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Applications of a New Element Model of Solid Bodies in Plasticity

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Abstract. Based on the deformations under the equivalent external forces are the same, a new element model of elastic perfectly-plastic and orthotropic solid bodies is presented, and the elastic perfectly-plastic and isotropic materials is a special subclass. Furthermore, the method for determining the displacements, the stresses and the strains of a body under the action of applied forces has been given. A new method for predicting the engineering elastic constants of a fiber-reinforced composite material is also presented. It can be found that the precision by using the new element method is good, and the limit analysis has the highest precision in all methods.

Introduction

At present, the solving methods for solid mechanics problems include the analytical method, the finite element method, etc. But these methods are all very complex [1-4]. Based on the generalized Hooke's law and the superposition principle, a new element model of isotropic and linearly elastic solid bodies is presented in the literature [5]. Further, the author proposed a new element model of linearly elastic and orthotropic solid bodies in the literatures [6, 7]. The paper will extend the new element model to elastic perfectly-plastic and orthotropic materials, and the elastic perfectly-plastic and isotropic materials is a special subclass. For solving the strains and the stresses of a solid body under the action of applied forces by the new element model, two methods have been proposed.

New Element Model

The new element model of an elastic perfectly-plastic and orthotropic solid body is same with the literature [7], but there are two differences. First, the dimensions in each coordinate axis direction of the new element model must be equal. Second, the each bar in the new element model is given a yield stress ("The yield stresses in tension and compression" of a bar can also adopt the different values.). Based on the deformations in uniaxial tension (or in uniaxial compression) under the equivalent external forces are the same, the yield stresses in tension (or in compression) of bars are given. Plasticity includes spatial problems, plane stress and plane strain problems, thus the new element model is established for the three problems respectively. For orthotropic materials, assume that the yield stresses in uniaxial tension (or in uniaxial compression) in the 1, 2, and 3 principal material directions are σ_{01}, σ_{02}, σ_{03} respectively (for isotropic materials, $\sigma_{01}=\sigma_{02}=\sigma_{03}=\sigma_0$), and the yield stresses in uniaxial tension and compression can also adopt the different values.

Spatial Problem. If the engineering elastic constants and the yield stresses of a material satisfy the three conditions: $(1-v_{21})\sigma_{01}=(E_1/E_2-v_{21})\sigma_{02}$, $(1-v_{31})\sigma_{01}=(E_1/E_3-v_{31})\sigma_{03}$, $(1-v_{32})\sigma_{02}=(E_2/E_3-v_{32})\sigma_{03}$ (For an isotropic material, it is clear that the three conditions can be satisfied.), then in the new element model, the yield stresses are σ_{01} for the four bars that the cross-sectional areas are A_1, the yield stresses are $\sigma_{02}E_1/E_2$ for the four bars that the cross-sectional areas are A_2, the yield stresses are $\sigma_{03}E_1/E_3$ for the four bars that the cross-sectional areas are A_3, the yield stresses are $0.5(1-v_{21})\sigma_{01}$ for the twenty bars that the cross-sectional areas are A_4, A_7, A_{10}, A_{11}, the yield stresses are $0.5(E_1/E_3-v_{31})\sigma_{03}$ for the twenty bars that the cross-sectional areas are A_5, A_8, A_{12}, A_{13}, the yield stresses are $0.5E_1/E_2(1-v_{32})\sigma_{02}$ for the twenty bars that the cross-sectional areas are A_6, A_9, A_{14}, A_{15} (If the space truss element model of an isotropic material in the literature [5] is used, then in the new element model, the yield stresses are σ_0 for the twelve bars that the cross-sectional areas are A_1, A_2, A_3, the yield stresses are $0.5(1-v)\sigma_0$ for the twelve bars that the cross-sectional areas are A_4, A_5, A_6, the yield stresses are $(1-2v)\sigma_0/3$ for the other twenty bars.).

