Now $\mathbb{C}_{21(m)} = d\underline{D} \cdot \underline{\tau} + \underline{\tau} \cdot d\underline{D}$

$$\left[\mathbb{C}_{21(m)}\right]_{ij} = dD_{kl}\, \delta_{ik} \tau_{lj} + \tau_{ik}\, dD_{kl}\, \delta_{lj}$$
$$= \left(\delta_{ik} \tau_{lj} + \tau_{ik} \delta_{lj}\right) dD_{kl}$$

Denote $\mathbb{S} = \left(\delta_{ik} \tau_{lj} + \tau_{ik} \delta_{lj}\right) \underset{\sim}{e}_i \otimes \underset{\sim}{e}_j \otimes \underset{\sim}{e}_k \otimes \underset{\sim}{e}_l$

Then $\mathbb{C}_{21(m)} = \mathbb{S} : d\underline{D}$

Need to Calculate $\mathbb{C}_1$.

$$\mathbb{C}_1 = \sum_A d\tau^A\, \underset{\sim}{n}_e^A \otimes \underset{\sim}{n}_e^A$$

Now $\underset{\sim}{n}_e^A = \underline{n}_*^A$

$$\therefore \mathbb{C}_1 = \sum_A d\tau^A\, \underset{\sim}{n}_*^A \otimes \underset{\sim}{n}_*^A$$

Now,

$$\tau^A = 2\left[\left(\frac{\partial U}{\partial \bar{I}_1} + \bar{I}_1 \frac{\partial U}{\partial \bar{I}_2}\right)\bar{\mu}_e^A - \frac{\partial U}{\partial \bar{I}_2}(\bar{\mu}_e^A)^2 + \frac{J}{2}\frac{\partial U}{\partial J} - \frac{1}{3}\frac{\partial U}{\partial \bar{I}_1}\bar{I}_1 - \frac{2}{3}\frac{\partial U}{\partial \bar{I}_2}\bar{I}_2\right]$$

From the update formula;

$$\varepsilon_e^B = \varepsilon_*^B - \frac{\Delta\gamma\, s^B}{\sqrt{2}\,\sigma'}$$

Now $\dfrac{s^B}{\sigma'} = \dfrac{\tau'^B}{\tau'}$

So $$\varepsilon_e^B = \varepsilon_*^B - \frac{\Delta\gamma\, \tau'^B}{\sqrt{2}\,\tau'}$$

Since we finally need $d\tau^A$, phrase in terms of $d\tau^A$.

To do so we need $\dfrac{\partial \varepsilon_e^B}{\partial \tau^A} =: \mathcal{A}_A^B$ and

$$\frac{\partial \tau'^B}{\partial \tau^C} = \frac{\partial}{\partial \tau^C}\left(\tau^B - \frac{1}{3}\sum_D \tau^D\right).$$

$$\therefore \frac{\partial \tau'^B}{\partial \tau^C} = \delta^B_C - \frac{1}{3} i^B i^C.$$

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or $\tau^A = \hat{\tau}^A\left(\varepsilon_e^B, J(\varepsilon_e^B) \right)$

$$d\tau^A = \sum_B \left(\frac{\partial \hat{\tau}^A}{\partial \varepsilon_e^B} + J \frac{\partial \hat{\tau}^A}{\partial J} i_B \right) d\varepsilon_e^B =: \sum_B \cancel{A}^A_B \, d\varepsilon_e^B$$

$$\frac{\partial \hat{\tau}^A}{\partial \varepsilon_e^B} = 2 \Bigg[\frac{\partial U}{\partial \bar{I}_1} 2 (\bar{\lambda}_e^A)^2 \delta_B^A + \frac{\partial U}{\partial \bar{I}_2} \left\{ 2(\bar{\lambda}_e^B)^2 (\bar{\lambda}_e^A)^2 + \bar{I}_1 2 (\bar{\lambda}_e^A)^2 \delta_B^A - 4 (\bar{\lambda}_e^A)^4 \delta_B^A \right\}$$

$$+ (\bar{\lambda}_e^A)^2 \left\{ \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_1} 2 (\bar{\lambda}_e^B)^2 + \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_2} \left[\bar{I}_1 - (\bar{\lambda}_e^B)^2 \right] 2 (\bar{\lambda}_e^B)^2 \right\}$$

$$+ \left\{ (\bar{\lambda}_e^A)^2 \bar{I}_1 - (\bar{\lambda}_e^A)^4 \right\} \left\{ \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_1} 2 (\bar{\lambda}_e^B)^2 + \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2} \left[\bar{I}_1 - (\bar{\lambda}_e^B)^2 \right] 2 (\bar{\lambda}_e^B)^2 \right\}$$

$$+ \frac{J}{2} \left\{ \frac{\partial^2 U}{\partial J \partial \bar{I}_1} 2 (\bar{\lambda}_e^B)^2 + \frac{\partial^2 U}{\partial J \partial \bar{I}_2} \left[\bar{I}_1 - (\bar{\lambda}_e^B)^2 \right] 2 (\bar{\lambda}_e^B)^2 \right\}$$

$$- \frac{1}{3} \frac{\partial U}{\partial \bar{I}_1} 2 (\bar{\lambda}_e^B)^2 - \frac{1}{3} \bar{I}_1 \left\{ \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_1} 2 (\bar{\lambda}_e^B)^2 + \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_2} \left[\bar{I}_1 - (\bar{\lambda}_e^B)^2 \right] 2 (\bar{\lambda}_e^B)^2 \right\}$$

$$- \frac{2}{3} \frac{\partial U}{\partial \bar{I}_2} \left[\bar{I}_1 - (\bar{\lambda}_e^B)^2 \right] 2 (\bar{\lambda}_e^B)^2$$

$$- \frac{2}{3} \bar{I}_2 \left\{ \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_1} 2 (\bar{\lambda}_e^B)^2 + \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2} \left[\bar{I}_1 - (\bar{\lambda}_e^B)^2 \right] 2 (\bar{\lambda}_e^B)^2 \right\}$$

