Analytical methods: challenges

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Lorentz workshop Micro/Nanoscale Models for Tribology (µ/n-Tribo-Models)

Question to co-chair Jim Greenwood

- I basically want to start saying that Johnson's CM book is most of what we know even today, and it is not by chance it has 16 000 citations. I collect later a few of the top papers citing it

- I still have my personal copy of 1993 when I was an Erasmus student in Nottingham University, and I don't think I know it all.

- Maybe JimBarber has some suggestions on where we have made real progress in analytical methods? He is writing a new book in contact mechanics, but he is not here!

- I will also say that you (JimG) were fortunate to work with David Tabor and KLJ, the real fathers of contact mechanics, who always started from experiments, tried simple analytical solutions, and only as a last possibility, resorted to more complicated solutions or numerical ones.

KLJ chapters

- Non-Hertzian normal contact of elastic bodies. Small progress in many areas: asymptotic singular stress field at wedge in sliding contact (Hills), certainly some in anisotropic, layered, plates, shells. Biggest advancements probably on adhesion, on which KLJ has only 3 pages on JKR theory !!

- Normal contact of inelastic solids (core model of elasto-plastic indentation, viscoelastic cyclic indentation, etc.) –huge technical development in instrumented and nanoindentation (see highlycited papers by Oliver & Pharr)

- Tangential loading and cyclic contact – perhaps some development with Ciavarella-Jager theorem and developments, also fretting fatigue, crack analogues, notch analogues, CLNA model of Ciavarella

- Rolling contact of elastic bodies --- not much fundamentally new there, but a lot of computational work for railways, squeal (friction instabilities), RCF (rolling contact fatigue), wear, etc

- Rolling contact of inelastic bodies – shakedown, ratchetting, some development by Kapoor but perhaps KLJ least «clean» work, because ratchetting turned out to be terribly complicated and dependent on non-linear kinematic hardening over millions of cycles makes the problem ill-defined

- Calendering and lubrication – elastic-plastic rolling of strips, lubrication. I'm not an expert, certainly huge progress on additives nano-particles, etc

- Dynamic effects and impact – much work on rate-state friction law and Adams' instability, earthquake mechanics. Impact is a separate big area

- Thermoelastic contact – after big contributions by JR Barber on both static and sliding contact, whats new?

- Rough surfaces – after the famous GW, the most highly cited theory is that of Majumbdar and Bhushan (1990) (Korcak law), then comes Persson (2001) and Ciavarella et al (2000) using old Archard ideas. Persson well known gives a clear approximation in the entire range of pressure. But the real physical problems remain unanswered – too much emphasys today on nominally flat stationary self-affine fractals, very little work on «shape»

KLJ's book citing top papers

- [An improved technique for determining hardness and elastic modulus using load and displacement sensing indentation experiments](#)

- [WC Oliver](#), GM Pharr - Journal of materials research, 1992 - Cambridge Univ Press

- Abstract The indentation load-displacement behavior of six materials tested with a Berkovich indenter has been carefully documented to establish an improved method for determining hardness and elastic modulus from indentation load-displacement data. The materials

- [Cited by 16953](#)

Second should be (but did not cite book)

- [Surface energy and the contact of elastic solids](#)

- KL **Johnson**, K **Kendall**… - … of the Royal …, 1971 - rspa.royalsocietypublishing.org

- Abstract This paper discusses the influence of surface energy on the contact between elastic solids. Equations are derived for its effect upon the contact size and the force of adhesion between two lightly loaded spherical solid surfaces. The theory is supported by experiments

- [Cited by 6022](#)

Second is

- [Measurement of hardness and elastic modulus by instrumented indentation: Advances in understanding and refinements to methodology](#)
- [WC Oliver](#), GM Pharr - Journal of materials research, 2004 - Cambridge Univ Press
- Abstract The method we introduced in 1992 for measuring hardness and elastic modulus by instrumented indentation techniques has widely been adopted and used in the characterization of small-scale mechanical behavior. Since its original development, the

- [Cited by 3746](#) [Related articles](#) [All 10 versions](#) [Cite](#) [Save](#) [More](#)
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4)

- [Force measurements with the atomic force microscope: Technique, interpretation and applications](#)

- HJ Butt, B Cappella, M Kappl - Surface science reports, 2005 - Elsevier

- The atomic force microscope (AFM) is not only a tool to image the topography of solid surfaces at high resolution. It can also be used to measure force-versus-distance curves. Such curves, briefly called force curves, provide valuable information on local material

- [Cited by 2303](#)

6)

- **[BOOK]** Computational contact mechanics
- P Wriggers, TA Laursen - 2006 - Springer
- Contact mechanics has its application in many engineering problems. No one can walk without frictional contact, and no car would move for the same reason. Hence contact mechanics has, from an engineering point of view, a long history, beginning in ancient Egypt