Plane Stress Problem. If the engineering elastic constants and the yield stresses of a material satisfy the condition: $(1-v_{21})\sigma_{01} = (E_1/E_2 - v_{21})\sigma_{02}$, then in the new element model, the yield stresses are σ_{01} for the two bars that the cross-sectional areas are A_1, the yield stresses are $\sigma_{02}E_1/E_2$ for the two bars that the cross-sectional areas are A_2, the yield stresses are $0.5(1-v_{21})\sigma_{01}$ for the ten bars that the cross-sectional areas are A_3, A_4, A_5, A_6.

Plane Strain Problem. If the engineering elastic constants and the yield stresses of a material satisfy the condition: $(1-v_{21}-v_{13}v_{31}-v_{23}v_{31})\sigma_{01}/(1-v_{13}v_{31}) = (E_1/E_2 - v_{21} - v_{23}^2 E_1/E_3 - v_{23}v_{31})\sigma_{02}/(1-v_{23}v_{32})$, then in the new element model, the yield stresses are σ_{01} for the two bars that the cross-sectional areas are A_1, the yield stresses are $\sigma_{02}E_1/E_2$ for the two bars that the cross-sectional areas are A_2, the yield stresses are $0.5(1-v_{21}-v_{13}v_{31}-v_{23}v_{31})\sigma_{01}/(1-v_{13}v_{31})$ for the ten bars that the cross-sectional areas are A_3, A_4, A_5, A_6.

Assuming that "the yield stresses in uniaxial tension and compression" of an isotropic material are equal, if the yield state in pure shear of a new element model occurs, then the corresponding shear yield stress of the material is $\tau_0 = 0.5(1-v)\sigma_0/(1+v)$ for spatial problems and plane stress problems, and the corresponding shear yield stress of the material is $\tau_0 = (0.5-v)\sigma_0/(1-v^2)$ for plane strain problems.

For an isotropic material, the above new element model can be used. Furthermore, assume that the yield stress in uniaxial tension is σ_{t0}, the yield stress in pure shear is $\tau_0 = k\sigma_{t0}$, now the above new element model is extended as follows. For a spatial problem, the yield stresses in tension of bars are σ_{t0}, and the yield stresses in compression of bars are infinite for the twelve bars that the cross-sectional areas are A_1, A_2, A_3 ; the yield stresses in tension of bars are $0.5(1-v)\sigma_{t0}$, and the yield stresses in compression of bars are $(2k+2kv-0.5+0.5v)\sigma_{t0}$ for the sixty bars that the cross-sectional areas are $A_4 \sim A_{15}$. For a plane stress problem, the yield stresses in tension of bars are σ_{t0}, and the yield stresses in compression of bars are infinite for the four bars that the cross-sectional areas are A_1, A_2; the yield stresses in tension of bars are $0.5(1-v)\sigma_{t0}$, and the yield stresses in compression of bars are $(2k+2kv-0.5+0.5v)\sigma_{t0}$ for the ten bars that the cross-sectional areas are $A_3 \sim A_6$. For a plane strain problem, the yield stresses in tension of bars are σ_{t0}, and the yield stresses in compression of bars are infinite for the four bars that the cross-sectional areas are A_1, A_2; the yield stresses in tension of bars are $(0.5-v)\sigma_{t0}/(1-v)$, and the yield stresses in compression of bars are $[2k+2kv-(0.5-v)/(1-v)]\sigma_{t0}$ for the ten bars that the cross-sectional areas are $A_3 \sim A_6$.

Calculation of Stress and Strain

The nodal displacements can be obtained by calculating this truss structure that consists of the new elements, thus the node displacement is used for the displacement of the corresponding point in the solid body, and how to calculate the strain and the stress of each point in the solid body under the action of applied forces, two simple methods are given below. In Fig.1(A) there is four new elements, which is composed of $4\times14=56$ bars.

