3-D Finite Element Analysis of Instrumented Indentation of Transversely Isotropic Materials

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Abstract:

In this study finite element computations are conducted using Abaqus to simulate indentation of transversely isotropic materials. Indentation simulation is carried out both perpendicular and parallel to the plane of isotropy using a 70.3 degree cone half-angle indenter. Using dimensional analysis dimensionless functions are constructed to establish a frame of relations between the materials properties and the indentation parameters. The dimensionless functions are verified for their validity and robustness. Finally an analysis is carried out to determine the sensitivity of the relations to slight errors in input values.

Keywords: Instrumented indentation, Simulation, Dimensional analysis, Transversely isotropic materials, Abaqus.

1. Introduction

Instrumented indentation as a tool for material characterization has become increasingly popular owing to its simplicity and versatility. The advantages of the indentation technique are: (i) it is non-destructive, (ii) it is location specific, (iii) it is scalable (from the nano-scopic to the microscopic length scale), and (iv) it requires very simple sample preparation. Consequently, indentation finds wide application in the determination of mechanical properties of small volumes such as thin films and coatings. Instrumented indentation involves an indenter being pressed down on a sample of a material to a certain depth and then withdrawn back. The whole process is typically treated as quasi-static. However, rate-effects can also be investigated under dynamic indentation conditions as well. Monitoring the indentation load P as a function of indentation depth h, into a substrate during both loading and unloading yields a Load-displacement (P-h) curve. Instrumented indentation can be performed using conical, spherical or pyramidal indenters.

Finite element analysis to predict the Load-displacement response of isotropic power law hardening materials in an indentation experiment has been extensively explored [1-7]. Each indentation experiment yields a P-h curve with at most three independent parameters. Modeling of indentation of isotropic materials can be done using an axisymmetric model as the model is symmetric about a single axis. One of the major goals of modeling indentation is to establish a relation between the materials properties and its indentation response. In the case of isotropic materials this has been extensively studied and numerous techniques have been formulated to establish such a relationship. However modeling of indentation on anisotropic materials and establishing a relationship between a materials properties and its indentation response still have many avenues left to be explored. The present work is an effort to model instrumented indentation on transversely isotropic materials and to establish a methodology to relate the materials properties with their indentation response.

2. Indentation Theory

Instrumented indentation involves the loading and unloading cycle of a conical, pyramidal or spherical indenter and measuring the complete force - depth relation during this process.
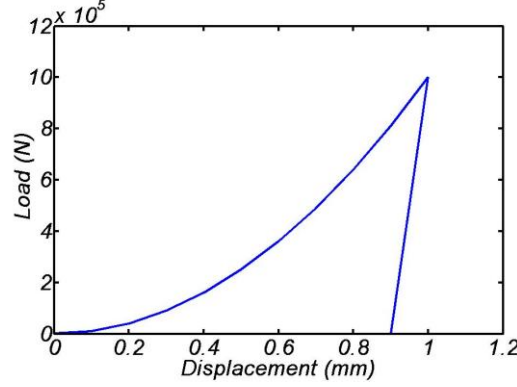
Figure 1 Load-Displacement response of a power-law hardening material

Fig.1 shows a typical Load-displacement response of a power law hardening elasto-plastic material, subject to sharp indentation. The loading response, for power law hardening materials is governed by kicks law,

$$P = Ch^2$$

Equation 1

where, C is the loading curvature and is constant for a given material and indenter cone angle. The maximum indentation depth h_m occurs at the maximum load of P_m. The unloading slope at maximum depth is defined as,

$$S_m = \frac{dP_u}{dh}|h_m$$

where, P_u is the unloading force. The residual depth of the indent, after complete unloading, is termed as h_r. The area under the loading curve is the total work done and is denoted by W_t, also the area under the unloading curve is the recovered elastic work done by the system and is denoted as W_e. The plastic work done can then be calculated as:

$$W_p = W_t - W_e$$

Equation 2

And the plastic to total work ratio is then defined as:

$$R_w = \frac{W_p}{W_t}$$

Hence we see that the loading curvature, C, the unloading slope at the maximum depth, S_m, the maximum and residual indentation depths, h_m and h_r and the areas under the loading and the unloading curves, W_p and W_e, are the parameters that can be obtained from a single indentation experiment. However it can be easily shown that not all these parameters are independent. Only three of these parameters namely, C, S_m and R_w may be treated as independent for most cases.