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Now $$J\frac{\partial \hat{\tau}^{A}}{\partial J}=2\Bigg[\left\{-\frac{1}{3}\bar{I}_1+\left(\bar{\lambda}^{A}_{e}\right)^2\right\}\left\{J\frac{\partial^2 U}{\partial \bar{I}_1\partial J}+\frac{\partial^2 U}{\partial \bar{I}_1\partial \bar{I}_1}\left(-\frac{2}{3}\right)\bar{I}_1+\frac{\partial^2 U}{\partial \bar{I}_1\partial \bar{I}_2}\left(-\frac{4}{3}\right)\bar{I}_2\right\}$$

$$+\frac{\partial U}{\partial \bar{I}_1}\left\{\left(-\frac{1}{3}\right)\left(-\frac{2}{3}\right)\bar{I}_1+\left(-\frac{2}{3}\right)\left(\bar{\lambda}^{A}_{e}\right)^2\right\}$$

$$+\left\{\bar{I}_1\left(\bar{\lambda}^{A}_{e}\right)^2-\left(\bar{\lambda}^{A}_{e}\right)^4-\frac{2}{3}\bar{I}_2\right\}\left\{J\frac{\partial^2 U}{\partial \bar{I}_2\partial J}+\frac{\partial^2 U}{\partial \bar{I}_2\partial \bar{I}_1}\left(-\frac{2}{3}\right)\bar{I}_1+\frac{\partial^2 U}{\partial \bar{I}_2\bar{I}_2}\left(-\frac{4}{3}\right)\bar{I}_2\right\}$$

$$*\ \frac{\partial U}{\partial \bar{I}_2}\left\{\left(\bar{\lambda}^{A}_{e}\right)^2\left(-\frac{2}{3}\right)\bar{I}_1+\bar{I}_1\left(-\frac{2}{3}\right)\left(\bar{\lambda}^{A}_{e}\right)^2-\left(-\frac{4}{3}\right)\left(\bar{\lambda}^{A}_{e}\right)^4-\frac{2}{3}\left(-\frac{4}{3}\right)\bar{I}_2\right\}$$

$$+\frac{J}{2}\frac{\partial U}{\partial J}+\frac{J}{2}\left\{J\frac{\partial^2 U}{\partial J\partial J}+\frac{\partial^2 U}{\partial J\partial \bar{I}_1}\left(-\frac{2}{3}\right)\bar{I}_1+\frac{\partial^2 U}{\partial J\partial \bar{I}_2}\left(-\frac{4}{3}\right)\bar{I}_2\right\}\Bigg]$$

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$$\therefore \left(\mathcal{A}^{B}{}_{A}\right)^{-1} = \mathcal{B}^{A}{}_{B}$$

$$= 2\Bigg[\delta^{AB}\left(2\bar{\mu}_e^A \frac{\partial U}{\partial \bar{I}_1} + 2\bar{I}_1 \bar{\mu}_e^A \frac{\partial U}{\partial \bar{I}_2} - 4\frac{\partial U}{\partial \bar{I}_2}(\bar{\mu}_e^A)^2\right)$$

$$+ \bar{\mu}_e^A \bar{\mu}_e^B \left(2\frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_1} + 2\bar{I}_1 \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_2} + 2\bar{I}_1 \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_1} + 2\frac{\partial U}{\partial \bar{I}_2} + 2\frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2}\bar{I}_1^2\right)$$

$$+ \left(\bar{\mu}_e^A \bar{\mu}_e^{B\,2} + \bar{\mu}_e^B \bar{\mu}_e^{A\,2}\right)\left(-2\frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_2} - 2\frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2}\bar{I}_1\right)$$

$$+ \bar{\mu}_e^{A\,2} \bar{\mu}_e^{B\,2}\left(+2\frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2}\right)$$

$$+ \left(\bar{\mu}_e^A + \bar{\mu}_e^B\right)\Bigg(-\frac{2}{3}\frac{\partial U}{\partial \bar{I}_1} - \frac{2}{3}\bar{I}_1 \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_1} - \frac{4}{3}\bar{I}_2 \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_1}$$

$$- \frac{2}{3}\bar{I}_1^2 \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_1} - \frac{4}{3}\bar{I}_1 \bar{I}_2 \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2} - \frac{4}{3}\frac{\partial U}{\partial \bar{I}_2}\bar{I}_1\Bigg)$$

$$+ \left(\bar{\mu}_e^{A\,2} + \bar{\mu}_e^{B\,2}\right)\left(\frac{2}{3}\bar{I}_1 \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_2} + \frac{4}{3}\bar{I}_2 \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2} + \frac{4}{3}\frac{\partial U}{\partial \bar{I}_2}\right)\Bigg]$$

$$+2\Bigg[i^A i^B \Bigg(-\frac{1}{3}\bar{I}_1 \left\{ \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_1}\left(-\frac{2}{3}\right)\bar{I}_1 + \frac{\partial^2 U}{\partial \bar{I}_1 \partial \bar{I}_2}\left(-\frac{4}{3}\right)\bar{I}_2 \right\}$$

$$+ \frac{2}{9}\frac{\partial U}{\partial \bar{I}_1}\bar{I}_1$$

$$- \frac{2}{3}\bar{I}_2 \left\{ \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_1}\left(-\frac{2}{3}\right)\bar{I}_1 + \frac{\partial^2 U}{\partial \bar{I}_2 \partial \bar{I}_2}\left(-\frac{4}{3}\right)\bar{I}_2 \right\}$$

$$+ \frac{8}{9}\frac{\partial U}{\partial \bar{I}_2}\bar{I}_2$$

$$+ \frac{J}{2}\frac{\partial U}{\partial J} + \frac{J^2}{2}\frac{\partial^2 U}{\partial J \partial J}\Bigg]$$

In the above, we have assumed that the cross deriv. $\dfrac{\partial U}{\partial \bar{I}_i \partial J} = 0$.