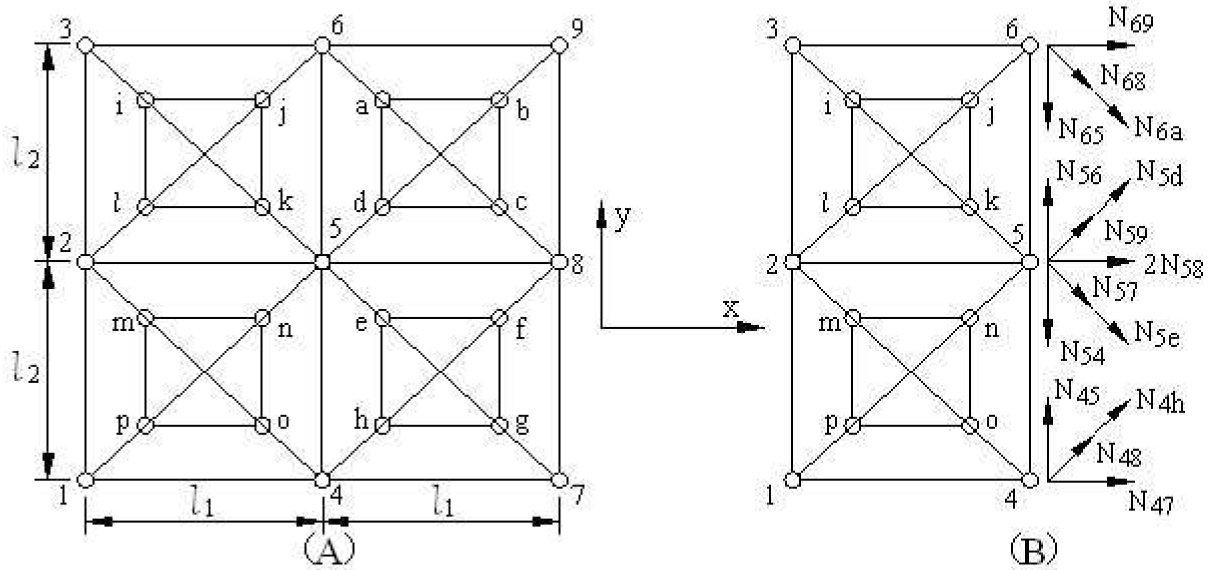

Fig. 1 Analysis of strain and stress at a point: (A) four new elements around node 5; (B) stresses from axial forces of bars

Method 1: Only it is applicable that the material is in elastic phase. The linear strains of bars can be obtained by calculating this truss structure, thus the linear strain is used for the linear strain in the corresponding direction at the point of the solid body. The shear strain at a point of the solid body is determined according to the relation between shear strain and linear strain at the point. The average method around a node can be used in order to improve the precision of the strain solutions, i.e., for the new elements around a same node, the normal strains (or shear strains) at the same node are averaged as the normal strain (or shear strain) at the node. Moreover, the strains can be also given from the nodal displacements. Finally, the stresses can be given by the generalized Hooke's law stress-strain relationships.

The relation [3] between shear strain and linear strain at a point can be given, for plane problems,

$$\varepsilon_{PN} = l^2\varepsilon_1 + m^2\varepsilon_2 + lm\gamma_{12} \tag{1}$$

For spatial problems,

$$\varepsilon_{PN} = l^2\varepsilon_1 + m^2\varepsilon_2 + n^2\varepsilon_3 + mn\gamma_{23} + nl\gamma_{31} + lm\gamma_{12} \tag{2}$$

where, ε_{PN} is the linear strain at point P along PN direction, $\varepsilon_1,\varepsilon_2,\varepsilon_3,\gamma_{23},\gamma_{31},\gamma_{12}$ are three normal strains and three shear strains, l,m,n is the cosine of the angle between line PN and the 1,2,3 coordinate axes respectively.