- Cited by 2023

7)

- **[BOOK]** [Factors Affecting Nanoindentation Test Data](#)
- AC Fischer-Cripps - 2000 - Springer
- In conventional indentation tests, the area of contact between the indenter and the specimen at maximum load is usually calculated from the diameter or size of the residual impression after the load has been removed. The size of the residual impression is usually considered

- [Cited by 1622](#)

- **[BOOK]** [Three-dimensional elastic bodies in rolling contact](#)

- JJ Kalker - 2013 - books.google.com

- This book is intended for mechanicians, engineering mathematicians, and, generally for
  theoretically inclined mechanical engineers. It has its origin in my Master's Thesis (J 957),
  which I wrote under the supervision of Professor Dr. R. Timman of the Delft TH and Dr. Ir. AD

- [Atomistic mechanisms and dynamics of adhesion, nanoindentation, and fracture](#)

- U Landman, WD Luedtke, [NA Burnham](#), RJ Colton - Science, 1990 - search.proquest.com

- Atomistic Mechanisms and Dynamics of Adhesion, Nanoindentation, and Fracture UZI LANDMAN, WD LUEDTKE, NANCY A. BURNHAM, RICHARD J. COLToN Molecular dynamics simulations and atomic force micros- copy are used to investigate the atomistic mechanisms

- [Cited by 1161](#)

- On the significance of the H/E ratio in wear control: a nanocomposite coating approach to optimised tribological behaviour

- A Leyland, A Matthews - Wear, 2000 - Elsevier

- Although hardness has long been regarded as a primary material property which defines wear resistance, there is strong evidence to suggest that the elastic modulus can also have an important influence on wear behaviour. In particular, the elastic strain to failure, which is

- Cited by 1142

- **[BOOK]** [Multibody dynamics with unilateral contacts](#)

- F Pfeiffer, [C Glocker](#) - 1996 - books.google.com

- As mechanical systems become more complex so do the mathematical models and simulations used to describe the interactions of their parts. One area of multibody theory that has received a great deal of attention in recent years is the dynamics of multiple contact

- [Cited by 1055](#)

- [Scratching the surface: fundamental investigations of tribology with atomic force microscopy](#)

- [RW Carpick](#), [M Salmeron](#) - Chemical Reviews, 1997 - ACS Publications

- A few years after the invention of the scanning tunneling microscope (STM), the atomic force microscope (AFM) was developed. 1 Instead of measuring tunneling current, a new physical quantity could be investigated with atomic-scale resolution: the force between a small tip

- [Cited by 1019](#)

- **[BOOK]** Nonsmooth mechanics

- B Brogliato, B Brogliato - 1999 - Springer

- Thank you for opening the third edition of this monograph. The first edition [202] was published in 1996 in the Lecture Notes in Control and Information Sciences series (vol. 220), and the second edition [203] in 1999 in the Communications and Control Engineering

- Cited by 995

- Fractal model of elastic-plastic contact between rough surfaces
- A Majumdar, B Bhushan - ASME J. Tribol, 1991 - …
.asmedigitalcollection.asme.org
- Roughness measurements by optical interferometry and scanning tunneling microscopy on a magnetic thin-film rigid disk surface have shown that its surface is fractal in nature. This leads to a scale-dependence of statistical parameters such as rms height, slope and

- Cited by 921…………..
- More than Theory of rubber friction and **contact mechanics** BNJ **Persson** - The Journal of Chemical Physics, 2001 - aip.scitation.org Cited by 707

- [Biomechanics and biophysics of cancer cells](#)

- S Suresh - Acta Materialia, 2007 - Elsevier

- The past decade has seen substantial growth in research into how changes in the biomechanical and biophysical properties of cells and subcellular structures influence, and are influenced by, the onset and progression of human diseases. This paper presents an

- [Cited by 910](#)

- [Impact on laminated composite materials](#)

- [S Abrate](#) - Appl Mech Rev, 1991 - … .asmedigitalcollection.asme.org

- Laminated composite materials are used extensively in aerospace and other applications. With their high specific modulus, high specific strength, and the capability of being tailored for a specific application, these materials offer definite advantages compared to more

- [Cited by 884](#)

- **[BOOK]** [Impact mechanics](#)
- WJ Stronge - 2004 - books.google.com
- Impact mechanics is concerned with the reaction forces that develop during a collision and
  the dynamic response of structures to these reaction forces. The subject has a wide range of
  engineering applications, from designing sports equipment to improving the

- [Cartilage and diarthrodial joints as paradigms for hierarchical materials and structures](#)

- [VC Mow](#), A Ratcliffe, AR Poole - Biomaterials, 1992 - Elsevier

- Abstract The anatomic forms of diarthrodial joints are important structural features which
provide and limit the motions required for the joint. Typically, the length scale of topographic
variation of anatomic forms ranges from 0.5 to 15 cm. Articular cartilage is the thin layer of