Note that $\mathcal{A}^A_B$ is symmetric.

$$\therefore \sum_A A^B_A \, d\tau^A = d\mathcal{E}^B_* - \frac{1}{\sqrt{2}\,\tau'}\left(d\Delta\gamma\, \tau'^B + \Delta\gamma\, d\tau'^B\right)$$

$$+ \frac{\Delta\gamma\, \tau'^B}{2\sqrt{2}\,\tau'} \sum_C \frac{1}{\cancel{2}} \cdot \cancel{2}\, \tau'^C d\tau'^C$$

$$= d\mathcal{E}^B_* - \frac{\tau'^B}{\sqrt{2}\,\tau'} d\Delta\gamma - \frac{\Delta\gamma}{\sqrt{2}\,\tau'} d\tau'^B + \frac{\Delta\gamma\, \tau'^B}{2\sqrt{2}\,\tau'} \sum_C \tau'^C \sum_D \left(\delta^C_D - \frac{1}{3} i^C_i \right) d\tau^D$$

$$= d\mathcal{E}^B_* - \frac{\tau'^B}{\sqrt{2}\,\tau'} d\Delta\gamma - \frac{\Delta\gamma}{\sqrt{2}\,\tau'} d\tau'^B + \frac{\Delta\gamma\, \tau'^B}{2\sqrt{2}\,\tau'} \left[\sum_C \tau'^C d\tau^C - \frac{1}{3}\sum_D d\tau^D \sum_C \overset{0}{\tau'^C}\right]$$

$$= d\mathcal{E}^B_* - \frac{\tau'^B}{\sqrt{2}\,\tau'} d\Delta\gamma - \frac{\Delta\gamma}{\sqrt{2}\,\tau'} \sum_C \delta^B_C \, d\tau^C \left(-\frac{1}{3} i^B_C\right) + \frac{\Delta\gamma}{2\sqrt{2}\,\tau'} \sum_C \tau'^B \tau'^C d\tau^C$$

missed a term

Now

$$\Delta\gamma = \hat{c}_1 \left[\sqrt{\frac{I^p_1}{3}} - 1\right]^{c_2} \sigma'^m$$

$$\therefore d\Delta\gamma = \hat{c}_1 c_2 \left[\sqrt{\frac{I^p_1}{3}} - 1\right]^{c_2 - 1} \frac{1}{2}\left(\frac{I^p_1}{3}\right)^{-\frac{1}{2}} \frac{1}{3}\, dI^p_1 \, \sigma'^m$$

$$+ \hat{c}_1 \left[\sqrt{\frac{I^p_1}{3}} - 1\right]^{c_2} d\left(\frac{\tau'^m}{J^m}\right)$$

$$\text{Now } d\left(\frac{\tau'^m}{J^m}\right) = \frac{1}{J^m}\, m\, \tau'^{(m-1)} \frac{1}{2\tau'} \sum_C \tau'^C d\tau^C$$

$$- \frac{m}{\cancel{J}} \frac{\tau'^m}{J^m} \sum_C \cancel{J}\, d\mathcal{E}^c_e$$

$$\therefore \boxed{\begin{aligned} d\Delta\gamma = {} & \frac{\Delta\gamma}{\left[\sqrt{\frac{I_1^p}{3}-1}\right]} \cdot \frac{c_2}{6} \cdot \frac{1}{\left(\frac{I_1^p}{3}\right)^{1/2}}\, dI_1^p \\ & + \frac{m}{2}\frac{\Delta\gamma}{\tau'^2} \sum_C \tau'^c\, d\tau^c \\ & - m\,\Delta\gamma \sum_C \sum_D A^c_D\, d\tau^D . \end{aligned}}$$

$\therefore$ Now $\quad I_1^p = \mathbb{I} : F_n^{pT} \cdot \left(\sum_A \frac{\mu_*^{A\,2}}{\mu_e^A} \frac{\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A}{\mu_*^A} \right) \cdot F_n^p$

$$\begin{aligned} dI_1^p = \mathbb{I} : F_n^{pT} \cdot \Bigg(\sum_A \Bigg[& \left(\frac{2}{\mu_e^A} \frac{\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A}{\mu_*^A} \right) \left(2\mu_*^A \underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A \right) : d\underset{\sim}{C}^* \\ & - \left(\frac{\mu_*^A}{\mu_e^A} \right)^2 \frac{\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A}{\mu_*^A}\, 2\mu_e^A\, d\varepsilon_e^A \\ & + \frac{\mu_*^{A\,2}}{\mu_e^A}\, d\left(\frac{\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A}{\mu_*^A} \right) \Bigg] \Bigg) \cdot F_n^p \end{aligned}$$

$$\boxed{\begin{aligned} dI_1^p = \mathbb{I} : F_n^{pT} \cdot \Bigg(\sum_A \Bigg[& \frac{2}{\mu_e^A} \left(\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A \right) \left(\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A : d\underset{\sim}{C}^* \right) \\ & - 2 \left(\frac{\mu_*^A}{\mu_e^A} \right) \left(\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A \right) d\varepsilon_e^A \\ & + \frac{\mu_*^{A\,2}}{\mu_e^A}\, d\left(\frac{\underset{\sim}{N}_*^A \otimes \underset{\sim}{N}_*^A}{\mu_*^A} \right) \Bigg] \Bigg) \cdot F_n^p . \end{aligned}}$$

$$\therefore \sum_A \mathcal{A}^B_A \, d\tau^A = d\mathcal{E}^B_*$$

$$- \frac{\tau'^B}{\sqrt{2}\,\tau'} \left[\frac{c_2}{6} \frac{\Delta\gamma}{\left(\frac{I_1^p}{3}\right)^{1/2}\left[\sqrt{\frac{I_1^p}{3}}-1\right]} \right] \left\{ \sum_D -\underline{B}^p_n : 2\left(\frac{\mu^D_*}{\mu^D_e}\right)\left(\underset{\sim}{N}^D_* \otimes \underset{\sim}{N}^D_*\right) \sum_C \mathcal{A}^D_C \, d\tau^C \right.$$

$$+ \sum_D \underline{B}^p_n : \left(\underset{\sim}{N}^D_* \otimes \underset{\sim}{N}^D_*\right) \frac{1}{\mu^D_e \mu^D_*} \, d\mathcal{E}^*_D$$

$$\left. + \sum_D \frac{\mu^{D\,2}_*}{\mu^D_e} \underline{B}^p_n : d\left(\frac{\underset{\sim}{N}^D_* \otimes \underset{\sim}{N}^D_*}{\mu^D_*}\right) \right\}$$

$$- \frac{\tau'^B}{\sqrt{2}\,\tau'} \frac{m}{2} \frac{\Delta\gamma}{\tau'^2} \sum_C \tau'^C \, d\tau^C \quad \checkmark$$

$$+ \frac{\tau'^B}{\sqrt{2}\,\tau'} m \Delta\gamma \sum_D \sum_C \boxed{\mathcal{A}^D_C} \, d\tau^C \quad \checkmark$$

$$- \frac{\Delta\gamma}{\sqrt{2}\,\tau'} \sum_C \delta^B_C \, d\tau^C \quad \checkmark$$

$$+ \frac{\Delta\gamma}{2\sqrt{2}\,\tau'} \sum_C \tau'^B \tau'^C \, d\tau^C$$

Define $-\underline{B}^p_n : 2\left(\frac{\mu^D_*}{\mu^D_e}\right)\left(\underset{\sim}{N}^D_* \otimes \underset{\sim}{N}^D_*\right) =: \alpha^D$

& $\sum_D \mathcal{A}^D_C =: \xi^C$.

