Taut helical strand bending stiffness

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Among other mechanical properties, an important feature for a cable-like system of wires (a helical strand) is its low bending stiffness. However, in many situations that stiffness has to be taken into account. This paper provides a review of various mechanical models for circular section wire strands which can be found in the literature. Each one of them yields a more or less accurate evaluation of that quantity in terms of the system parameters. As each model has to be based on a set of simplifying assumptions, a classification is proposed. Being a complement as well as an update to a previous paper (Cardou and Jolicoeur, 1997), several relevant references already discussed in that paper, have not been included in this review.

Introduction

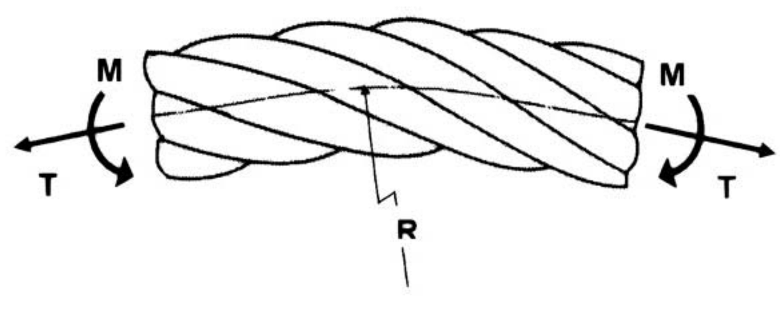
Understanding and deriving the relationship which may exist between the mechanical properties of a taut helical strand (rope, cable, electrical conductor etc.) and those of its constituting fibres or wires is an ancient problem (Galileo, 1638). The simplest case, dealing with the elastic response under axial load, has been dealt with in various ways, and generally satisfactory results have been obtained, at least for simple strands (Cardou and Jolicoeur, 1997), and far away from the ends of a specimen. As for the bending behaviour, a much more complex problem, several models are also available. No one is completely satisfactory, their degree of validity depending very much on the application being considered: either stress evaluation, or prediction of stiffness, internal damping, fatigue strength etc.

The objective of this paper is to present a review of the literature dealing in one way or another on bending stiffness evaluation for a taut helical strand made of circular section wires, thus completing and updating for that particular problem the review paper by Cardou and Jolicoeur (1997). Thus, except in a few instances, papers already referenced in that review have not been included in the present work. Besides analytical models, some papers on experimental bending stiffness measurement are also analyzed.

Calculation of a helical strand bending stiffness (cable, electrical conductor etc.) is of interest, both from a practical and theoretical point of view. For practical applications, engineers did not wait for a rigorous theory to be available to perform an approximate numerical evaluation of strength and stiffness of such a mechanical element. Instead, they used empirical formulas based on parameters obtained through experimentation. In particular, being of primary importance in numerous hoisting devices, the bending «resistance» (that is, stiffness) of ropes has been the subject of several early publications, such as those by Amontons (1699), Coulomb (1785), and Morin (1845). In his paper, the latter also gives a critical account of a contribution made earlier by the famous elastician Navier. That contribution, which was based on Coulomb's experimental results, was apparently made in his Lecture notes at École Nationale des Ponts et Chaussées, in Paris. All these early works on rope bending stiffness try to obtain empirical formulas allowing one to calculate the resistance to winding of a given rope, as a function of a few parameters : its own diameter, the diameter of the pulley or hoisting drum, the axial tension it is subjected to, and, last but not least, its degree of wear.

In the XIXth Century, a more rigorous approach becomes necessary in order to evaluate the load carrying ca-

Fig. 1 Calculation of a helical strand bending stiffness



pacity of the newly invented wire ropes. A formula is first proposed by the German mechanicist Franz Reuleaux (1861), allowing the calculation of the bending stress in a wire rope wound over a pulley of diameter D :

$$\sigma = \frac{d}{D} E \qquad (1)$$

in which d is a wire diameter, and E the material Young's modulus. The formula does not take into account a wire helix angle, and it assumes that the cable diameter is small compared with that of the pulley. That is, the exact position of a wire within the wire rope is not taken into account: all wires are assumed to undergo more or less the same bending.

As this formula overestimates the bending stress, practitioners have had to introduce an empirical constant $K < 1$. Constant K is based on empirical considerations, such as replacing the actual Young's modulus by a cable's equivalent modulus in traction, $E_c < E$. In his monograph on aerial cableways, Schneigert (1964) gives a list of such practical formulas. Technical books on cables generally include a chapter on bending stress calculations: see for example the book by Ernst (1950 and three subsequent editions), on hoisting machines and systems, and the one by Feyrer (1994 and 2000), in which fatigue problems are given a special attention. In these monographs, proposed formulas are based on theoretical models which will be presented below. However, several papers dealing with bending stresses only, and not with bending stiffness evaluation, are not included in the present review.

Theoretical models: general considerations

Strand global behaviour

In a 1997 review paper, Cardou and Jolicoeur have described the difficulties that arise when trying to develop a theoretical model for a helical strand bending behaviour, as well as available approaches. They may be summarized as follows.