3. Transversely isotropic materials

Transversely isotropic materials exhibit isotropy along a transverse plane and anisotropy along the longitudinal direction. Within the transverse plane the material exhibits isotropic behavior and perpendicular to the plane the material exhibits different properties.

The properties and approximations described by Nakamura, Gu[8] are used here to define a transversely isotropic material. There are five independent elastic properties namely E_L, E_T, ν_T, ν_{TL}, G_L where, the subscripts L and T denote the longitudinal and transverse directions respectively. Also the plastic domain of the material can be defined by 5 independent parameters namely σ_L, σ_T, τ_L, τ_T and n, where n is the strain hardening exponent.

However, determining relations between the material properties and the indentation response parameters in the most general case of a transversely isotropic material where the transversely isotropic material is characterized by ten properties (elastic and plastic) is extremely cumbersome. Hence, following Nakamura and Gu[8] several approximations were invoked to the number of independent properties that can reasonably describe the features of a transversely isotropic material to 5, i.e., E_L, E_T, σ_L, σ_T and n.

4. Computational model

As previously mentioned, modeling of a conical indentation experiment on an isotropic material can be achieved using an axisymmetric model as the system is symmetric about an axis. Similarly, while modeling indentation perpendicular to the plane of isotropy of a transversely isotropic material an axisymmetric model would suffice as the symmetry about an axis still exists. However, for the case of indenting parallel to the plane of isotropy a three dimensional model is imperative as the symmetry about the vertical axis no longer exists. As the present work involves indentation both perpendicular and parallel to the plane of isotropy, a 3-D model is used. The finite element package Abaqus was used as the platform to perform all simulations.

The indenter was modeled as a rigid body with cone angle of 70.3 degrees and the contact as frictionless. Also large deformation computations were performed. The mesh design used for the analysis and a typical von mises stress contour after indentation are shown in Fig.2.
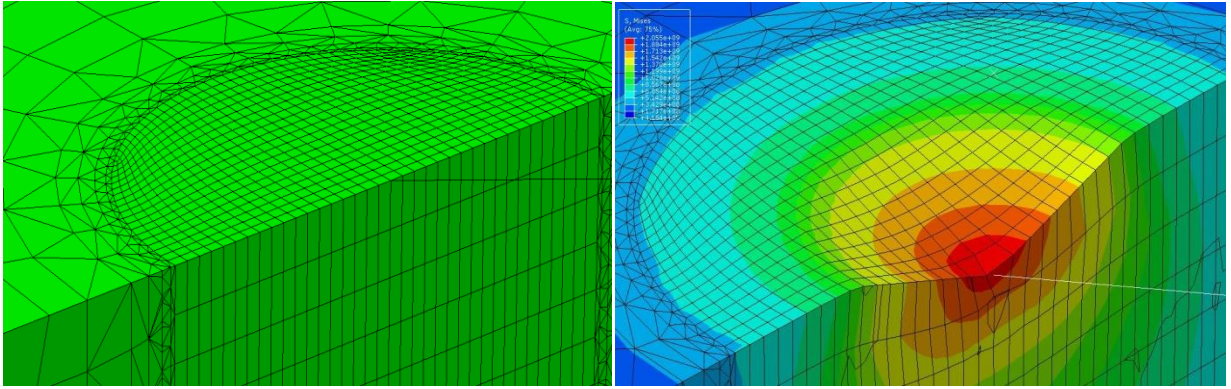
Figure 2: Mesh design and von mises stress contour of an indentation

The model comprised of 12000 eight-noded, hex C3D8R elements with a fine mesh near the contact region and a coarser mesh further away to ensure numerical accuracy. The fine inner meshing and the coarser boundary meshing are connected by 30000 four-noded tetrahedral C3D4 elements. It was ensured that at the maximum load, the minimum number of contact elements in the contact zone cross-section was no less than 30 in each FEM computation. The mesh was determined to be insensitive to far field boundary conditions and well tested for convergence.