For solving a plane problem, computing the normal strain and the shear strain at node 5 (Fig.1) by the new element model are given as follows:

$\varepsilon_{5x} = (\varepsilon_{52} + \varepsilon_{58})/2$, $\varepsilon_{5y} = (\varepsilon_{56} + \varepsilon_{54})/2$, $\gamma_5 = (\gamma_{5(1)} + \gamma_{5(2)} + \gamma_{5(3)} + \gamma_{5(4)})/4$,

where, $\gamma_{5(1)} = (\varepsilon_{59} - l_{59}^2\varepsilon_{58} - m_{59}^2\varepsilon_{56})/(l_{59}m_{59})$, $\gamma_{5(2)} = (\varepsilon_{53} - l_{53}^2\varepsilon_{52} - m_{53}^2\varepsilon_{56})/(l_{53}m_{53})$,

$\gamma_{5(3)} = (\varepsilon_{51} - l_{51}^2\varepsilon_{52} - m_{51}^2\varepsilon_{54})/(l_{51}m_{51})$, $\gamma_{5(4)} = (\varepsilon_{57} - l_{57}^2\varepsilon_{58} - m_{57}^2\varepsilon_{54})/(l_{57}m_{57})$,

ε_{5x} is the normal strain in the x-direction at node 5, γ_5 is the shear strain at node 5, ε_{52} is the linear strain of bar 52, l_{59} is the cosine of the angle between bar 59 and the x coordinate axis.

Method 2: It is applicable that the material is in elastic or plastic phases. The axial forces of bars can be obtained by calculating this truss structure. At a point of the solid body, in order to get the normal stress (or shear stress) acting on a plane, first, according to the axial forces of these bars that located in the one side of the plane and intersected at the point, compute the projection of these axial forces in the direction of the normal (or the tangent) to the plane, then divided by the action area at the point, thus the normal stress (or shear stress) at the point can be obtained. Finally, the calculation of the strains is same with the "Method 1". If the material is in elastic phase, then the strains can be also given by the generalized Hooke's law strain-stress relationships.

For solving a plane problem, computing the normal stress and the shear stress at a node (Fig.1) by the new element model are given as follows:

$\sigma_{5x} = (2N_{58,x} + N_{59,x} + N_{5d,x} + N_{57,x} + N_{5e,x})/(l_2 h)$, $\tau_5 = (\tau_{5xy} + \tau_{5yx})/2$,

$\sigma_{5y} = (2N_{56,y} + N_{59,y} + N_{5d,y} + N_{53,y} + N_{5k,y})/(l_1 h)$, $\sigma_{6x} = (N_{69,x} + N_{68,x} + N_{6a,x})/(0.5 l_2 h)$,

where, $\tau_{5xy} = (N_{56,y} + N_{59,y} + N_{5d,y} + N_{57,y} + N_{5e,y} + N_{54,y})/(l_2 h)$,

$\tau_{5yx} = (N_{58,x} + N_{59,x} + N_{5d,x} + N_{53,x} + N_{5k,x} + N_{52,x})/(l_1 h)$,

σ_{5x} is the normal stress in the x-direction at node 5, τ_5 is the shear stress at node 5, $N_{59,x}$ is the projection of the axial force of bar 59 in the x-direction, h is the thickness of the plane problem.

A New Method for Predicting the Engineering Elastic Constants

The formula of the new element model in the literatures [7] expresses the relationship between the cross-sectional area of a bar and the engineering elastic constants of a material. Thus, a new method for determining the engineering elastic constants of a fiber-reinforced composite material can be obtained. A fiber-reinforced composite material is composed of the fibers and the matrix. According to the formula of the new element model, a truss model can be obtained from the engineering elastic constants of the matrix material. Then, the fibers (a fiber is regarded as a bar) are superposed with the truss model, thus becomes a new truss model. Now, the cross-sectional area of each bar in the new truss model is known. Therefore, the engineering elastic constants of the fiber-reinforced composite material can be deduced by using the formula of the new element model in the literatures [7]. In order to improve the precision of determining the shear modulus of the composite material, "the shear modulus obtained previously" plus "the reduced shear modulus of the fiber" (namely, multiply the shear modulus of the fiber by the relative cross-sectional areas of fibers) is the final shear modulus.