& $\frac{c_2}{6} \frac{\Delta\gamma}{\left(\frac{I_1^p}{3}\right)^{1/2}\left[\sqrt{\frac{I_1^p}{3}}-1\right]} =: q$. ; $\nu^D := \frac{\mu^{D\,2}_*}{\mu^D_e}$

$$\therefore \sum_A \Big[\mathcal{A}_A^B + \frac{\Delta\gamma}{\sqrt{2}\tau'}\delta_A^B + \Big(\frac{m}{2\sqrt{2}}\frac{\Delta\gamma}{\tau'^3} - \frac{\Delta\gamma}{2\sqrt{2}\tau'}\Big)\tau'^B\tau'^A - \frac{m\Delta\gamma}{\sqrt{2}\tau'}\tau'^B\xi^C + q\frac{\tau'^B}{\sqrt{2}\tau'}\sum_D \alpha^D \mathcal{B}_A^D \Big] d\tau^A$$

Asymmetric due to Kanelay (?) tension dep. of eff. stress

Asymmetric due to I_1^p dependence

$$= \sum_D \Big(\delta_D^B - q\frac{\tau'^B}{\sqrt{2}\tau'}\beta_D\Big) d\varepsilon_*^D$$

$$- \frac{\tau'^B}{\sqrt{2}\tau'} q \sum_D \nu^D B_n^D : d\Big(\frac{N_*^D \otimes N_*^D}{\mu_*^D}\Big)$$

Denote $$\mathcal{H}_B^A := \Big[\mathcal{A}_A^B + \frac{\Delta\gamma}{\sqrt{2}\tau'}\delta_A^B + \frac{\Delta\gamma}{2\sqrt{2}\tau'}\Big(\frac{m}{\tau'^2} - 1\Big)\tau'^B\tau'^A - \frac{m\Delta\gamma}{\sqrt{2}\tau'}\tau'^B\xi^C + q\frac{\tau'^B}{\sqrt{2}\tau'}\sum_D \alpha^D \mathcal{B}_A^D \Big]^{-1}$$

Then $$d\tau^A = \sum_B \mathcal{H}_B^A \Big[\sum_D \Big(\delta_D^B - \frac{q}{\sqrt{2}\tau'}\tau'^B\beta_D\Big) d\varepsilon_*^D - \frac{q}{\sqrt{2}\tau'}\sum_D \tau'^B \nu^D B_n^D : d\Big(\frac{N_*^D \otimes N_*^D}{\mu_*^D}\Big)$$

$$\therefore \quad \tau_1 : \delta\underline{D}$$

$$= \sum_A d\tau^A (n_*^A \otimes n_*^A) : \delta\underline{D}$$

Asymmetry due to q / I_1^p dep.

Asymmetry due to q / I_1^p dep.

$$= \sum_A \delta\underline{D} : (n_*^A \otimes n_*^A) \sum_B \mathcal{H}_B^A \Big[\sum_D \Big(\delta_D^B - \frac{q}{\sqrt{2}\tau'}\tau'^B\beta_D\Big)(n_*^D \otimes n_*^D) : d\underline{D} - \frac{q}{\sqrt{2}\tau'}\sum \tau'^B \nu^D B_n^D : d\Big(\frac{N_*^D \otimes N_*^D}{\mu_*^D}\Big)$$

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$$\underline{B}_n^p : d\left(\frac{\underset{\sim}{N}_*^D \otimes \underset{\sim}{N}_*^D}{\mu_*^D}\right) = \mathbb{I} : \underline{F}_n^{p^T} \cdot d\left(\frac{\underset{\sim}{N}_*^D \otimes \underset{\sim}{N}_*^D}{\mu_*^D}\right) \cdot \underline{F}_n^p .$$

Consult fig. 21.

Now $d\overset{*}{\underline{C}} = 2\, \overset{*}{\underline{F}}^T \cdot dD \cdot \overset{*}{\underline{F}}$.

Now $\mathbb{I}_{ijkl} = \delta_{ik}\delta_{jl}$

$$B^p_{nij}\, \delta_{ir}\delta_{js}\, \overset{*}{F}_{kr}\, dD_{kl}\, \overset{*}{F}_{ls}$$

$$= B^p_{n(rs)}\, \overset{*}{F}_{kr}\, \overset{*}{F}_{ls}\, dD_{kl}$$

Now $\overset{*}{F}_{kr} = F_{ka} F^{p^{-1}}_{nar}$

$B^p_{nrs} = F^p_{nrb} F^p_{nsb}$

$$\therefore \quad B^p_{n(rs)}\, \overset{*}{F}_{kr}\, \overset{*}{F}_{ls} = F^p_{n(rb)} F^p_{n(sb)}\, F_{ka} F^{p^{-1}}_{n(ar)}\, F_{lc} F^{p^{-1}}_{n(cs)}$$

$$= \delta_{ab}\delta_{cb}\, F_{ka} F_{lc} .$$

$$= F_{kb} F_{lb} = B_{kl}$$

$$\therefore \quad \boxed{\underline{B}_n^p : \mathbb{I} : d\overset{*}{\underline{C}} = 2\, \underline{B} : d\underline{D}}$$

$$\boxed{\underline{B}^b_n : \mathbb{I} \otimes \mathbb{I} : d\overset{*}{\underline{C}} = 2\,\mathrm{tr}(\underline{B}^b_n)\, \overset{*}{\underline{B}} : d\underline{D}}.$$

$$\underline{B}^b_{n(ij)}\, \overset{*}{C}{}^{-1}_{ij}\, \overset{*}{C}{}^{-1}_{rs}\, \overset{*}{F}{}^{T}_{\cancel{rk}}\, dD_{ke}\, F_{es}$$

$$B^b_{n(ij)}\, \overset{*}{C}{}^{-1}_{ij}\, \delta_{ke}\, dD_{ke}$$

Now $\quad \underline{F}^b_n \cdot \underline{F}^{bT}_n : \overset{*}{\underline{F}}{}^{-1} \cdot \overset{*}{\underline{F}}{}^{-T}$

$$= \underline{F}^b_n \cdot \underline{F}^{bT}_n : \underline{F}^b_n \cdot \underline{F}^{-1} \cdot \underline{F}^{-T} \cdot \underline{F}^{bT}_n$$

$$= \underline{C}^b_n \cdot \underline{C}^b_n : \underline{C}^{-1} \qquad \longrightarrow \text{no good.}$$