- [Robotic grasping and contact: A review](#)

- [A Bicchi](#), [V Kumar](#) - … ICRA'00. IEEE International Conference on, 2000 - ieeexplore.ieee.org

- Centro "E. Piaggio University of Pisa 56126 Pisa Italy bicchi@ing.unipi.it **…** Abstract In this paper, we survey the field of robotic grasping and the work that has been done in this area over the last two decades, with a slight bias toward the develop- ment of the theoretical framework

- [Cited by 732](#)

- [Frictional characteristics of atomically thin sheets](#)

- [C Lee](#), [Q Li](#), W Kalb, [XZ Liu](#), H Berger… - …, 2010 - science.sciencemag.org

- Abstract Using friction force microscopy, we compared the nanoscale frictional characteristics of atomically thin sheets of graphene, molybdenum disulfide (MoS 2), niobium diselenide, and hexagonal boron nitride exfoliated onto a weakly adherent

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- **[HTML]** [Determination of elastic moduli of thin layers of soft material using the atomic force microscope](#)

- EK Dimitriadis, F Horkay, [J Maresca](#), [B Kachar](#)… - Biophysical journal, 2002 - Elsevier

- We address three problems that limit the use of the atomic force microscope when measuring elastic moduli of soft materials at microscopic scales. The first concerns the use of sharp cantilever tips, which typically induce local strains that far exceed the linear material

- [Cited by 681](#)

- [Discrete particle simulation of particulate systems: a review of major applications and findings](#)

- HP Zhu, [ZY Zhou](#), [RY Yang](#), [AB Yu](#) - Chemical Engineering Science, 2008 - Elsevier

- Understanding and modelling the dynamic behaviour of particulate systems has been a major research focus worldwide for many years. Discrete particle simulation plays an important role in this area. This technique can provide dynamic information, such as the

- [Cited by 660](#)

- [Indentation of ceramics with spheres: a century after Hertz](#)

- [BR Lawn](#) - Journal of the American Ceramic Society, 1998 - Wiley Online Library

- Abstract In this article we review the nature and mechanics of damage induced in ceramics by spherical indenters, from the classical studies of Hertz over a century ago to the present day. Basic descriptions of continuum elastic and elastic–plastic contact stress fields are first

- [Mechanics of fretting fatigue](#)

- DA Hills - Wear, 1994 - Elsevier

- Abstract Several aspects of the mechanics of cracks originating at sites of fretting are considered. It is argued that the problem may be distilled into three separate parts: the contact problem itself in full or partial slip, the initiation of a crack from a surface suffering

- [Elastic–plastic contact analysis of a sphere and a rigid flat](#)

- I Etsion - ASME J. Appl. Mech, 2002 - …
  .asmedigitalcollection.asme.org

- The elastic-plastic contact of a sphere and a flat is a fundamental problem in contact mechanics. It is applicable, for example, in problems such as particle handling 1, or sealing, friction, wear, and thermal and electrical conductivity between contacting rough surfaces.

- [Cited by 604](#)

- [A simple predictive model for spherical indentation](#)

- JS Field, [MV Swain](#) - Journal of Materials Research, 1993 - Cambridge Univ Press

- Abstract A simple model is described with which the entire force versus penetration behavior of indentation with a sphere, during loading and unloading, may be simulated from knowledge of the four test material parameters, Young's modulus, Poisson's ratio, flow stress

- [Nano-indentation of polymeric surfaces](#)

- BJ Briscoe, [L Fiori](#), E Pelillo - Journal of Physics D: Applied …, 1998 - iopscience.iop.org

- Abstract This paper presents results of normal hardness, plasticity index and elastic modulus for a selection of organic polymers (a poly (methylmethacrylate), PMMA, a poly (styrene), PS, a poly (carbonate), PC, and an ultra-high molecular weight poly (ethylene), UHMWPE)

- [Cited by 582](#)

- [On the nature of surface roughness with application to contact mechanics, sealing, rubber friction and adhesion](#)

- [BNJ Persson](#), O Albohr, U Tartaglino… - Journal of Physics: …, 2004 - iopscience.iop.org

- Abstract Surface roughness has a huge impact on many important phenomena. The most important property of rough surfaces is the surface roughness power spectrum C (q). We present surface roughness power spectra of many surfaces of practical importance, obtained

- [Cited by 573](#)

- [Direct observation of frictional contacts: New insights for state-dependent properties](#)

- JH Dieterich, [BD Kilgore](#) - Pure and Applied Geophysics, 1994 - Springer

- Rocks and many other materials display a rather complicated, but characteristic, dependence of friction on sliding history. These effects are well-described by empirical rate-and state-dependent constitutive formulations which have been utilized for analysis of fault