A Simple Example

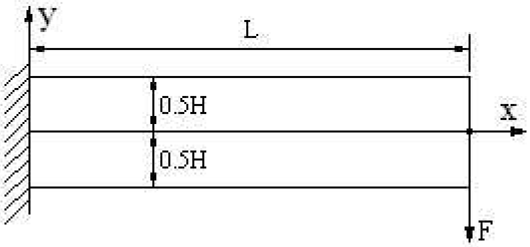

Fig. 2 Bending of a cantilever

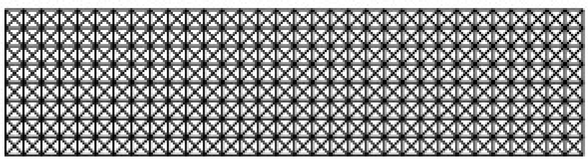

Fig. 3 The cantilever modeled using the new element

Consider a cantilever having a rectangular cross section by a force F=10000*N* applied at the end (Fig. 2), the dimensions are L =320 mm, H=80 mm, and thickness h=1mm. Let elastic modulus $E = 2\times10^5$ N/mm^2, and poisson ratio v=1/3. Finally, the problem is solved with a mesh of 256 new elements; the calculation model of the new element method is shown in Fig. 3. For each new element, let the dimensions in the x- and y-directions $l_1 = l_2 = 10mm$, cross-sectional areas of bars $A_1 = A_2 = 3.75mm^2$, and $A_3 = 5.3033mm^2$. Moreover, the problem is solved with a mesh of 256 plane stress elements, the dimensions in the x- and y-directions of the plane stress element are same with the new element. Using Abaqus software, the cantilever has been solved by using the new element method and the finite element method respectively. Moreover, it has been solved by using the analytical method. The displacement contours in the x-direction of the cantilever are shown in Fig. 4, and the results of the three methods are shown in Table 1-4.

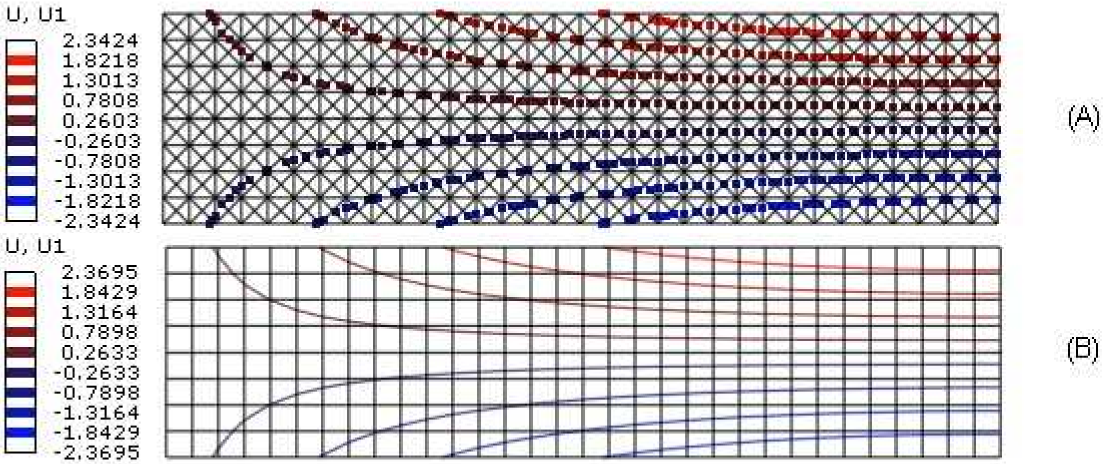

Fig. 4 Displacement contours in the x-direction of the cantilever: (A) the new element method; (B) the finite element method

Table 1 The contrast of the displacements in the x- and y-directions for the three methods

Coordinates of point(x,y)/mm	UX/mm			UY/mm		
	NEM	FEM	AM	NEM	FEM	AM
(100,0)	0	0	0	-1.791	-1.812	-1.68
(100,20)	0.609	0.617	0.587	-1.808	-1.829	-1.697
(100,40)	1.24	1.255	1.195	-1.859	-1.88	-1.748
(200,0)	0	0	0	-6.141	-6.212	-5.938
(200,20)	1	1.012	0.985	-6.15	-6.222	-5.947
(200,40)	2.022	2.046	1.992	-6.178	-6.25	-5.975

Note: "NEM" expresses "new element method", "FEM" expresses "finite element method", "AM" expresses "analytical method".