~~So~~ Keep. $\quad \boxed{\underline{B}^b_n : \overset{*}{\underline{C}}{}^{-1} \otimes \overset{*}{\underline{C}}{}^{-1} : d\overset{*}{\underline{C}} = 2\left(\underline{B}^b_n : \overset{*}{\underline{C}}{}^{-1}\right)\mathbb{I} : d\underline{D}}$

$$\underline{B}^b_n : \partial_{\overset{*}{\underline{C}}}\, \overset{*}{C}{}^{-1} : d\overset{*}{\underline{C}}$$

$$= -\underline{B}^b_n : \mathbb{I}_{\overset{*}{\underline{C}}} : d\overset{*}{\underline{C}}$$

$$= -2B^b_{n(ij)}\, \overset{*}{C}{}^{-1}_{im}\, \overset{*}{C}{}^{-1}_{jn}\, \overset{*}{F}{}^{T}_{mk}\, dD_{ke}\, \overset{*}{F}_{en}$$

$$= -2\, B^b_{n(ij)} \left(\overset{*}{\underline{C}}{}^{-1} \cdot \overset{*}{\underline{F}}{}^{T}\right)_{ik} \left(\overset{*}{\underline{C}}{}^{-1} \cdot \overset{*}{\underline{F}}{}^{T}\right)_{je} dD_{ke}$$

$$= -2\, B^b_{n(ij)}\, \overset{*}{F}{}^{-1}_{ik}\, \overset{*}{F}{}^{-1}_{je}\, dD_{ke}$$

$$= -2\, F^b_{n(ir)}\, F^b_{n(jr)}\, F^b_{n(is)}\, F^{-1}_{sk}\, F^b_{n(jt)}\, F^{-1}_{te}\, dD_{ke}$$

$$=$$ $\qquad\qquad\qquad \longrightarrow$ no ~~good~~

So
$$\boxed{\begin{aligned} &\underline{B}^b_n : \partial_{\overset{*}{\underline{C}}} \overset{*}{\underline{C}}^{-1} : d\overset{*}{\underline{C}} \\ &= -2\, \underline{B}^b_n : \mathbb{I}_{(\overset{*}{\underline{C}}^{-1} \cdot \overset{*}{\underline{F}}^{T})} : d\underline{D} \end{aligned}}$$

Now
$$\boxed{\begin{aligned} &\underline{B}^b_n : \mathbb{I} \otimes \underline{M}^A_* : d\overset{*}{\underline{C}} \\ &= 2\, tr(\underline{B}^b_n)\, \mu^A_*\, \underset{\sim}{m}^A_* : d\underline{D} \end{aligned}}$$

$$\boxed{\begin{aligned} &\underline{B}^b_n : \underline{M}^A_* \otimes \mathbb{I} : d\overset{*}{\underline{C}} \\ &= 2\, (\underline{B}^b_n : \underline{M}^A_*)\, \overset{*}{\underline{B}} : d\underline{D} \end{aligned}}$$

$$\boxed{\begin{aligned} &\underline{B}^b_n : \underline{M}^A_* \otimes \underline{M}^A_* : d\overset{*}{\underline{C}} \\ &= 2\left(\underline{B}^b_n : \underline{M}^A_*\right) \mu^A_*\, \underset{\sim}{m}^A_* : d\underline{D} \end{aligned}}$$

$$\boxed{\begin{aligned} &\underline{B}^b_n : \overset{*}{\underline{C}}^{-1} \otimes \underline{M}^A_* : d\overset{*}{\underline{C}} \\ &= 2\left(\underline{B}^b_n : \overset{*}{\underline{C}}^{-1}\right) \mu^A_*\, \underset{\sim}{m}^A_* : d\underline{D} \end{aligned}}$$

$$\boxed{\begin{aligned} &\underline{B}^b_n : \underline{M}^A_* \otimes \overset{*}{\underline{C}}^{-1} : d\overset{*}{\underline{C}} \\ &= 2\left(\underline{B}^b_n : \underline{M}^A_*\right) \mathbb{I} : d\underline{D} \end{aligned}}$$

Hence $\quad \underline{B}^{b}_{n} : d\left(\dfrac{\underset{\sim}{N}^{A}_{*} \otimes \underset{\sim}{N}^{A}_{*}}{\mu^{2}_{*}} \right)$

$$= \frac{2}{D^{A}_{*}} \Bigg[\underline{B} - tr\left(\underline{B}^{b}_{n}\right) \underline{\overset{*}{B}}$$
$$+ \frac{\overset{*}{I}_3}{\mu^{*}_{A}} \left\{ \left[\underline{B}^{b}_{n} : \underline{\overset{*}{C}}^{-1} \right] \mathbb{I} - \underline{B}^{b}_{n} : \mathbb{I}_{(\overset{*}{\underline{C}}^{-1} \overset{*}{\underline{F}}^{T})} \right\}$$
$$+ tr\left(\underline{B}^{b}_{n}\right) \mu^{A}_{*} \, \underline{\underline{m}}^{A}_{*} + \left(\underline{B}^{b}_{n} : \underline{M}^{A}_{*}\right) \underline{\overset{*}{B}}$$
$$+ \left(\overset{*}{I}_1 - 4\mu^{A}_{*} + I_3 / \mu^{A\,2}_{*} \right) \underline{\underline{m}}^{A}_{*} \left(\underline{B}^{b}_{n} : \underline{M}^{A}_{*} \right)$$
$$- \frac{\overset{*}{I}_3}{(\mu^{A}_{*})^2} \left\{ \mu^{A}_{*} \left(\underline{B}^{b}_{n} : \underline{\overset{*}{C}}^{-1} \right) \underline{\underline{m}}^{A}_{*} + \left(\underline{B}^{b}_{n} : \underline{M}^{A}_{*} \right) \mathbb{I} \right\} \Bigg] : \underline{dD}$$