5. Dimensional Analysis

The theory used in the dimensional analysis of isotropic materials is extended to transversely isotropic materials. For a given cone angle the load required to indent a power law hardening transversely isotropic material can be written as,

$$P = P\left(h, E_0, \sigma_0, \frac{E_L}{E_T}, \frac{\sigma_L}{\sigma_T}, n\right)$$
<div align="right">Equation 3</div>

where E_0 and σ_0 are the average Young's modulus and yield stress of the material, h is the depth of indentation and n the strain hardening exponent.

Using dimensional analysis Eq. (3) becomes

$$P = \sigma_0 h^2 \Pi_1 \left(\frac{E_0}{\sigma_0}, \frac{E_L}{E_T}, \frac{\sigma_L}{\sigma_T}, n \right)$$

Equation 4

And from Eq. (1)

$$C = \frac{P}{h^2} = \sigma_0 \Pi_1 \left(\frac{E_0}{\sigma_0}, \frac{E_L}{E_T}, \frac{\sigma_L}{\sigma_T}, n \right)$$

Equation 5

for a 70.3 degree cone half angle.

Similarly, the unloading slope, S_m, at the maximum depth of indentation, h_m, is given as:

$$S_m = h_m \sigma_0 \Pi_2 \left(\frac{E_0}{\sigma_0}, \frac{E_L}{E_T}, \frac{\sigma_L}{\sigma_T}, n \right);$$

Equation 6

The plastic work ratio R_w can be represented as:

$$R_w = \frac{W_p}{W_t} = \Pi_3 \left(\frac{E_0}{\sigma_0}, \frac{E_L}{E_T}, \frac{\sigma_L}{\sigma_T}, n \right);$$

Equation 7

6. Computations

The two cases considered for transversely isotropic materials are:

- Indentation perpendicular to the plane of isotropy and,
- Indentation parallel to the plane of isotropy.

The analysis conducted for these two cases is identical with the exception of the change in indentation direction with respect to the plane of isotropy. The process is described as follows:

A comprehensive parametric study of 120 cases was made representing a wide range of mechanical behavior. The values of mean Young's modulus E_0 ranged from 50 to 200 GPa, mean yield strength σ_0 from 210 to 1400 MPa, strain hardening exponent n from 0 to 0.3, and Poisson's ratio ν was fixed at 0.3. Also the ratios of E_L/E_T and σ_L/σ_T were varied from 1 to 2 for each of these cases. The finite element model described previously was used to obtain computational results. The exact forms of the dimensionless functions that link the indentation response parameters with the material properties were then obtained.

To confirm the robustness of the form and curve fit of the dimensionless functions, the 120 initial materials were used and the actual simulated curve properties were compared with the curve properties predicted by the dimensionless equations. A good match between the two was found for both the indentation orientations considered and the average error for each curve property for each indenter is shown in Table 1 for the two indentation orientations.

Table I: Average error for 120 initial materials in comparison with finite element obtained values.

Direction of indentation	C	S_M	R_W
Perpendicular Error %	0.255	0.209	0.06
Parallel Error %	0.736	0.327	0.148

Table II: Average error for ten materials in comparison with finite element obtained values.

Direction of indentation	C	S_M	R_W
Perpendicular Error %	1.652	0.343	0.376
Parallel Error %	2.2	1.225	0.481

To further verify the accuracy of the proposed algorithm, ten sets of material properties spread out in the entire domain of the database were chosen. The *P-h* response for each of the indenter angles was computed through finite element analysis simulation using Abaqus and then matched with those obtained using the dimensionless equations. The error in C, S_m and R_w, in comparison with the finite element P-h response, for the two indentation orientations, is shown in Table 2.

7. Sensitivity Analysis

The 120 materials used previously were used to confirm the low sensitivity of the analysis. It was observed that a ±5% change in any one parameter leads to a variation of no more than ±7.5**% in** the indentation response for indentation perpendicular to the plane of isotropy and ±10**%** for indentation parallel to the plane of isotropy.

8. Conclusions

In this study finite element computations were conducted using Abaqus to simulate indentation of transversely isotropic materials. Indentation simulation was carried out both perpendicular and parallel to the plane of isotropy using a 70.3 degree cone half-angle indenter. Using dimensional analysis dimensionless functions were constructed to establish a frame of relations between the materials properties and the indentation parameters. The dimensionless functions were verified for their validity and robustness. Finally an analysis was carried out to determine the sensitivity of the relations to slight errors in input values. Separate relations were established for indentations perpendicular and parallel to the plane of isotropy.

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