E_1, E_2, E_3= Young's moduli in the 1-, 2-, and 3-directions(Note that the 1-, 2-, and 3-directions shown in Fig. 1 are three principal material directions), v_{ij} =Poisson's ratio, i.e., the negative of the transverse strain in the i-direction over the strain in the j-direction when stress is applied in the j-direction, i.e., $v_{ij} = -\varepsilon_i / \varepsilon_j$ for $\sigma = \sigma_j$ and all other stresses are zero, G_{23}, G_{31}, G_{12}=shear moduli in the 2-3, 3-1, and 1-2 planes.

Plane Truss Element

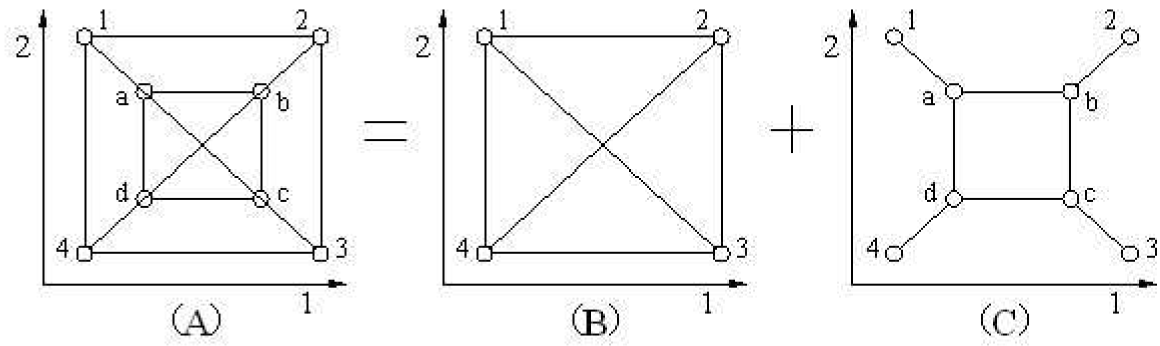

Fig. 2 Plane truss element and its components

The plane truss element (Fig. 2(A)) is a rectangular truss consisting of fourteen bars, it is a symmetric model, and its components are shown in Fig. 2(B) and Fig. 2(C). The moduli of elasticity of all bars are E_1 (Young's modulus in the 1-direction). The bars include six types with different cross-sectional areas, i.e., $A_1 = A_{12} = A_{43}$, $A_2 = A_{14} = A_{23}$, $A_3 = A_{13} = A_{24}$, $A_4 = A_{1a} = A_{2b} = A_{3c} = A_{4d}$, $A_5 = A_{ab} = A_{cd}$, $A_6 = A_{ad} = A_{bc}$, where A is the cross-sectional area of bar. The dimensions in the 1-direction and 2-direction of the plane truss element are $l_1 = l_{12} = l_{43}$, $l_2 = l_{14} = l_{23}$ respectively, where l is the length of bar. Moreover, $l_{ab} = l_{cd} = m_1 l_1$, $l_{ad} = l_{bc} = m_1 l_2$, where, the range of value for m_1 is (0.01,0.99). Plane elastic problems include plane stress and plane strain problems, thus the plane truss element model is set up for the two problems respectively, assuming that the direction of thickness is in the 3-direction, and the thickness of solid body is h .

Plane Stress Problem. According to the above method, the cross-sectional area of bar can be written as

$$A_1 = \frac{(l_2^2 - v_{12} l_1^2) h}{2(1 - v_{12}^2 E_1 / E_2) l_2} , \qquad A_2 = \frac{(l_1^2 E_2 / E_1 - v_{12} l_2^2) h}{2(1 - v_{12}^2 E_1 / E_2) l_1} ,$$

$$A_3 = \frac{G_{12} h (l_1^2 + l_2^2)^{1.5}}{2 E_1 l_1 l_2} , \qquad A_4 = \left(\frac{v_{12}}{1 - v_{12}^2 E_1 / E_2} - \frac{G_{12}}{E_1} \right) \frac{h (l_1^2 + l_2^2)^{1.5}}{2 l_1 l_2} ,$$

$$A_5 = \frac{l_1 A_4}{(l_1^2 + l_2^2)^{0.5}} , \qquad A_6 = \frac{l_2 A_4}{(l_1^2 + l_2^2)^{0.5}} \tag{2}$$

Plane Strain Problem. According to the above method, the cross-sectional area of bar can be written as

$$A_1 = \frac{l_2^2 (1 - v_{23}^2 E_2 / E_3) - l_1^2 (v_{12} + v_{13} v_{23} E_2 / E_3)}{2(1 - v_{12}^2 E_1 / E_2 - v_{13}^2 E_1 / E_3 - v_{23}^2 E_2 / E_3 - 2 v_{12} v_{13} v_{23} E_1 / E_3) l_2} h ,$$

$$A_2 = \frac{l_1^2 (1 - v_{13}^2 E_1 / E_3) E_2 / E_1 - l_2^2 (v_{12} + v_{13} v_{23} E_2 / E_3)}{2(1 - v_{12}^2 E_1 / E_2 - v_{13}^2 E_1 / E_3 - v_{23}^2 E_2 / E_3 - 2 v_{12} v_{13} v_{23} E_1 / E_3) l_1} h ,$$

$$A_3 = \frac{G_{12} (l_1^2 + l_2^2)^{1.5}}{2 E_1 l_1 l_2} h ,$$

$$A_4 = \left(\frac{v_{12} + v_{13} v_{23} E_2 / E_3}{1 - v_{12}^2 E_1 / E_2 - v_{13}^2 E_1 / E_3 - v_{23}^2 E_2 / E_3 - 2 v_{12} v_{13} v_{23} E_1 / E_3} - \frac{G_{12}}{E_1} \right) \frac{(l_1^2 + l_2^2)^{1.5}}{2 l_1 l_2} h \ ,$$

$$A_5 = \frac{l_1 A_4}{(l_1^2 + l_2^2)^{0.5}} \ , \qquad A_6 = \frac{l_2 A_4}{(l_1^2 + l_2^2)^{0.5}} \tag{3}$$

Method of solution

The method of solution by the new model is simple. A solid body is considered to consist of many such truss elements, the neighboring truss elements being connected through their common nodes. In this way, the solid body becomes a truss structure, its displacements can be obtained by calculating this truss structure, and the stresses and strains of the solid body can be given by simple method. By the way, the lengths of the twelve bars that are located at the centre of the space truss element are all cl in the literature [9], the range of value for c is (0.01, 0.99), and the cross-sectional areas of the twelve bars are all $3(4v-1)l^2/[8(1+v)(1-2v)]$.Using the new method and the finite element method of Elasticity, some solid bodies have been analyzed, and the results of the two methods are in good agreement.

Conclusions

The paper provides a truss element model for analyzing linearly elastic and orthotropic solid bodies, and it is very simple. Furthermore, it is useful for understanding the physical mechanism of a solid body under the action of applied forces.

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