So need to calculate

(1) $tr\left(\underline{B}^{b}_{n}\right) = B^{b}_{n\,ii}$

(2) $\underline{B}^{b}_{n} : \underline{\overset{*}{C}}^{-1} = B^{b}_{n\,ij} \, \overset{*}{C}^{-1}_{ij}$

$\underline{B}^{b}_{n} : \mathbb{I}_{\underline{F}^{-1}} = B^{b}_{n(ij)} \overset{*}{F}^{-1}_{ik} \overset{*}{F}^{-1}_{jl}$

(3) $\underline{B}^{b}_{n} : \mathbb{I}_{(\overset{*}{\underline{C}}^{-1} \overset{*}{\underline{F}}^{T})}$

Calculate $\quad \overline{\overset{*}{C}}_{kl} = \overset{*}{C}^{-1}_{kr} \overset{*}{F}_{lr} = \overset{*}{F}^{-1}_{kl}$

Then $\underline{B}^{b}_{n} : \mathbb{I}_{(\overline{\underline{\overset{*}{C}}})} = B^{b}_{n\,ij} \, \overline{\overset{*}{C}}_{ij}$

(4) $\underline{B}^{b}_{n} : \underline{M}^{A}_{*} = B^{b}_{n\,ij} \, N^{A}_{*i} \, N^{A}_{*j}$

Quadratic	Linear	
13.79	0.099	0.28
18.00 ?	0.0136	0.30
19.72.	0.0019	0.32
72.022		

$\delta D_{ij}\ \underbrace{C_{ijkl}}\ dD_{kl}$

				4		7	
1	C_{1111}	C_{1122}	C_{1133}	C_{1112}	C_{1113}	C_{1123}	
	C_{2211}	C_{2222}	C_{2233}	C_{2212}	C_{2213}	C_{2223}	
	C_{3311}	C_{3322}	C_{3333}	C_{3312}	C_{3313}	C_{3323}	
2	C_{1211}	C_{1222}	C_{1233}	C_{1212}	C_{1213}	C_{1223}	8
	C_{1311}	C_{1322}	C_{1333}	C_{1312} 5	C_{1313}	C_{1323}	
3	C_{2311}	C_{2322}	C_{2333}	C_{2312} 6	C_{2313}	C_{2323}	9

C_{ijkl}

7) $\left\{ \begin{matrix} krow = 1,3 \\ kcol = 6 \end{matrix} \right\}$ $(krow, krow, 2, 3) = ijkl$

9) $\left\{ \begin{matrix} krow = 6 \\ kcol = 6 \end{matrix} \right\}$ $(2, 3, 2, 3) \equiv \underline{ijkl}$

1 $\left\{ \begin{matrix} Krow\ (ij) = 1\ to\ 3 \\ Kcol\ (kl) = 1\ to\ 3 \end{matrix} \right\}$ $(krow, krow, kcol, kcol) \equiv (ijkl)$

2 $\left\{ \begin{matrix} krow = 4,5 \\ kcol = 1\ to\ 3 \end{matrix} \right\}$ $(1, (krow-2), kcol, kcol) \equiv (ijkl)$

3 $\left\{ \begin{matrix} krow = 6 \\ kcol = 1\ to\ 3 \end{matrix} \right\}$ $(2, 3, kcol, kcol) \equiv (ijkl)$

4 $\left\{ \begin{matrix} krow = 1\ to\ 3 \\ kcol = 4,5 \end{matrix} \right\}$ $\{krow, krow, 1, (kcol-2)\} \equiv (ijkl)$

5 $\left\{ \begin{matrix} krow = 4,5 \\ kcol = 4,5 \end{matrix} \right\}$. $\{1, (krow-2), 1, (kcol-2)\} \equiv (ijkl)$

6 $\left\{ \begin{matrix} krow = 6 \\ kcol = 4,5 \end{matrix} \right\}$ $(2, 3, 1, (kcol-2)\} \equiv (ijkl)$

8) $\left\{ \begin{matrix} krow = 4,5 \end{matrix} \right\}$ $(1, (krow-2), 2, 3] \equiv (ijkl)$

6/18/98 ①

$$d\left(\frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{\mu^A}\right)$$

$$= \frac{1}{\mu^A}\left(d\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A\right) + \frac{1}{\mu_A}\left(\underset{\sim}{N}^A \otimes d\underset{\sim}{N}^A\right) - \frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{(\mu^A)^2}\, d\mu^A.$$

(for teaching)

Now $\quad d\underset{\sim A}{N} = d\underline{\Omega}^L \cdot \underset{\sim}{N}_A$

Now $\quad \Omega^L_{ij} = \dfrac{\left(\underset{\sim}{n}_i \cdot \underline{dD} \cdot \underset{\sim}{n}_j\right) 2\lambda_i\lambda_j}{(\lambda_j^2 - \lambda_i^2)} \qquad i \neq j.$

$$= \frac{2\lambda_i\lambda_j}{(\lambda_i+\lambda_j)(\lambda_j-\lambda_i)}\left(\underset{\sim}{n}_i \otimes \underset{\sim}{n}_j\right) : \underline{dD} \qquad i \neq j$$

$$\Omega^L_{ij} = 0 \qquad\qquad i = j.$$

$$\sum_{i,j=1}^{3} \frac{1}{\mu^A}\Omega^L_{ij}\left(\underset{\sim}{N}_i \otimes \underset{\sim}{N}_j\right) \cdot \underset{\sim}{N}_A \otimes \underset{\sim}{N}_A$$

$$+ \sum_{\substack{i,j=1 \\ i \neq j}}^{3} \frac{1}{\mu^A}\Omega^L_{ij}\, \underset{\sim}{N}_A \otimes \left[\left(\underset{\sim}{N}_i \otimes \underset{\sim}{N}_j\right) \cdot \underset{\sim}{N}_A\right]$$

$$- \frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{(\mu^A)^2}\left(\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A\right) : d\underline{C}$$

For each A,

$$\sum_{i=1}^{3} \frac{1}{\mu_A} \Omega^L_{iA}\, \underset{\sim}{N}_i \otimes \underset{\sim}{N}_A$$

$$+ \sum_{i=1}^{3} \frac{1}{\mu_A} \Omega^L_{iA}\, \underset{\sim}{N}_A \otimes \underset{\sim}{N}_i$$

$$- \frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{(\mu^A)^2} \left(\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A\right) : d\underset{\sim}{C}$$