It is well known that a helical strand (subsequently called "a strand") in bending (on which a curvature is imposed) has two limiting behaviours; a) solid behaviour; b) independent wire behaviour. Both limiting cases are easily analyzed (at least within a certain set of assumptions). This is the reason why most authors arrive at about the same values for the corresponding bending stiffness :

- Solid behaviour: all wires in the strand are assumed to act as a single unit (contact effects are neglected). Bending stiffness $B_{max} = (EI)_{max}$ is an upper bound to the actual strand bending stiffness.
- Independent wire behaviour: all wires are assumed to act independently, as if there was no friction in the strand. Normal contact forces, which are necessary to impose the given curvature, do no work in the bending process. One can write independent bending equations for each wire being considered as a helical rod. The corresponding bending stiffness, $B_{min} = (EI)_{min}$ is a lower bound to the actual strand bending stiffness.

As the lay angle (with respect to the strand axis) is small, it is often neglected. Thus, calculations are made easier. By doing so, values obtained for B_{max} and B_{min} are slightly higher than the "exact" values. Also, wire diameter being much smaller than that of the strand, one may neglect a wire bending (and twisting) stiffness when calculating B_{max} . The corresponding value is of course slightly lower than the "exact" value.

It is found that the bending stiffness actual value lies between these two values. This result is usually explained as follows. A given strand, initially straight, is subjected to an axial force T. Through some process, it is assumed that a constant radius of curvature R is imposed upon it. It is possible to obtain such a uniform radius by having the strand pass over a pulley. However, this is not exactly equivalent to the assumed process since this radius is obtained only at the end of a process in which each point on the strand axis goes separately from zero to final curvature. In order to have the whole strand specimen go from zero to final curvature in a global fashion, one should invent some kind of a variable radius pulley. Thus, it is realized that such a loading process is purely theoretical (the more so with the axial load T being applied).

The necessary bending moment which must be imposed on the strand is assumed to be related to curvature by an equation such as M = B/R. By definition, B is the bending stiffness for the given curvature (1/R). Starting with a straight strand (zero curvature), one assumes that all wires act as a single unit, like in a solid. This solid behaviour is obtained because of inter-wire friction: at first, tangential forces are too small to allow relative displacement at contact points. Indeed, because of inter-wire pressure (due to axial load T), and assuming Coulomb's Law, such relative displacements may occur only after a certain value has been attained by the local tangential force. It will be explained below that this model is not accurate. Yet, most models are based on this assumption. Accordingly, bending stiffness at that stage is B_{max} and it remains constant, as in a solid beam up to curvature $(1/R_0)$ (point A in Fig. **2**). At that stage, inter-wire slip is assumed to initiate at one particular contact point of each wire in the outer layer. As curvature is increased beyond $(1/R_0)$, slip propagates and bending stiffness B decreases. Relationship between M and (1/R) becomes non linear. As it is defined, B is the slope of straight line OB. It is thus a "secant modulus" B_s. At each point of the M vs (1/R) curve, one can also define a tangent modulus B_t such that increments in bending moment and curvature are related by : $dM = B_t d(1/R)$. At curvature $(1/R_1)$ (point B, Fig. **2**), the slip regime is complete. According to Coulomb's Law, tangential forces are constant and bending moment is again a linear function of curvature, of slope B_1, limit value of the tangent modulus. Their relationship is given by:

$$M = M_1 + \left(\frac{1}{R} - \frac{1}{R_1}\right) B_1 \qquad (2)$$

It is generally assumed that $B_1 = B_{min}$. However, some models have found that full slip is never reached, and thus $B_1 > B_{min}$ (Hong et al., 2005).

Even in the simple theoretical situation of a uniform curvature, it is difficult to build an analytical model of the transition, partial slip, region AB. This is the reason why several authors only consider the limit cases OA and BC. Actually, these regions are linear only if more complex phenomena are neglected. For example, even for small curvature, each wire is subjected to an internal torque which tends to roll the outer layer wires over the adjacent layer (Sathikh et al., 2000). Besides, it is well known that contact points have a certain elastic tangential compliance, which depends on contact conditions and thus on the applied axial load T. This makes region OA slightly non-linear, and the initial slope at point O must be smaller than B_{max} (Hardy and Leblond, 2003). For large curvature, using the original helix angle becomes more and more questionable and, also, transverse deformation of the strand cross-section should be considered (in particular if the strand goes over a pulley).