Table 2 The contrast of the normal strain ε_x and the normal strain ε_y for the three methods

Coordinates of point(x,y)/mm	ε_x			ε_y		
	NEM	FEM	AM	NEM	FEM	AM
(100,0)	0	0	0	0	0	0
(100,20)	5.057E-03	5.116E-3	5.156E-03	-1.686E-03	-1.705E-3	-1.719E-03
(100,40)	1.011E-02	1.023E-2	1.031E-02	-2.950E-03	-2.984E-3	-3.438E-03
(200,0)	0	0	0	0	0	0
(200,20)	2.758E-03	2.790E-3	2.813E-03	-9.193E-04	-9.301E-4	-9.375E-04
(200,40)	5.517E-03	5.581E-3	5.625E-03	-1.609E-03	-1.627E-3	-1.875E-03

Table 3 The contrast of the shear strain γ and the normal stress σ_x for the three methods

Coordinates of point(x,y)/mm	γ			σ_x / N/mm^2		
	NEM	FEM	AM	NEM	FEM	AM
(100,0)	-2.452E-03	-2.422E-03	-2.500E-03	0	0	0
(100,20)	-1.839E-03	-1.802E-03	-1.875E-03	1011.4	1023.2	1031
(100,40)	-6.875E-04	-4.844E-04	0	2054.1	2078.5	2063
(200,0)	-2.451E-03	-2.422E-03	-2.500E-03	0	0	0
(200,20)	-1.839E-03	-1.802E-03	-1.875E-03	551.6	558.1	562.5
(200,40)	-6.900E-04	-4.844E-04	0	1120.7	1133.7	1125

Table 4 The contrast of the normal stress σ_y and the shear stress τ for the three methods

Coordinates of point(x,y)/mm	σ_y / N/mm^2			τ / N/mm^2		
	NEM	FEM	AM	NEM	FEM	AM
(100,0)	0	0	0	-183.9	-181.6	-187.5
(100,20)	0	0	0	-137.9	-135.1	-140.6
(100,40)	94.7	95.9	0	-51.6	-36.3	0
(200,0)	0	0	0	-183.8	-181.6	-187.5
(200,20)	0	0	0	-137.9	-135.1	-140.6
(200,40)	51.8	52.3	0	-51.8	-36.3	0

From Fig. 4 and Table 1-4, the results of the three methods agree well.

Plastic limit analysis: Assuming that the yield stresses of the material in tension and compression are 3000N/mm^2, then the cantilever is solved by using the new element method, with a mesh of 16 new elements. For each new element, let the dimensions in the x- and y-directions $l_1 = l_2 = 40mm$, cross-sectional areas of bars $A_1 = A_2 = 15mm^2$, and $A_3 = 21.2132mm^2$, the yield stresses in tension and compression are 3000N/mm^2 for the four bars that the cross-sectional areas are A_1 and A_2, the yield stresses in tension and compression are 1000N/mm^2 for the two bars that the cross-sectional areas are A_3. The cantilever has been solved by using Ansys software, the yield ultimate load of the cantilever is 15kN, it is completely accord with the exact solution of Plasticity. If the appropriate mesh size for solving beams and frames is adopted, then the limit analysis by using the new element method is extremely good, and the new element method has the highest precision in all methods.

Conclusions

The paper provides a new element model for analyzing elastic perfectly-plastic and orthotropic solid bodies. The two methods for determining the displacements, the stresses and the strains of a body under the action of applied forces have been given. It can be found that it is very simple, and the precision of the new element method is good. Furthermore, a new method for predicting the engineering elastic constants of a fiber-reinforced composite material is presented, and using the new element model can guide the composite material design.

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