Let A & B represent the two equal eigenval
Then what is required is a method to deal with the terms

$$\sum_{i=1}^{3} \frac{\tau^A}{\mu_A} \Omega^L_{iA}\, \underset{\sim}{N}_i \otimes \underset{\sim}{N}_A + \sum_{i=1}^{3} \frac{\tau^B}{\mu_B} \Omega^L_{iB}\, \underset{\sim}{N}_i \otimes \underset{\sim}{N}_B$$

$$+ \sum_{i=1}^{3} \frac{\tau^A}{\mu_A} \Omega^L_{iA}\, \underset{\sim}{N}_A \otimes \underset{\sim}{N}_i + \sum_{i=1}^{3} \frac{\tau^B}{\mu_B} \Omega^L_{iB}\, \underset{\sim}{N}_B \otimes \underset{\sim}{N}_i$$

1 & 4

$$\frac{\tau^A}{\mu_A}\Omega^L_{CA}\,\underset{\sim}{N}_C \otimes \underset{\sim}{N}_A + \frac{\tau^B}{\mu_B}\Omega^L_{CB}\,\underset{\sim}{N}_B \otimes \underset{\sim}{N}_C$$

$$+ \frac{\tau^A}{\mu_A}\Omega^L_{BA}\,\underset{\sim}{N}_B \otimes \underset{\sim}{N}_A + \frac{\tau^B}{\mu_B}\Omega^L_{AB}\,\underset{\sim}{N}_B \otimes \underset{\sim}{N}_A.$$

2 & 3

$$\frac{\tau^B}{\mu_B}\Omega^L_{CB}\,\underset{\sim}{N}_C \otimes \underset{\sim}{N}_B + \frac{\tau^A}{\mu_A}\Omega^L_{CA}\,\underset{\sim}{N}_A \otimes \underset{\sim}{N}_C$$

$$+ \frac{\tau^B}{\mu_B}\Omega^L_{AB}\,\underset{\sim}{N}_A \otimes \underset{\sim}{N}_B + \frac{\tau^A}{\mu^A}\Omega^L_{BA}\,\underset{\sim}{N}_A \otimes \underset{\sim}{N}_B.$$

$$= \frac{\tau^A}{\mu_A}\Omega^L_{CA}\left(\underset{\sim}{N}_C \otimes \underset{\sim}{N}_A + \underset{\sim}{N}_A \otimes \underset{\sim}{N}_C\right) + \frac{\tau^B}{\mu_B}\Omega^L_{CB}\left(\underset{\sim}{N}_B \otimes \underset{\sim}{N}_C + \underset{\sim}{N}_C \otimes \underset{\sim}{N}_B\right)$$

$$+ \Omega^L_{AB}\left(\frac{\tau^B}{\mu_B} - \frac{\tau^A}{\mu_A}\right)\underset{\sim}{N}_B \otimes \underset{\sim}{N}_A$$

$$+ \Omega^L_{AB}\left(\frac{\tau^B}{\mu_B} - \frac{\tau^A}{\mu_A}\right)\underset{\sim}{N}_A \otimes \underset{\sim}{N}_B$$

$$= \boxed{\frac{\tau^A}{\mu_A}\Omega^L_{CA}\left(\underset{\sim}{N}_C \otimes \underset{\sim}{N}_A + \underset{\sim}{N}_A \otimes \underset{\sim}{N}_C\right) + \frac{\tau^B}{\mu_B}\Omega^L_{CB}\left(\underset{\sim}{N}_B \otimes \underset{\sim}{N}_C + \underset{\sim}{N}_C \otimes \underset{\sim}{N}_B\right) + \Omega^L_{AB}\left(\frac{\tau^B}{\mu_B} - \frac{\tau^A}{\mu_A}\right)\left(\underset{\sim}{N}_B \otimes \underset{\sim}{N}_A + \underset{\sim}{N}_A \otimes \underset{\sim}{N}_B\right)}$$

Now
$$\Omega^L_{AB} = \frac{2\lambda_A\lambda_B}{(\lambda_A+\lambda_B)(\lambda_B-\lambda_A)}\left(\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B\right) : \underline{dD}.$$

$$\therefore \quad \Omega^L_{AB}\left(\frac{\tau^B}{\mu_B} - \frac{\tau^A}{\mu_A}\right)$$

$$= D_{AB} \quad \frac{2\lambda_A\lambda_B}{(\lambda_A+\lambda_B)(\lambda_B-\lambda_A)} \quad \frac{\mu_A\tau^B - \mu_B\tau^A}{\mu_B\mu_A}$$

$$= D_{AB} \quad \frac{2}{\lambda_A\lambda_B} \quad \frac{\mu_A\tau^B - \mu_B\tau^A}{\mu_B - \mu_A}.$$

$$\text{Lt } \lambda_A \to \lambda_B \qquad = -\frac{2\tau}{\mu}\left(\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B\right) : \underline{dD}$$

$$\therefore \quad \sum_{A=1}^{3} \tau^A d\left(\frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{\mu^A}\right)$$

$$= \tau^C d\left(\frac{\underset{\sim}{N}^C \otimes \underset{\sim}{N}^C}{\mu^C}\right)$$

term from eq. 1

$$- \tau^A \frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{(\mu^A)^2} (\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A) : d\underline{C} - \tau^B \frac{\underset{\sim}{N}^B \otimes \underset{\sim}{N}^B}{(\mu^B)^2} (\underset{\sim}{N}^B \otimes \underset{\sim}{N}^B) : d\underline{C}$$

$$+ \frac{\tau^A}{\mu_A} \Omega^L_{CA} (\underset{\sim}{N}_C \otimes \underset{\sim}{N}_A + \underset{\sim}{N}_A \otimes \underset{\sim}{N}_C)$$

$$+ \frac{\tau^B}{\mu_B} \Omega^L_{CB} (\underset{\sim}{N}_B \otimes \underset{\sim}{N}_C + \underset{\sim}{N}_C \otimes \underset{\sim}{N}_B)$$

$$- \frac{2\tau}{\mu} (\underset{\sim}{N}_B \otimes \underset{\sim}{N}_A + \underset{\sim}{N}_A \otimes \underset{\sim}{N}_B) (\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B) : \underline{dD}$$