Fig. 2 Individual wire behaviour

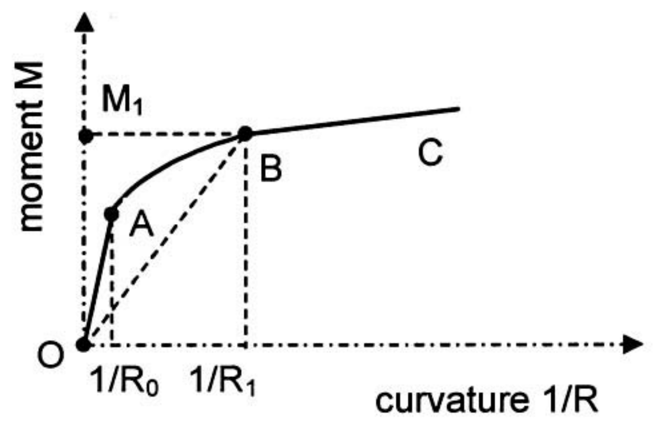
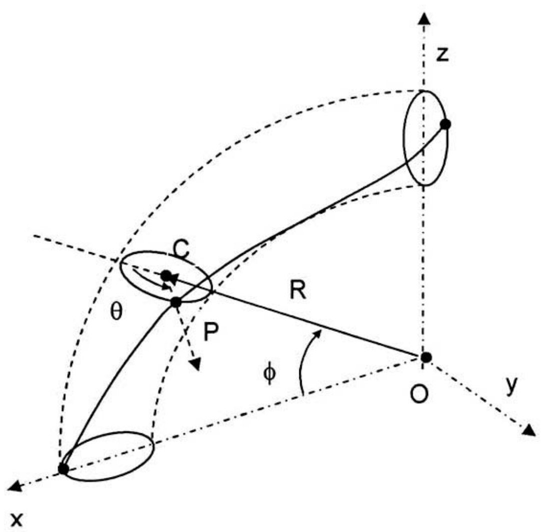


Fig. 3



Individual wire behaviour

In order to obtain a mathematical model for a strand bending stiffness B, based on individual wire behaviour, one faces the following main difficulties.

– There is, first, a purely geometrical problem. Even with the uniform curvature case, the shape actually taken by a wire centerline is not known (Fig. 3). When strand curvature is small enough, one may assume that this centerline keeps its original circular helix shape. However, in particular for stress evaluation, in order to take into account the fact that the centerline is wrapped on a circular torus, rather than on a circular cylinder, two hypotheses have been used : a) a wire centerline is a torus geodesic; b) that centerline is a parametric curve, with position of a point P being defined by two angles θ and ϕ proportional to one another. Such curve is also called a "toroidal spiral" (von Seggern, 1990), and will be noted T.S. in the sequel.

– One also has to make hypotheses on the type of contact prevailing between wires. Indeed, two types of contact mode have to be considered. The first type may be called the "circumferential" contact, in which wires belonging to the same layer are pressed against each other, along line contacts. The second type may be called the "radial" contact, in which contact takes place between adjacent layers. In real strands, it is probable that both types occur simultaneously, depending on the geometrical parameters of the strand. However, in order to develop a mathematical model, one has to choose which type of contact mode is the prevailing one.

– Finally, for multilayer strands, one has to evaluate how pressure is transmitted from one layer to the next, as this parameter is critical to obtain limit slip values. Two limit cases are found: a) no transmission at all: pressure on one given layer comes only from the adjacent outer layer; b) pressure is fully transmitted from one layer to the next, starting with the external one.

Theoretical models: early and most recent publications

Several models for a strand bending behaviour can already be found in the literature. Each one is based on a number of hypotheses regarding the preceding points, and the review by Cardou and Jolicoeur (1997) tried to present a synthetic view of the results thus obtained. However, it has been found that some early important contributions, published in German, French or Italian, should be added to that list, if anything, to acknowledge their anteriority.

While earlier authors were mostly interested in stress and strength evaluation, Isaachsen (1907) obtains the theoretical curve taken by a cable going over a pulley, in the entrance and exit regions. He assumes a constant bending stiffness B, which is calculated with the simple assumption that wires are straight and independent.

Baticle (1912) considers that, in the bent strand, a wire centerline lies on a torus and coincides with a geodesic on that surface. Using the mathematical properties for such a curve, he derives an approximate expression of curvature and torsion of a wire centerline in the deformed state. Using these values in standard Strength of Materials formulas, he finds the bending and twisting moments on a given wire cross-section. These moments are then projected onto an axis perpendicular to the bending plane (containing the deformed strand centerline). Adding for all the wires, he gets the resulting bending moment necessary to impose the corresponding strand curvature. Since he does not take into account inter-wire friction, Baticle thus obtains a value for a strand minimum bending stiffness B_{min}.

Carstarphen (1931) considers an independent helical wire. A constant curvature is then imposed to the axis of the cylinder on which the undeformed wire is wound. Helix angle variation is neglected. Treatment is similar to Timoshenko's (1956) for the calculation of a bent helical spring, and the formula for the curvature of an independent wire is the same. Using this value, Carstarphen obtains the bending stress in a wire. It would be easy to go a step further and calculate the bending stiffness B_{min}, which is just the sum of individual wire stiffness, as contact effects are not considered.