- [Cited by 525](#)

- **[BOOK]** [Nonlinear computational structural mechanics: new approaches and non-incremental methods of calculation](#)

- [P Ladevèze](#) - 2012 - books.google.com

- Mechanical Engineering, an engineering discipline borne of the needs of the in dustrial revolution, is once again asked to do its substantial share in the call for industrial renewal. The general call is urgent as we face profound issues of pro ductivity and competitiveness

- [Cited by 530](#)

- **[BOOK]** [Multi-level methods in lubrication](#)

- CH Venner, AA Lubrecht - 2000 - books.google.com

- Efficient numerical solution of realistic and, therefore, complex equation systems occupies many researchers in many disciplines. For various reasons, but mainly in order to approximate reality, a very large number of unknowns are needed. Using classical

- [Cited by 515](#)

- [Literature survey of contact dynamics modelling](#)

- G Gilardi, I Sharf - Mechanism and machine theory, 2002 - Elsevier

- Impact is a complex phenomenon that occurs when two or more bodies undergo a collision. This phenomenon is important in many different areas—machine design, robotics, multi-body analysis are just a few examples. The purpose of this manuscript is to provide an

- [Cited by 504](#)

- [Graded materials for resistance to contact deformation and damage](#)

- S Suresh - Science, 2001 - science.sciencemag.org

- Abstract The mechanical response of materials with spatial gradients in composition and structure is of considerable interest in disciplines as diverse as tribology, geology, optoelectronics, biomechanics, fracture mechanics, and nanotechnology. The damage and

- [An adhesion map for the contact of elastic spheres](#)

- KL Johnson, JA Greenwood - Journal of colloid and interface science, 1997 - Elsevier

- Several continuum mechanics models of the adhesion between elastic spheres have found application to compliant materials such as rubber and to fine particles in the air or in colloidal suspension. More recently they are being used in connection with experimental

- [Nanometre-scale rolling and sliding of carbon nanotubes](#)

- MR Falvo, RM Taylor Ii, A Helser, V Chi, FP Brooks Jr… - Nature, 1999 - nature.com

- Abstract Understanding the relative motion of objects in contact is essential for controlling macroscopic lubrication and adhesion, for comprehending biological macromolecular interfaces, and for developing submicrometre-scale electromechanical devices 1, 2. An

- [Atomic force microscopy probing of cell elasticity](#)

- TG Kuznetsova, [MN Starodubtseva](#), NI Yegorenkov… - Micron, 2007 - Elsevier

- Atomic force microscopy (AFM) has recently provided the great progress in the study of micro-and nanostructures including living cells and cell organelles. Modern AFM techniques allow solving a number of problems of cell biomechanics due to simultaneous evaluation of

- [Cited by 444](#)

- [A theoretical model for the stick/bounce behaviour of adhesive, elastic-plastic spheres](#)

- C Thornton, [Z Ning](#) - Powder technology, 1998 - Elsevier

- The paper considers the normal impact of elastic-perfectly plastic spheres, with and without interface adhesion, and presents an analytical solution for the coefficient of restitution which is expressed in terms of the impact velocity, the critical sticking velocity and the velocity

- [Cited by 437](#)

- [Adhesion of elastic spheres](#)

- JA Greenwood - Proceedings of the Royal Society of …, 1997 - rspa.royalsocietypublishing.org

- Abstract Bradley (1932) showed that if two rigid spheres of radii R_1 and R_2 are placed in contact, they will adhere with a force $2\pi\Delta R\gamma$, where R is the equivalent radius $R_1 R_1/(R_1 + R_2)$ and $\Delta\gamma$ is the surface energy or 'work of adhesion'(equal to $\gamma_1 + \gamma_2 - \gamma_{12}$). Subsequently

- [A general equation for fitting contact area and friction vs load measurements](#)

- [RW Carpick](#), [DF Ogletree](#), [M Salmeron](#) - Journal of Colloid and Interface ..., 1999 - Elsevier

- The variation of contact area with load between adhesive elastic spheres depends upon the effective range of attractive surface forces. Relatively simple forms to describe limiting cases exist, but the general intermediate case requires a more complex analysis. Maugis, using a

- [Cited by 360](#)

Where are the challenges then in analytical methods ……… for the next 5-10 years??

- More indentation?
- More roughness? Using asperity models?
- More friction instabilities?
- More thermal effects?

Big challenge is what can we do analytically!

- The emphasis on computation is sometimes misleading: in adhesion, for example, with just nano-to-micro scales which can be covered, paradoxical results emerge (like Pastewka – Robbins conclusion that stickiness does not depend on rms amplitude of roughness, contrary to all existing evidence!), because we are still limited in scales

- Of course some relatively new phenomena can be expected to be understood even analytically

- Old analytical theories can find always new applications