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$$\therefore \quad \underset{\sim}{F} \cdot \sum_{A=1}^{3} \tau^A \, d\left(\frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{\mu^A}\right) \cdot F^T$$

$$= \quad \tau^C \underset{\sim}{F} \cdot d\left(\frac{\underset{\sim}{N}^C \otimes \underset{\sim}{N}^C}{\mu^C}\right) \cdot F^T$$

$$-2\tau^A \underline{(\underset{\sim}{n}^A \otimes \underset{\sim}{n}^A)(\underset{\sim}{n}^A \otimes \underset{\sim}{n}^A)} : dD \; -2\tau^B \, (\underset{\sim}{n}^B \otimes \underset{\sim}{n}^B)(\underset{\sim}{n}^B \otimes \underset{\sim}{n}^B) : dD$$

$$+ \quad \tau^A \, 2 \frac{\sqrt{\mu_C}\sqrt{\mu_A}}{\mu_A - \mu_C} \left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_A + \underset{\sim}{n}_A \otimes \underset{\sim}{n}_C\right)\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_A\right) : dD$$

$$+ \quad \tau^B \, 2 \frac{\sqrt{\mu_C}\sqrt{\mu_B}}{\mu_B - \mu_C} \left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_B + \underset{\sim}{n}_B \otimes \underset{\sim}{n}_C\right)\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_B\right) : dD$$

$$- \quad 2\tau \left(\underset{\sim}{n}_B \otimes \underset{\sim}{n}_A + \underset{\sim}{n}_A \otimes \underset{\sim}{n}_B\right)\left(\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B\right) : dD.$$

Symmetrized form

$$
\begin{aligned}
& -2\tau\left[\left(\underset{\sim}{n}^A \otimes \underset{\sim}{n}^A\right)\left(\underset{\sim}{n}^A \otimes \underset{\sim}{n}^A\right)+\left(\underset{\sim}{n}^B \otimes \underset{\sim}{n}^B\right)\left(\underset{\sim}{n}^B \otimes \underset{\sim}{n}^B\right)\right] \\
& +\frac{\tau\sqrt{\mu_c}\sqrt{\mu}}{\mu-\mu_c}\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_A+\underset{\sim}{n}_A \otimes \underset{\sim}{n}_C\right)\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_A+\underset{\sim}{n}_A \otimes \underset{\sim}{n}_C\right) \\
& +\frac{\tau\sqrt{\mu_c}\sqrt{\mu}}{\mu-\mu_c}\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_B+\underset{\sim}{n}_B \otimes \underset{\sim}{n}_C\right)\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_B+\underset{\sim}{n}_B \otimes \underset{\sim}{n}_C\right) \\
& -\tau\left(\underset{\sim}{n}_B \otimes \underset{\sim}{n}_A+\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B\right)\left(\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B+\underset{\sim}{n}_B \otimes \underset{\sim}{n}_A\right) \\
& +\tau^c F \cdot d\left(\frac{\underset{\sim}{N}^c \otimes \underset{\sim}{N}^c}{\mu^c}\right) \cdot \underline{F}^T
\end{aligned}
$$

6/19/98 (1)

Need

$$\underline{B}_n^p : \sum_{D=1}^{3} \nu^D d\left(\frac{\underset{\sim}{N}_*^D \otimes \underset{\sim}{N}_*^D}{\mu_*^D}\right).$$

for the case of two equal eigenvalues.

$$\sum_{D=1}^{3} \nu^D d\left(\frac{\underset{\sim}{N}^D \otimes \underset{\sim}{N}^D}{\mu^D}\right)$$

$$= \nu^C d\left(\frac{\underset{\sim}{N}^C \otimes \underset{\sim}{N}^C}{\mu^C}\right)$$

$$- \nu^A \frac{\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A}{(\mu_A)^2}\left(\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A\right) : d\underline{C} - \nu^B \frac{\underset{\sim}{N}^B \otimes \underset{\sim}{N}^B}{(\mu_B)^2}\left(\underset{\sim}{N}^B \otimes \underset{\sim}{N}^B\right) : d$$

$$+ \frac{\nu^A}{\mu_A} \frac{2\sqrt{\mu_C}\sqrt{\mu_A}}{\mu_A - \mu_C}\left(\underset{\sim}{N}_C \otimes \underset{\sim}{N}_A + \underset{\sim}{N}_A \otimes \underset{\sim}{N}_C\right)\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_A\right) : d$$

$$+ \frac{\nu^B}{\mu_B} \frac{2\sqrt{\mu_B}\sqrt{\mu_C}}{\mu_B - \mu_C}\left(\underset{\sim}{N}_B \otimes \underset{\sim}{N}_C + \underset{\sim}{N}_C \otimes \underset{\sim}{N}_B\right)\left(\underset{\sim}{n}_C \otimes \underset{\sim}{n}_B\right) : d$$

$$- \frac{2\nu}{\mu}\left(\underset{\sim}{N}_B \otimes \underset{\sim}{N}_A + \underset{\sim}{N}_A \otimes \underset{\sim}{N}_B\right)\left(\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B\right) : d\underline{D}$$

$$= \nu^c\, d\left(\frac{\underset{\sim}{N}^c \otimes \underset{\sim}{N}^c}{\mu^c}\right)$$

$$- \frac{\nu^A}{\mu_A}\left[\underline{B}^b_n : (\underset{\sim}{N}^A \otimes \underset{\sim}{N}^A)\right] 2\,(\underset{\sim}{n}_A \otimes \underset{\sim}{n}_A) : d\underline{D}$$

$$- \frac{\nu^B}{\mu_B}\left[\underline{B}^b_m : (\underset{\sim}{N}^B \otimes \underset{\sim}{N}^B)\right] 2\,(\underset{\sim}{n}_B \otimes \underset{\sim}{n}_B) : d\underline{D}$$

$$+ \frac{\nu\sqrt{\mu_c}\sqrt{\mu}}{\mu(\mu-\mu_c)}\, 2\left[\left[\underline{B}^b_n : (\underset{\sim}{N}_c \otimes \underset{\sim}{N}_A)\right](\underset{\sim}{n}_c \otimes \underset{\sim}{n}_A) + \left[\underline{B}^b_n : (\underset{\sim}{N}_c \otimes \underset{\sim}{N}_B)\right](\underset{\sim}{n}_c \otimes \underset{\sim}{n}_B)\right]$$

$$- \frac{2\nu}{\mu}\, 2\left[\underline{B}^b_n : (\underset{\sim}{N}_A \otimes \underset{\sim}{N}_B)\right](\underset{\sim}{n}_A \otimes \underset{\sim}{n}_B)$$