Apparently, Ernst (1933, 1934) is the first author to study progressive wire slip during the bending process. In his thesis, he studies the behaviour of a taut "spiral" cable (that is, a simple strand) under a transverse load applied at mid-span, as in the case of a small radius roller. First, he explains why radial contact should be assumed, rather than circumferential contact. Analysis is based on the original helical curves. As this is a case with variable curvature (also called "free bending"), he is mostly interested in the critical section, right under the applied transverse load. He shows that slip starts in this section, in the neighbourhood of the strand "neutral axis". As the load is increased, slip propagates from wire to wire in that section, in the external layer, up to the point where all wires in that layer are in the slip regime. He proposes a model to describe slip propagation along a given wire, from the critical section to the point where it crosses the strand "neutral axis". A difficulty arises from the type of load being imposed to the strand, which yields an unknown variable curvature for the bent cable. Assuming that pressure is transmitted integrally from one layer to the next, he shows that slip in the second layer (the external one being the first) can start only after slip is complete in the first. The same holds for the third layer etc. This requires a calculation of the corresponding inertia or stiffness at each phase. This calculation takes into account, after full slip of a layer, of a constant so-called "friction moment". The author is mostly interested in calculating bending stresses, called "secondary stresses", while "primary stresses" are those stresses arising from the axial load. In particular, he is not interested in studying bending stiffness (or bending moment) vs curvature relationship. Ernst's model is given a detailed account in a book on cableways, by Czitary (1962).

Similarly, Panetti (1944) includes tangential interactions by considering sliding and friction forces occurring beyond a certain value of curvature. He stresses the fact that two contact modes can exist: "circumferential" (or "tangential") and radial. He chooses to base his model on the "circumferential" contact mode. Based on this hypothesis, he shows that sliding starts (in a given section) between wires most remote from the strand neutral axis and, as curvature increases, that it propagates toward the neutral axis. He also departs from Ernst's approach by assuming that a constant curvature is imposed onto the strand. He gets an M vs 1/R diagram similar to the one in Fig. 2. Taking a four-layer strand (plus center wire), a coefficient of friction of 0.15, and helix angle of 17° (the same for all four layers), he gets a limiting value for the slope of straight line OB (Fig. 2) of 3.8 B_{min}.

The same problem is revisited by Rebuffel (1949) and Lehanneur (1949). Rebuffel studies the complete bending process OABC (Fig. 2). He determines the value $1/R_{01}$ at which sliding starts in the outer layer. His model is based on the radial contact mode. In his analysis, he assumes that a wire centerline keeps its original, helical shape. He finds that sliding starts slightly below the strand neutral axis (on the compression side). As curvature is increased, it propagates toward the highest and lowest wire positions (in a given cross section), corresponding to the convex (extrados) and concave (intrados) sides of the bent strand. Because of symmetry, slip cannot occur at these extreme positions. Rebuffel assumes that slip reaches these upper and lower points simultaneously at curvature $1/R_{11}$. The outer layer is then completely unlocked (except for these points), and the same operation is repeated for inner layers, one at a time. He assumes that there is no interaction between layers: sliding of upper layers does not influence an inner layer behaviour, and pressure on one layer comes only from the immediately adjacent outer one. When the innermost layer is completely unlocked, the strand is at point B (Fig. 2).

Lehanneur (1949) starts with Rebuffel's approach. However, he studies the shape taken by a wire centerline on a torus. After showing that Baticle's geodesic hypothesis is incorrect, he adopts the T.S. curve. He drops Rebuffel's hypothesis that sliding reaches extreme points simultaneously. He also gives detailed equations for some parameters, such as the strand bending stiffness for independent wires. This expression uses a wire bending and twisting stiffness, and their respective contribution to the strand bending stiffness depending on its position in the cross section. The resulting stiffness is very similar to Costello's (1990). For example, taking a lay angle of 17°, and Poisson's ratio of 0.3, values differ by 0.3%. However, when complete slip has occurred, Lehanneur, like Rebuffel, takes also into account a "friction moment" which continues to act if curvature is increased beyond that point (moment M_1 in Fig. 2).

It is interesting to compare results obtained with the Rebuffel/Lehanneur model (R/L model) with those obtained with a model by Lanteigne (1985), in which the slip mechanism (region AB, Fig. 2) is quite different, leading to a staircase type of curve. Let us consider Lanteigne's example in which the strand is the ACSR Bersimis overhead electrical conductor, with a seven wire steel core and three aluminium wire layers. The conductor is subjected to a 62 kN axial load (40% of its RTS or Rated Tensile Strength). With a coefficient of friction of 0.7, Lanteigne's model predicts full slip with a 2 m radius of curvature. At that stage, bending moment is 98 N.m. With the R/L model one has to add a 63 N.m "friction moment", independent of the subsequent curvature increase. In the same example, slip between the first (or external) and second layer is predicted to occur at a radius of 32 m with the Lanteigne model, and at a 75 m radius with the R/L model. The latter model thus predicts a much earlier slip and a shorter OA domain (Fig. 2).

While Czitary's analysis (1962) follows Ernst's for the wire slip conditions, he studies in detail the variation in curvature of a given wire centerline after strand bending. His hypothesis is the same as Lehanneur's, that is, proportional coordinate angles (θ,ϕ) at a given point P (Fig. 3). He thus obtains a more accurate evaluation of wire bending stress than Ernst's.

Schmidt (1965) is mostly interested in the evaluation of bending stresses when a cable enters and exits a pulley. However, he improves Ernst's evaluation of inter-layer pressure transmission and of slip limit condition. His result is not very far from Ernst's. For example, for strands with cross-winding layers, he finds a proportionality factor of 2.93, as compared with Ernst's value of 3. In fact, like Czitary (1962), he adopts this rounded off value. In his stress calculations, rather than using the notion of bending stiffness B, he uses B_{max} and a "friction moment". He also follows Czitary (1962) in the calculation of a wire deformed shape and curvature variation. However, with a numerical example, he shows that the influence on bending stress of that variation is rather small.

Schimmerl (1967) considers that the worst case for wire curvature variation is when that wire is just a string with no bending stiffness. Thus, following Baticle (1912), he assumes that a wire centerline deforms into a torus geodesic. Assuming that inter-wire friction is negligible, he calculates corresponding upper bounds for the maximum stresses in a wire due to the variation in wire curvature.

Leider (1974, 1977) is also interested in bending stresses, assuming zero friction. In his first paper (1974), his approach is similar to Schmidt's. In his second paper (1977), he concentrates on obtaining a better evaluation of secondary bending stresses, taking into account a wire curvature variation, for which he gets the same value as Czitary (1962). The resulting calculated stress variation is different, however. An interesting feature of the paper is a comparison, in a particular case, of wire curvature variation with the geodesic curve assumption vs T.S. curve assumption (Schimmerl's model). The geodesic curve yields a much larger variation of wire centerline curvature, meaning larger secondary stresses, and larger bending strain energy. Hence, according to the principle of least potential energy, the T.S. curve assumption should be closer to the real curve (a complete proof has also to take into account the strain energy contribution from a wire tensile force).

In a monograph by Feyrer (1994) on cable calculation and cable strength, there are two chapters on cable bending. The first one (Chapter 3), deals with calculation of stress when a cable is wound over a pulley, in which he uses the proportionality between angles (θ,ϕ). The second one (Chapter 4), deals with local bending under a transverse force (as with a roller). In this chapter, the analysis is based on a constant stiffness hypothesis. The required stiffness is obtained from an empirical equation derived by Wang (1990), one of his collaborators. This equation was obtained from tests performed on two "spiral" cables. It should also be added that both chapters include a large number of references on the subject.

More recently, Papailiou (1995, 1997) has proposed a model similar to the R/L model with the following differences:

a) Variation of wire curvature after bending of strand is neglected (same hypothesis as Rebuffel).
b) On a particular wire, slip starts at a point on strand neutral axis, and stress at that particular point is assumed to keep its zero bending value, that is the stress coming from the axial load only (the primary stress). This assumption drastically simplifies further calculations. The critical curvature at which there is impending slip differs from that given by the R/L model by a term in the denominator (φ = lay angle, μ = coefficient of friction)
c) Wire stiffness in torsion is neglected by Papailiou in the calculation of independent wire bending stiffness with respect to strand neutral axis.
d) Pressure between layers is fully transmitted from the outer layers to the inner ones. This is of course an important point as this pressure is the controlling parameter in inter-layer slippage. As mentioned above, for the slip condition between two layers, the R/L model considers only the pressure coming from the outer one of the pair.

Hypothesis (b) had already been made by Leider (1974) and, earlier, by Schmidt (1965), who found experimentally that it is a legitimate approximation, as slip starts near the strand neutral axis (as also shown by Lehanneur). Hypothesis (c) is also reasonable in multi-layer strands, where wire diameter is much smaller than strand diameter. Hypothesis (d) is probably much more realistic than the one made in the R/L model (Ernst (1933, 1934), and Panetti (1944), make the same assumption as Papailiou). When it comes to the study of slip propagation along a given wire, the various models, including Papailiou's, assume that contact between a wire and the layer underneath is a line contact. Of course, it is a simplification, since adjacent layers are generally wound in opposite direction (lay angles of opposite signs), thus leading to pointwise contact between wires, except for the innermost layer, which is in contact with the core wire. Apparently, Papailiou is the only one giving a quantitative justification of this assumption.

As already explained, for a multilayer strand, the radial contact mode generally implies point contact between adjacent wires in adjacent layers. Trying to introduce these multiple points of contact in the analysis is very cumbersome. It seems more effective to replace these discrete contact conditions by continuous ones. This has been done by Raoof (1992) in his orthotropic layers models, in which he used the elastic tangential stiffness at a contact point to define an equivalent stiffness for his layer. Papailiou (1995) first studies the variation of tensile force along a given wire under pointwise contact conditions. Then, he shows that the same variation may be obtained with sufficient accuracy by taking a line contact. However, his model is the simple stick/slip Coulomb model, and tangential elasticity is not considered.

True point contact has been considered by Siegert et al. (1997), in order to study cable fretting fatigue near a clamp. As far as bending stiffness is concerned, a model has been proposed recently by Hardy and Leblond (2003) in which wire motion in the outer layer is calculated taking into account contact point elasticity (which includes micro-slip in the contact region, well before gross slip takes place). Thus, when uniform curvature is imposed onto the strand, its equivalent bending stiffness is a non-linear function of the imposed curvature. For the given example, they find that initial bending stiffness (at zero curvature) is roughly 60% of B_{max}. Their study is limited to the behaviour of the outer layer. If inner layer tangential elasticity had been included, it would even be lower. Thus, one can see that, in Fig. 2, curve OA should be non-linear, and its slope at origin is in fact lower than B_{max}.

Papailiou's model has been used, and slightly modified, by Hong et al. (2005). The main difference is that pressure transmission from outer layers to inner ones is revisited. Indeed, following Ernst (1933) and Schmidt (1965), Papailiou (1997) neglects the fact that layer radii, helix angle, and number of wires, vary. This has an influence on pressure transmission. This fact had already been considered by Lanteigne (1985) and even by Papailiou (1995). An interesting consequence of this variation is that, according to the numerical results of Hong et al. (2005), full slip is never obtained when curvature is increased. On the strand convex side (the tension side, in terms of beam bending), slip stops at a certain angle θ, that angle depending on the axial load T, and on the friction coefficient. From the selected numerical example, that limit angle is apparently the same for each layer. Therefore, whatever the curvature, there seems to be a no-slip sector in the strand cross-section. Also, slip in one layer starts before the limit angle is reached in the outer layers. Another consequence is that the minimum tangent bending stiffness B_1 (Eq.2), is much larger than B_{min}. For the case reported by Hong et al. (2005), $B_{min} = 70$ N.m^2, while $B_1 \cong 1000$ N.m^2.

Finally, one should mention recent analytical contributions to single-layer strand bending theory. First, Labrosse (1998) re-derives Love's equations (1944) for curved rods. Like Costello (1990), he makes the radial contact assumption and includes possible wire pivoting distributed couples. His analytical model, combined with Coulomb's Law, is later applied to the development of a specific finite element. Examples of constant radius and free bending are solved numerically under various contact conditions. That finite element model is used by Nawrocki and Labrosse (2000) to study the bending problem of a one-pitch length specimen, with equal and opposite imposed rotation at both ends. Free-slip, with zero friction is assumed all over, thus corresponding to $B = B_{min}$ bending stiffness, while no-slip is assumed at the ends, which are considered as rigidly held. Thus, locally, $B = B_{max}$. Tensile force at both ends is obtained in each of the six outer wires.

Also starting with Love's equations (1944), Sathikh et al. (2000) obtain a single-layer strand bending stiffness when constant curvature is imposed. While that study is restricted to the no-slip regime, it takes into account all possible internal forces and moments, as well as a possible rolling motion of outer wires on the core. Comparison with other models allows one to estimate the influence of respective hypotheses and simplifications. Single-layer strand bending is also studied by Rawlins (2005), with the same curved rod equations. However, his analysis is based on the "circumferential" contact mode hypothesis, where outer wires are in contact, but not with the core wire. The problem being considered is that of free bending near a clamped end, a situation similar to the case of a conductor at a suspension clamp. While Rawlins' model is also a no-slip model, he takes into account the tangential elasticity between contacting wires. Reported results do not consider bending stiffness but, rather, strand displacements. While single-layer strand modelling presents a definite practical interest (Labrosse, 1998), and also for testing the various assumptions, extension of these models to more general multilayer strands seems a challenging task.

Experimental work

As already emphasized, the problem of uniform curvature bending of a strand is rather theoretical. Indeed, even if such a situation is found with a cable-pulley system, it requires contact forces which further complicate the problem (Wyss, 1949). However, many experimental data have been published based on that situation, where outer layer stress have been obtained using strain gages glued on some of the wires of that layer. As may be expected, bending stresses being found are not much different from those given by Reuleaux's equation (Eq. 1).

Strain measurements on the strand outer wires may give an indication on the validity of some theoretical models such as the one by Hong et al. (2005). As mentioned previously, in this model it is found that wires contained in one sector will not slip whatever the strand curvature. That sector corresponds to points most remote from the strand "neutral axis", on the convex (or tensile) side. For a strand wrapped around a pulley of diameter D, bending stress σ can still be calculated using Reuleaux's equation (Eq. 1), in which wire diameter d has to be replaced by the strand diameter. This yields a much larger value of σ which does not seem to agree with experimental data.

Another experimental approach is based on free bending (Papailiou, 1995). In this approach, a strand is put under axial load, which is generally kept constant. Then, a transverse load is imposed, generally at the center section of the specimen, starting from zero up to a given maximum value. Strand deflexion may be recorded at one or more point and an equivalent bending stiffness B_{eq} can thus be calculated, using beam theory equations.

Raoof (1989) uses such an experimental set-up with strain gages in the vicinity of a clamped end of the specimen, thus yielding wire stress. Rather than the applied transverse load, it is the transverse displacement at the loading point which is recorded. He finds that stress is almost a sine function with respect to wire section polar angle, not far from solid section behaviour. For wires in the vicinity of the cable "neutral axis" strain vanishes under small transverse displacement. However, for larger displacements, strain is found to increase on one side of the cable, even at the "neutral axis". This shows that solid behaviour is no longer valid (at least on the outer layer), due to wire slip. Indeed, the initial sine curve becomes more and more distorted and non symmetric, as the imposed transverse displacement increases. These tests cannot be used here to obtain quantitative information, as the author's model calculates an *a priori* bending stiffness using cable axial stiffness properties. In a recent paper, Raoof and Davies (2004) obtain a bending stiffness estimate using the "corkscrew shape" that takes a cable when it is unwrapped from its turret. The helical ripple has a certain radius r. Then, an axial force is applied on the cable, thus decreasing the ripple radius. The authors show that it is possible to relate that decrease to the current cable bending stiffness.

McConnell and Zemke (1980) also use a free bending experimental setup. A straight electrical conductor specimen is put under mechanical tension and a transverse load is applied at mid-point. They record the resulting transverse displacement at that point. Using equations based on standard beam behaviour (clamped-clamped beam under bending and axial load), they obtain an equivalent bending stiffness B_{eq} . Results from tests performed on nine different conductors yield values of B_{eq} which are slightly larger, and in some cases, even smaller, than B_{min}. Values smaller than B_{min} are difficult to explain, and they tend to show that the elastic beam equations on which the analysis is based are not satisfactory. For example, a "rope" effect, which is usually neglected in beam bending calculation, might be involved, similar to the "membrane" effect that is included in some plate calculations. Besides, such small values for B_{eq} are puzzling when one considers that mid-point deflection depends on the specimen global behaviour. Specimens of length L being assumed to be clamped at both ends, there are inflexion points in the vicinity of (L/4) and (3L/4) and, in these zero-curvature regions, bending stiffness should rather be closer to B_{max} having thus an upward influence on B_{eq}.

These results by McConnell and Zemke can also be questioned following those obtained by Zeitler (1994). He performs the same type of tests on four different electrical conductors, with specimens having three different lengths: L = 1.0 m, 1.5 m, and 2.0 m. They are put under five levels of axial load F_1. Zeitler imposes a deflection d at mid-point and he records the corresponding transverse load F_q. Data obtained for one of the conductors only are reported in the paper. These results tend to indicate a linear relationship between F_q and d, when d varies from 0 to 60 mm. These straight lines are compared with theoretical ones based on two limit cases: firstly, a zero bending stiffness conductor (which is then a flexible cable, or a string), and, secondly, a clamped-clamped beam. Unfortunately, in the latter case, the axial force has not been included in the analysis. An empirical equation is derived in which the transverse force F_q is taken as being proportional to d (as it should be for small deflections) and where the slope is given by $(c_1F_1+c_0)$. A very good fit is obtained by assuming that c_1 is proportional to (1/L) while c_0 is proportional to $(1/L^2)$. Zeitler concludes that the beam bending model is not acceptable as it implies a $(1/L^3)$ dependency, and accordingly, stiffness B cannot be obtained from this type of test. In order to check this conclusion, as the axial force was not considered in his beam bending equation, it is interesting to try fitting McConnell and Zemke's equation to Zeitler's data. It is found that such a fit is impossible in several cases. The applicability of this equation to that type of test is thus questionable, even when the fit is possible. Zeitler's results tend to show that the "rope effect" cannot be neglected and, as already mentioned, this may explain why some B values obtained by McConnell and Zemke are smaller than B_{min}.

As a practical application for his theoretical model, Papailiou (1997) performs the same type of test on ACSR Cardinal conductor specimens. However, instead of just measuring mid-point deflection, he measures deflections at a number of points, 3 mm apart, over half the span of the specimen, from one end to center point. In the specimen, bending moment and bending stiffness vary from point to point. Thus, using the finite element approach, a local stiffness B is calculated. This is done by incrementing the transverse load step by step and, at each step, fitting the calculated curve with the observed one. For each beam element, the stiffness B is obtained with Papailiou's analytical model for uniform curvature. For example, for a L = 1 m specimen, under a 40 kN axial load and a 4 kN transverse load, it is found that local stiffness B goes from 420 N.m^2 at clamp and specimen center point to B_{max} = 1800 N.m^2 in a 100 mm region centered on the inflexion point, at L/4. The calculated 420 N.m^2 value found in that case is much larger than the minimum theoretical value B_{min} = 33 N.m^2.

Papailiou's data also include mid-point deflection versus imposed transverse load. These curves are slightly non-linear and show a narrow hysteresis cycle in the unloading phase. While he does not calculate an equivalent bending stiffness, this can easily be done in order to compare results with those obtained by McConnell and Zemke (1980). Using Papailiou's data, one finds B_{eq} = 1170 N.m^2 for the 1 m specimen under a 40 kN axial load, and B_{eq} = 583 N.m^2 for the 2.65 m specimen under 80 kN axial load. It is found that B_{eq} , which should increase because of the much higher imposed axial load, has in fact been divided by 2. Such a result can only come from the increase in length. Had the test been performed under a 40 kN axial load, the decrease of B_{eq} would have been probably even larger. It thus appears that, as the specimen length increases, the "rope effect" becomes more important and the bending behaviour secondary. Hence, when calculating B_{eq} one should take that effect into account. This underlines how questionable it is to use such B_{eq} in order to characterize a strand bending stiffness. In spite of this uncertainty, and since it is fairly easy to implement, that test is often used. For example, in his discussion of Papailiou's paper (1997), Poffenberger (1997) refers to two other earlier reports where Mc-

Connell and Zemke's method has been used.

Another method which is available in order to obtain an experimental value for a strand bending stiffness consists on vibrating a taut specimen. It is generally considered as clamped at both ends and it is put in transverse vibration through an electromagnetic vibrator. Excitation frequency is adjusted to obtain resonance at a certain mode. Resulting antinode amplitude is generally small which implies that wire slip within the strand is also small. As curvature varies along the specimen, the degree of slip may vary from section to section, with a corresponding variation in strand bending stiffness. According to Papailiou's model (1995), each section undergoes a fully reversed bending moment cycle which should give a narrow hysteresis cycle whose mean slope should be more or less the same as that of line OA (Fig. 2). Comparing the strand eigenfrequency with that of the corresponding taut flexible cable, an equivalent bending stiffness B_{eq} may be calculated. It is found that, in general, B_{eq} is about half of B_{max}. This is probably due to the tangential compliance at inter-wire contact points (Hardy and Leblond, 2003).

Rather than calculating B_{eq}, Scanlan and Swart (1968) have used this dynamic approach to obtain a local evaluation of strand bending stiffness. They use an ACSR Pheasant specimen, length 60 in (1.524 m), clamped at both ends after an axial load of 6400 lb (28.5 kN) has been imposed. They put it into vibration at its first mode. Transverse displacement v(x) is measured at 15 locations distributed over half the specimen span. From these data, they calculate approximate values of slope and curvature . Even if this is not always the case, as it depends on vibration amplitude (Fig. 2), here, bending moment and curvature are assumed to be proportional. Thus, using the differential dynamic equation, the local bending stiffness can be calculated. A maximum value of about $0.4B_{max}$ is found near the clamped ends. It has a minimum of $0.16B_{max}$ at center of span. At x = L/4, which is a point of inflexion, stiffness is found to have an intermediate value which is difficult to justify, as curvature vanishes in that region, rendering numerical calculations rather dubious. These results differ markedly from those obtained by Papailiou under quasi-static bending conditions. The discrepancy might come from the relatively smaller amplitude used by Scanlan and Swart and, also, from their assumption of proportionality between bending moment and curvature.

Conclusion

At this stage, it would seem that the best available model in order to predict the static free bending stiffness of a multi-layer strand under axial tension is the model first proposed by Rebuffel (1949), then revised by Lehanneur (1949) and independently extended and implemented by Papailiou (1995, 1997). Its main hypotheses may be summarized as follows:

a) Radial contact is assumed between wires
b) Contact points are replaced by line contact
c) Wire torsion is neglected, thus departing from helical rod models
d) Uniform Coulomb friction between layers is assumed
e) Full pressure transmission from one layer to the next is assumed. However, an appropriate transmission factor should be used to take into account the parameters in each layer
f) Tangential elastic compliance at contact points is neglected
g) Incipient slip is assumed to start at strand "neutral" axis. Wire tension is kept constant at that point and assumed to be equal to the nominal tension due to axial load
h) Strand centerline curvature is assumed to be small enough so that variation of wire curvature, helix angles, strand cross-section, inter-layer pressures, can be neglected

As already explained, tests based on measuring the mid-span deflection of a taut specimen, are difficult to interpret. Many papers have been published reporting strain measurements performed on outer layer wires. Such data are useful, of course, to evaluate bending stresses in these wires. However, they are difficult to use in global strand behaviour studies. Thus, it would seem that the first convincing experimental validation has been given by Papailiou himself. Unfortunately, such a validation requires the use of the finite element method, as the strand deformed shape has a variable curvature and a variable stiffness. That stiffness is calculated with the uniform curvature analytic model, which is assumed to apply even in the variable curvature case.

While this review concentrates on taut strands, it should be mentioned that slack (non-tensioned) cable elements are used in some applications, mostly as vibration dampers (e.g. Stockbridge dampers for overhead transmission power lines). Such damper modelling has been proposed by Sauter (2003), Sauter and Hagedorn (2002) and by Dastous (2005). In his work, Dastous uses Papailiou's analytical model, in conjunction with a commercial finite element package, this package allowing non-linear problems treatment. Another difference between both authors is that Dastous, at each step, uses the tangent modulus B_t, while Papailiou uses the secant modulus B_s, that is, the stiffness defined as the ratio between the current values of bending moment and curvature (Fig. 2). Dastous applies his model to specific static and dynamic problems: equilibrium configuration and transverse vibrations of a short specimen under zero mean axial load under cyclic axial load. In the vibration case, he calculates this axial load amplitude as a function of mid-span amplitude. Numerical results are compared with data obtained from low-frequency tests. Reported results for two types of electrical conductors are rather encouraging, thus showing that helical strand bending problems can best be solved using a combination of an analytical model with the finite element method, as also found in (Nawrocki and Labrosse, 2000).

For the latter problem, and for other applications, such as fatigue strength or internal damping evaluation, which depend basically on wire-wire contact conditions, it is clear that some of the above hypotheses have to be modified or even dropped altogether (Leblond and Hardy, 2005).

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